A simplified analysis of the face stability of tunnels with a preinstalled protective shell

Une analyse simplifiée de la stabilité du front de taille d'un tunnel creusé à l'aide d'une 'voûte parapluie' pré-installée

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ABSTRACT: An analysis of the stability conditions of the face of a circular tunnel with a protective "shell" or "umbrella" installed prior to excavation is proposed. It is assumed that instability can originate from the extrusion of the soil through the shell. Closed-form solutions, derived by the "slab method", for undrained and drained conditions, permit to determine the value of the average horizontal stress - σ_{ext} - which should act at the upstream section of the shell to cause the extrusion of the soil through the face. If the in situ horizontal stress σ_{h0} at that section does not exceed σ_{ext}, the face is stable, and the safety factor can be defined as σ_{ext}/σ_{h0}.

RÉSUME: Une analyse de la stabilité du front de taille d'un tunnel circulaire présoutenu par une "voûte parapluie" (coque) cylindrique ou tronconique enfoncée dans le terrain en avant du front de taille même. La voûte est supposée continue et raide. Le problème a été traité comme une phénomène d'extrusion du sol dans la coque. On a obtenu des solutions analitiques approchées, par le "slab method", qui permettent d'évaluer la pression σ_{ext} qu'il faut appliquer sur la section d'entrée amont de la coque pour causer l'extrusion du sol. Si la contrainte horizontale in situ σ_{h0}, sur la même section, est plus petit que σ_{ext}, le front est stable avec un coefficient de sécurité donné par le rapport σ_{ext}/σ_{h0}.

1 INTRODUCTION

The stability of the face is probably the most important single factor for the rate of construction progress of tunnels in soils. Indeed, the possibility itself of advancement is linked to the capability of the face to stand-up, at least for a short, but finite, time interval after excavation. It is not surprising, therefore, that a great variety of measures have been devised to prevent the instability of the face. The best way for avoiding ground losses and for assuring the stability of the face during the excavation of the tunnel probably consists in the installation *ahead of the face* of a proper length of lining or of a presupport. The aim is to insert into the soil mass a resistant structure before carrying out the excavation. To this purpose, the installation of successive protective fan arrays of structural elements, from the tunnel face, seems to be rather frequently adopted (Panet, 1995). The array is known as "umbrella" or protective "shell". It may be formed of almost contiguous subhorizontal micropiles, jet-grouted columns, etc.. The umbrella can also consist of thin contiguous, but not continuous, concrete slabs of short length, as in the mechanical precutting method. Recently a method has been proposed (Peila et al., 1995) for the construction of a continuous truncated cone ring cast in place in a slot, up to 12m long, excavated by means of a cutting chain (containing picks and discs) supported by a boom which tracks around a steel arch matching the tunnel profile. In any case a sort of cylindrical or truncated cone shell is built ahead of the tunnel face. The shell can generally resist tensile as well as shear stresses. Successive shells must, of course, overlap. The length of overlapping is critical for the stability of the face; it is selected, as a rule, on an empirical basis. Moreover, the stabilizing effects of the umbrella almost invariably are rated only during construction, according to a mere learn-as-you-go approach. A more rational approach is clearly needed.

A simplified analysis of the problem is proposed in the paper. The analysis is also extended to the case when, in addition to the shell, a central "core" is installed. Four different highly idealized systems have been studied. Closed-form solutions have been obtained in each case for undrained and drained conditions.

2 FORMULATION AND METHOD OF ANALYSIS

The protective umbrella (Fig. 1*a* and 1*b*) may be formed either by adjacent discrete subhorizontal structural members (Fig. 1*c*) or by a continuous concrete shell (Fig. 1*d*). In the first case the umbrella is generally open a the bottom - where it is difficult or unnecessary to install the structural elements - while in the second case it is closed also at the invert and truly is a long cylindrical or truncated-cone (splayed) shell.

For large tunnel cross sections a central core - also made of structural elements with a small spacing - may integrate the umbrella; concentric umbrellas may be built.

In the present study the umbrella is considered as a continuous shell; the latter term will be retained hereafter. The analysis has been carried out with reference to highly idealised schemes, under the following simplifying assumptions.

a) Ground: formed by soil, homogeneous and isotropic with regard to shear strength, not affected by discontinuities.
b) Tunnel: circular, horizontal, deep, full-face excavated.
c) Face: vertical.
d) Protective shell: continuous, axisymmetric, coaxial with the tunnel, cylindrical or truncated-cone shaped, inextensible both along the radial and the longitudinal directions. These assumption also apply to the core, if any.
e) Selfweight effects of the soil inside the shell can be neglected.

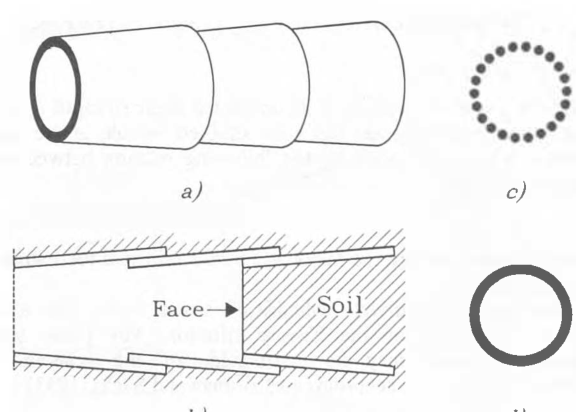

Fig. 1 Typical features of a protective "shell" or "umbrella". *a)* Axonometry. *b)* Vertical longitudinal section. *c)* Shell formed by micropiles or by jet-grouted columns (cross section). *d)* Continuous cast-in place concrete shell ("Pretunnel").

Besides, the shell does not undergo rigid displacements, being supported, at the tunnel face, by the previously built lining, or fixed to the preceding umbrella. The core is supposed to be firmly anchored in the soil mass ahead of the shell (and, of course, ahead of the tunnel face), so that it can withstand pull-out forces exerted on its skin by the soil inside the shell.

Under the above hypotheses an approximate solution to the problem of the face stability can be attempted, treating it as an extrusion process, according to the idea which Broms and Bennermark (1967) used to evaluate the face stability of the tunnel without protective shell. In fact, the excavation of the tunnel brings about the increase of deviatoric stresses in the soil around and ahead of the face; this increase is higher the higher the depth of the tunnel, i.e. the higher the stress release induced at the face by the excavation. If the shear stresses in the soil are sufficiently high, the soil inside the shell starts to yield and possibly to "flow" out from the face. The yielding soil enters the upstream section of the shell and leaves it through the face. The soil around the shell is prevented from yielding and from collapsing due to the presence of the preinstalled shell.

The analysis is developed under the further hypothesis that plastic flow has already attained steady state conditions. Many field observations show that extrusion can actually occur in soft soils (Broms and Bennermark, 1967; Terzaghi, 1942). Yielding of the soil inside the shell does not necessarily occur, except for high deviatoric stresses and for soft soils; hence, the foregoing hypothesis limits the applicability of the proposed solutions, which are however on the safe side.

The stability condition of the face may be evaluated by comparing the total horizontal pressure that should act at the upstream section of the shell to induce the extrusion of the soil, σ_{ext}, with the *in situ* overburden horizontal stress existing at the same section, σ_{ho}. If $\sigma_{ext} > \sigma_{ho}$ the face is stable and the safety factor may be expressed as the ratio σ_{ext}/σ_{ho}.

The analysis can be carried out using the "slab method" taking advantage of studies in the field of the theory of metal-forming processes (e.g. Hill, 1950; Hoffman and Sachs, 1953). In this method it is assumed that the normal stress on each plane perpendicular to the flow direction, x, is principal and uniform over the cross section of the shell, even when shear forces (usually frictional) act at the soil-shell interface. The governing differential equation is derived by imposing the equilibrium condition along the x direction to a slab, of thickness dx, normal to the flow direction. This equation includes as a rule both the minor as well as the major principal stress; it must be transformed taking into account the yield criterion of the soil. In the cases dealt with in the paper a first-order linear differential equation with separable variables is eventually obtained, which can be easily integrated. The "slab method" can be numbered with the limit equilibrium methods; clearly it leads to approximate solutions.

3 YIELD CRITERIA OF THE SOIL AND SHEAR RESISTANCE ALONG THE SHELL-SOIL INTERFACE

Undrained condition

The soil is characterized by its undrained shear strength c_u. The von Mises yield criterion has been adopted, which, in the cases under study is expressed by the following relation between the principal stresses:

$$\sigma_1 - \sigma_3 = 2\,c_u$$

where σ_1 and σ_3 are the major and minor principal total stresses respectively.

Due to the assumed "cylindrical" stress state, the same expression holds for the Tresca criterion. For plane strain condition (which may be postulated for the scheme S4, subsequently shown, according to Hoffman and Sachs, 1953):

$$\sigma_1 - \sigma_3 = \frac{4\,c_u}{\sqrt{3}}$$

The shear strength of the shell-soil interface, τ, can be expressed as $\tau = f c_u$, with $0 \le f \le 1$; f must be determined by experiment.

Drained condition

The Mohr-Coulomb yield criterion has been adopted for the soil; for the present analysis it may be expressed as

$$p' = g + c\sigma'_x$$

where: σ'_x is the effective normal stress on planes perpendicular to the x coordinate; p' is the major principal effective stress; it is assumed that p' coincides with the normal effective stress on the soil-shell interface;

$$g = 2c'\tan\left(\frac{\pi}{4} + \frac{\phi'}{2}\right); \qquad c = \tan^2\left(\frac{\pi}{4} + \frac{\phi'}{2}\right)$$

where: c' coesion intercept; ϕ' angle of shearing strength.

The shear strength of the shell-soil interface can be expressed by the relation

$$\tau = a + b\sigma'_x \qquad (\delta < \gamma)$$

where: $a = \dfrac{c'\tan\gamma}{\cot\phi'}$; $\qquad b = \tan\gamma$.

The angle γ depends upon the soil-shell angle of shearing strength δ (Randolph et al., 1991):

$$\cot\delta = \frac{1}{\cot\delta + 2\cot(\beta - \delta)}; \quad \beta = \arcsin\left(\frac{\sin\gamma}{\sin\phi'}\right); \quad (\delta < \phi').$$

4 RESULTS

Solutions obtained for cylindrical and for truncated-cone shells, with or without core, are reported below; the reference scheme, the yield criteria, the governing equation and the expression of the total stress σ_x (or of the effective stress σ'_x) are given. The value of σ_x at the upstream cross section of the shell is the extrusion pressure σ_{ext}. The subscript a refers to the tunnel face, the subscript b to the upstream section of the shell.

Scheme S1: cylindrical shell, Fig. 2.

Undrained condition

At the interface: $\tau = f\,c_u$

Differential equation: $$d\sigma_x = \left(\frac{4f}{D}c_u\right)dx \tag{1}$$

$$\sigma_x = 4f\,c_u\frac{x}{D} + \sigma_{xa} \qquad \text{for } (\sigma_x)_{x=0} = \sigma_{xa} \tag{2}$$

where σ_{xa} is the pressure applied to the face.

The average mobilized shear strength along the interface of the shell may be easily calculated.

Drained condition

At the interface: $\tau = a + b\sigma'_x$

Differential equation: $$\frac{4}{D}dx = \frac{d\sigma'_x}{a + b\sigma'_x} \tag{3}$$

$$\sigma'_x = \frac{a}{b}\left(e^{\frac{4bx}{D}} - 1\right) + \sigma'_{xa}\,e^{\frac{4bx}{D}} \qquad \text{for } (\sigma_x)_{x=0} = \sigma_{xa} \tag{4}$$

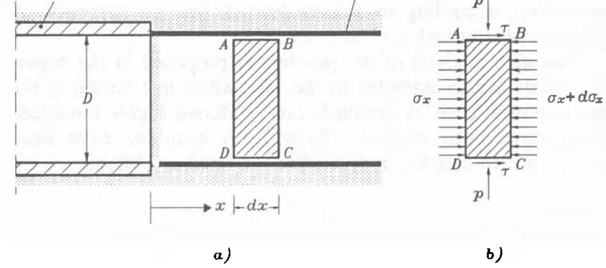

Fig. 2. Reference scheme S1 for the analysis of cylindrical shell. *a)* Longitudinal section. *b)* Soil "slab": free-body diagram.

where σ'_{xa} is the effective pressure applied to the face.

Drained condition, uniform horizontal seepage toward the face with constant piezometric gradient i

Differential equation: $$d\sigma'_x + i\,\gamma_w\,dx = \frac{4}{D}\left(a + b\sigma'_x\right)dx \qquad (5)$$

where γ_w is the specific weight of water.

$$\sigma'_x = \frac{1}{b}\left(a - \frac{i\gamma_w D}{4}\right)\left(e^{\frac{4bx}{D}} - 1\right) \qquad \text{for } \sigma'_{xa} = 0 \qquad (6)$$

Extrusion takes place if the effective horizontal in situ stress σ'_{h0} at the upstream section of the shell exceeds $\left(\sigma'_x\right)_{x=L} = \sigma'_{ext}$. Seepage toward the face considerably reduces the safety factor.

Scheme S2: cylindrical shell with core, Fig. 3.

Undrained condition

At the interface: $\tau = f\,c_u$

Differential equation: $$d\sigma_x = \left(\frac{2f}{h_0}c_u\right)dx \qquad (7)$$

$$\sigma_x = 2f\,c_u\frac{x}{h_0} + \sigma_{xa} \qquad \text{for } (\sigma_x)_{x=0} = \sigma_{xa} \qquad (8)$$

Drained condition

At the interface: $\tau = a + b\sigma'_x$

Differential equation: $$\frac{2}{h_0}dx = \frac{d\sigma'_x}{a + b\sigma'_x} \qquad (9)$$

$$\sigma'_x = \frac{a}{b}\left(e^{\frac{2bx}{h_0}} - 1\right) + \sigma'_{xa}\,e^{\frac{2bx}{h_0}} \qquad \text{for } (\sigma_x)_{x=0} = \sigma'_{xa} \qquad (10)$$

The value of σ'_x / a greatly depends on the values of b and h_0, Fig. 4.

Scheme S3: truncated-cone shell, Fig. 5

Undrained condition

At the interface: $\tau = f\,c_u$

Differential equation: $$d\sigma_x = 2c_u\left(\frac{f}{\tan\alpha} + 2\right)\frac{dD}{D} \qquad (11)$$

$$\sigma_x = 2\,c_u\left(\frac{f}{tg\alpha} + 2\right)\ln\left(\frac{D}{D_a}\right) \qquad \text{for } \sigma_{xa} = 0 \qquad (12)$$

or $$\sigma_x = 2\,c_u\left(\frac{f}{\tan\alpha} + 2\right)\ln\left(1 + 2\frac{(x - x_a)}{D_a}\tan\alpha\right) \qquad (13)$$

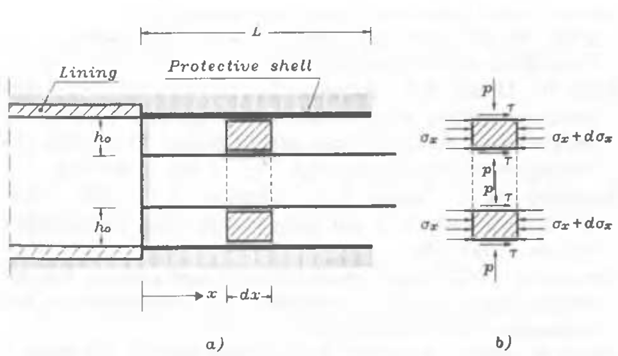

Fig. 3. Scheme S2 for the analysis of the cylindrical shell with a central "core". *a*) and *b*) as in Fig. 2.

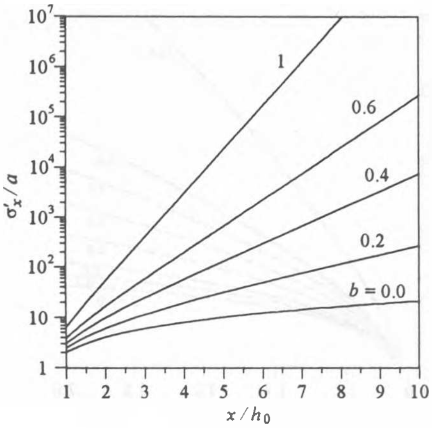

Fig. 4. Scheme S2, zero pore water pressures. Normalized extrusion pressure σ'_x / a against normalized length x/h_0 of the shell; h_0 thickness of the annular soil mass between the shell and the central core. *b*: see text.

Drained condition

At the interface: $\tau = a + b\sigma'_x$

Differential equation: $$2\frac{dD}{D} = \frac{d\sigma'_x}{a^* + B\sigma'_x} \qquad (14)$$

$$\sigma'_x = \frac{a^*}{B}\left[\left(\frac{D}{D_a}\right)^{2B} - 1\right] \qquad \text{for } (\sigma_x)_{x=0} = 0 \qquad (15)$$

or $$\sigma'_x = \frac{a^*}{B}\left[\left(1 + 2\frac{(x - x_a)}{D_a}\tan\alpha\right)^{2B} - 1\right] \qquad (16)$$

where: $a^* = \frac{a}{\tan\alpha} + g$; $B = \frac{b}{\tan\alpha} + c - 1$.

σ'_x / a strongly increases with B, other factors being equal, Fig.6.

Scheme S4: truncated-cone shell with a central core, Fig. 7.

Undrained condition

At the interface: $\tau = f\,c_u$

Differential equation: $$d\sigma_x = \frac{dh}{h}\left(\frac{2f\,c_u}{\varepsilon} + \frac{4c_u}{\sqrt{3}}\right) \qquad (17)$$

$$\sigma_x = 2c_u\left(\frac{f}{\varepsilon} + \frac{2}{\sqrt{3}}\right)\ln\left(1 + \frac{x}{h_0}\varepsilon\right) \qquad \text{for } \sigma_{xa} = 0 \qquad (18)$$

where: $\varepsilon = \tan\alpha - \tan\beta > 0$

Drained condition

At the interface: $\tau = a + b\sigma'_x$

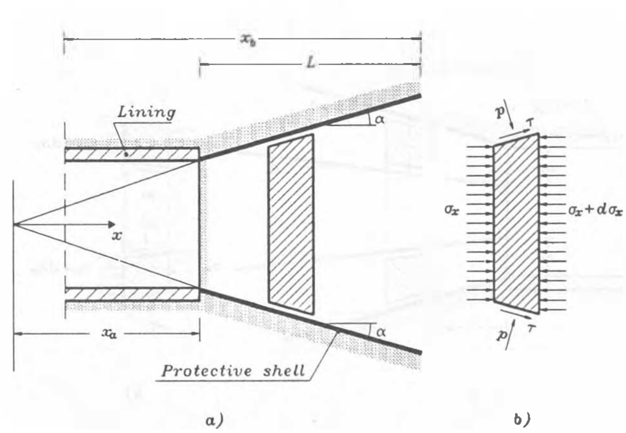

Fig. 5. Scheme S3 for the analysis of the truncated cone shell. *a*) and *b*) as in Fig. 2.

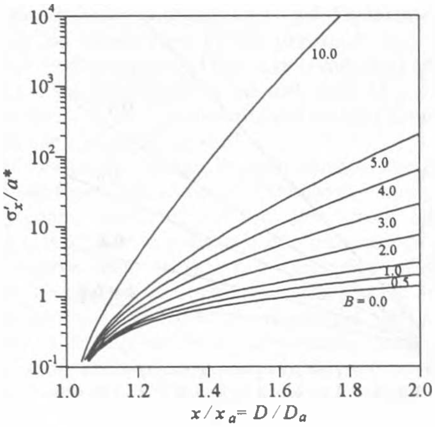

Fig. 6. Scheme S3, zero pore water pressures. Normalized extrusion pressure σ'_x/a^* against x/x_a (>1). Symbols: see text.

Differential equation: $$\frac{dh}{h} = \frac{d\sigma'_x}{a^* + B\sigma'_x} \qquad (19)$$

$$\sigma'_x = \frac{a^*}{B}\left[\left(1 + \frac{\varepsilon}{h_o}x\right)^B - 1\right] \quad \text{for} \quad (\sigma_x)_{x=0} = 0 \qquad (20)$$

or $$\sigma'_x = \frac{a^*}{B}\left[\left(\frac{h}{h_0}\right)^B - 1\right] \qquad (21)$$

where: $a^* = \frac{2a}{\varepsilon} + g;\ \ B = \frac{2b}{\varepsilon} + c - 1.$

5 DISCUSSION AND CONCLUSIONS

Undrained condition

The value that the horizontal total pressure at the upstream section of the shell must attain to cause the extrusion of the soil, σ_{ext}, strongly depends on the shear strength available at the shell-soil interface and on the length of the shell. Note that σ_{ext} is to be calculated with reference to the length of the part of the shell within which excavation has not yet been carried out (with a minimum equal to the length of overlapping of two consecutive shells). For cylindrical shells, σ_x is inversely proportional to the tunnel diameter D; for x/D <1.5 and f =1, σ_{ext} is lower than the value $6c_u$ (which does not contemplate the influence of the diameter of the tunnel) suggested by Broms and Bennermark (1967) and by Peck (1969) for unprotected faces. The proposed solutions show that the single most important factor for the stability of the face is the resistance of the shell-soil interface. When f = 0, the face is instable unless a stabilizing pressure σ_{xa} acts on it; the installation of the shell may affect adversely the face stability if the interface is smooth. And this must be taken as clear warning against umbrellas made of elements not properly bonded to the surrounding soil. The presence of the core increases σ_{ext}, especially when the ratio h_0/D is small. The solution obtained by the "slab method" is in good agreement with a limit analysis solution in the case of the truncated cone shell (Valore, 1993).

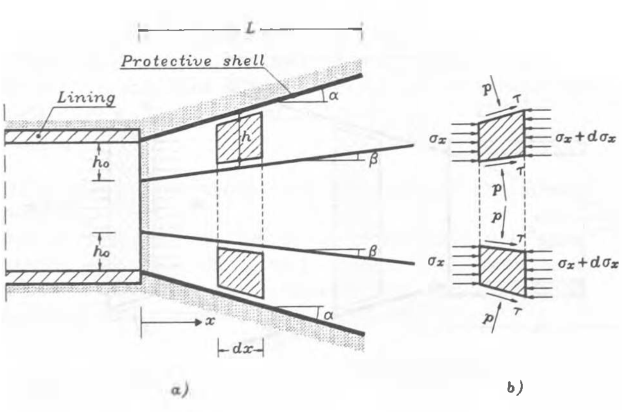

Fig. 7. Scheme S4 for the analysis of the truncated cone shell with a central core. *a*) and *b*) as in Fig. 2.

Drained condition

The proposed solutions refer to zero pore water pressure, except in the case of the cylindrical shell (scheme S1), for which seepage forces have been also considered.

For the system S1 the extrusion pressure σ_{ext} exponentially increases with $4bx/D$ (or with the length of the shell). However if the adhesion at the interface shell-soil vanishes (a = 0) and if there is no stabilizing pressure at the face (σ_{xa} = 0), the face can never be stable irrespective of the length of the shell. This result again points to the decisive importance of interface resistance, and of the stabilizing pressure on the face however small it may be. This result permits to understand, for instance, why even a thin veneer of shotcrete - properly fixed along the tunnel profile to the already installed lining or to the protective shell itself - proves often highly effective, if coupled with a preinstalled shell, in stabilizing the face of tunnels in cohesionless soils.

The presence of a central core considerably increases the stability of the face.

Seepage toward the face adversely affects stability, as expected.

Despite the many oversimplifications in the formulation of the problem and the significant departures from the actual soil-shell system, the proposed solutions permit to found on rational grounds the evaluation of the safety condition of the tunnel face protected by means of a shell installed in advance of excavation.

It is to be stressed that collapse mechanisms other than those investigated in the present paper can of course exist.

It is hoped that well documented case-histories become available to permit validation of the proposed solutions.

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REFERENCES

Broms B.B., Bennermark H. 1967. Stability of clay at vertical openings. *Jl. of Soil Mech. and Found. Div., Proc. ASCE*, Vol. 93, No. SM1, 71-94.

Hill R. *The Mathematical Theory of Plasticity.* Oxford University Press, 1950.

Hoffman O., Sachs G. *Introduction to the Theory of Plasticity for Engineers.* McGraw-Hill Book Company, 1953, New York.

Panet M. 1995. *Le calcul des tunnels par la méthode convergence-confinement*- Presses de l'Ecole National des Ponts et Chaussées, Paris.

Peck R. 1969. Deep excavations and tunnelling in soft ground. *Proc. 7th Int. Conf. Soil Mech. Found. Eng.*, Mexico City, State of-the-Art Volume, 225-290.

Peila D., Oreste P.P., Rabajoli G., Trabucco E. 1995. The pretunnel method, a new italian technology for full-face tunnel excavation: a numerical approach to design. *Tunnelling and Underground Space Technology*, Vol. 10, No. 3, 367-374.

Randolph M.F., Leong E.C., Houlsby G.T. 1991. One-dimensional analysis of soil plugs in pipe piles. *Géotechnique* 41, No. 4, 587-598.

Terzaghi K. 1942. Shield tunnels of the Chicago subway. *Jl. of the Boston Society of Civil Engineers.* In Contributions to Soil Mechanics 1941-1953, 67-114.

Valore C. 1993. Un metodo d'analisi della stabilità del fronte di gallerie con "ombrello" protettivo. *Atti XVIII Conv. Naz. di Geotecnica*, Vol. I, Ass. Geot. Ital. Rimini, 283-294.