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Panel discussion: Definition and determination of characteristic soil properties

Débat de spécialistes: Définition et détermination des valeurs caractéristiques des propriétés des sols

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ABSTRACT: The fundamental soil parameter used in EC7 "Geotechnics" is the characteristic value x_k, from which the design value x_d is derived. These soil parameters are defined and compared to todays practice. A simple approach, derived from fundamental statistics and probability concepts, is proposed for the selection of the characteristic values. The approach presented in this paper is entirely based on today's practice. It offers an extension to incorporate experience or judgmental knowledge as well as test values to produce characteristic values x_k. The resulting characteristic values are equally valid using the EC7-approach as well as for today's method. The proposed equation (6) is derived from fundamental concepts of statistics and probability, yet the design engineer does not need to acquire those concepts to successfully employ the method presented. An illustrative example to demonstrate the application of method is given.

1 INTRODUCTION

Civil engineering design is largely a matter of decision making under uncertainty. The loads and responses are not known exactly, the accuracy of the design method is uncertain, and the choice between alternative designs or construction procedures is rarely clear cut. At all stages from initial concept to final completion, decisions must be made using either incomplete information or super-abundant and inconsistent information. This inherent uncertainty is particularly true for geotechnical engineering since natural soils are extremely variable in their properties and the rational selection of suitable design parameters is generally one of the most difficult part of a design. Little guidance can be found in standard soil mechanics textbooks.

For geotechnical design tasks some kind of a conservatively chosen mean value, often based on local experience or subjective information, needs to be selected. If not entirely based on experience only, in practice the information on the mean properties is sometimes supplemented by relatively few test values.

In this paper, practical definitions of the fundamental soil parameters as used in EC7 "Geotechnics", such as the characteristic value x_k and the design value x_d, are presented. In addition, a simple approach, derived from fundamental statistics and probability concepts, is proposed for the selection of the characteristic values.

By means of this approach, the characteristic soil properties can be determined with subjective experience only, based on test results only, as well as with the combined information of the two. An illustrative example to demonstrate the application of method is given.

2 SAFETY IN GEOTECHNICAL DESIGN METHODS

2.1 *Traditional Approach: Global safety factors*

The traditional or current design method is based on the global safety concept. All variables such as loads, soil parameters, calculation procedures are used at a level of a cautiously selected mean for the design computations. A global factor of safety, accounting for the combined variability and uncertainty effects, is applied to arrive at a reasonable safety level of the final geotechnical structure.

The fundamental soil properties used for today's design computations are conservatively chosen mean values. Their degree of conservatism is generally not defined on a quantitative basis, but depends on the experience and judgment of the designer.

2.2 *Eurocode 7: Partial safety factors*

Eurocode 7 uses the limit state format with a set of fixed partial factors of safety, which is at present considered the most rational design procedure for geotechnical routine problems. Within this framework the characteristic values of soil properties have to be selected in a well defined way if a specific safety level shall be obtained. Variability or uncertainty in soil layers is only taken into account in selecting the characteristic soil values accordingly.

The characteristic value x_k is therefore the fundamental soil parameter in EC7

Both, the characteristic value x_k (EC7) as well as the conservatively chosen mean value (traditional approach) are very closely related and are assumed to be equivalent for practical purposes.

3. SUBSOIL MODELLING

There are two strongly interdependent tasks to be solved for each geotechnical problem:

- Selection of layers
  The subsoil is idealized into a few well defined and homogeneous soil layers, which constitute the basis of subsequent analyses. This is by no means a trivial step and generally involves an iteration process between the two points described here.
- Selection of soil properties for each layer
  The "averaged" soil properties (= characteristic values) are selected for each layer taking into account factors such as:
  - geological, hydrogeological and other pertinent background information e.g. from previous projects
  - The variability of the soil properties within each layer
  - The extent of the zone of ground influencing the behaviour of the geotechnical structure
  - The specific geotechnical problem to be solved, e.g. peak shear strength versus residual strength, etc.

In the following we will mainly focus on the selection and determination of "averaged" soil properties for each selected layer. The term "averaged" soil value, as used by Terzaghi (1940), today is synonymous with "conservatively chosen mean value".

4. CHARACTERISTIC SOIL PROPERTIES

In principle it would be possible to measure the soil properties everywhere in the affected volume and determine the statistical mean values of that zone. In this case we would know the true mean value of the selected layer and could use it for further analyses.

In practice, however this procedure is quite impossible. In reality, therefore the true mean properties of a layer do remain unknown and estimates of the true mean have to be made on the basis of experience or only small numbers of tests.

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4.1 *Definitions and practical meanings*

The different meanings of the above mentioned values as well as the soil parameters referred to in Eurocode 7, will be defined and also explained in practical terms:

- mean value x_m
- characteristic value x_k
- design value x_d

a) *Mean Value x_m*

The mean value of a soil property corresponds to the true statistical mean of a soil layer and is usually unkown due to limited sampling. It is however this mean value which we are targeting at, since it is a clearly defined fundamental soil parameter to be used for design computations.

The best we are able to do is to estimate the unknown mean value of the layer as exactly as possible with the best "limited" information at hand. This estimation does involve some uncertainty which needs to be accounted for accordingly.

The mean value itself is not used in Eurocode 7. Instead the characteristic value, being the best possible estimate of this mean value of the selected layer, is used in place of the mean.

b) *Characteristic Value x_k*

> The characteristic value x_k is the best estimate of the "unknown" statistical mean x_m of a soil layer

According to EC7: The characteristic value shall be selected with the aim that the probability of a more adverse (mean) value governing the behaviour of the soil and rock in the ground is not greater than 5%.

What exactly does this mean?

Characteristic value x_k	=	a conservatively chosen mean value of the layer
	or	
		a cautiously selected mean value of the selected soil layer

In EC7 the degree of conservatism or caution is further defined as follows:

- The probability of a more adverse real mean value governing the behaviour of the soil layer should not be higher than 5%.
- We want to be approximately 95% confident that the real statistical mean of the layer is superior to the selected x_k.

$\longrightarrow$ $x_k = x_m$ - standard error of means

c) *Design Value x_d*

The design values x_d for soil parameters are generally derived from the characteristic values x_k

$$x_d = \frac{x_k}{\gamma_m} \tag{1}$$

γ_m is the partial factor of safety as given in EC7 (1994), Table 2.1.

4.2 *Determination of characteristic soil properties*

Based on statistical sampling and estimation theories, the characteristic value x_k can be described by the following general expression:

$$x_k = x_m - f \cdot \frac{S_{sample}}{\sqrt{n}} \tag{2}$$

$$S_{Sample} = S_x = \sqrt{\frac{\Sigma\left(x_i - x_m\right)^2}{n-1}} \tag{3}$$

where:

S_{sample}	=	standard deviation of the test values
n	=	number of samples or test values x_i
f	=	statistical coefficient related to the type of distribution (Normal, Student's....), confidence limits and number of test values

Substituting the coefficient of variation, $V_x = S_{sample}/x_m$, into equation (2) gives

$$x_k = x_m \cdot \left(1 - \frac{f}{\sqrt{n}} \cdot V_x\right) \tag{4}$$

Although this statistical equation is correct for our problem at hand, it poses several difficulties to apply it directly in geotechnical practice. The most serious limitations are shown in Figure 1 below.

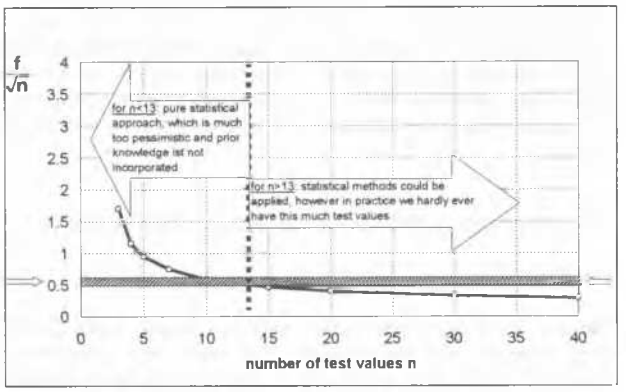

Figure 1

In the range of n = 0 - 1 test values, statistical methods cannot be used (standard deviation is not defined). This however is a fairly typical situation for smaller geotechnical projects.

In the range of n = 2 - 9 test values, statistical methods could

be used theoretically. As demonstrated in Figure 1 however, the coefficient on the left axis exhibits a value too high due to the small and uncertain statistical basis and stabilizes only after a larger number of n. In addition, the coefficient of variation V_x on purely statistical grounds will also be somewhat higher than typically encountered in soils. Since both these coefficients are multiplied, as seen in equation (4), the resulting characteristic value would be unreasonably low and pessimistic.

In the range of n > 10 test values, statistical methods can be applied sucessfully. It is however rarely the case to have such a large data base in geotechnical projects.

Comparative computations have shown that a good approximation to the above formula is achieved with

$$\frac{f}{\sqrt{n}} \cong \frac{1}{2} \tag{5}$$

This approximation does produce characteristic values within reasonable accuracy and is valid for several distributions typical for soils. It also compares very favourably with values typically used in the geotechnical practice. A major advantage is that the method can also be used when no test values at all are available as is often the case.

Proposed relationship for the determination of x_k

$$\boxed{x_k = x_m \cdot \left(1 - \frac{V_x}{2}\right)} \tag{6}$$

The equation means, that half of a standard deviation needs to be deducted from the mean value x_m to obtain the characteristic value x_k. As will be seen in chapter 4.2.a), it is advantageous to write the equation in terms of V_x and not with S_x.

This equation has been proposed in Switzerland 6 years ago (Schneider 1990). Its use in practical applications has proven its simplicity and more important, close correlation with values estimated by experienced geotechnical engineers.

Equation (6) can easily be applied e.g. to scattered data as a function of depth, since half of a standard deviation can either be determined visually or by means of statistical regression analyses.

4.3 *Determination of the values x_m and V_x*

As seen by inspection of the proposed equation (6), only the two soil parameters x_m and V_x are required for the assessment of the characteristic value x_k

In the following, 3 possibilities are presented to determine the required values x_m and V_x:

- No test values available
- Numerical test values available (e.g. several angles of internal friction ϕ)
- Combination of the test values + a priori information

a) *No test values available (n = 0)*

$$x_m \cong \frac{a + 4 \cdot b + c}{6} \tag{7}$$

$$V_x = \frac{S_x}{x_m} \cong \frac{c - a}{a + 4 \cdot b + c} \tag{8}$$

where: a = estimated minimum value
b = most likely value
c = estimated maximum value

The estimation of the values a, b and c can be based on personal experience or judgment, documented local or regional experience as well as published tables with typical values.

The value of x_m can reliably be estimated by the above method, since it is fairly insensitive to heterogeneity or scatter around the mean properties.

The coefficient of variation V_x however poses more difficulties since it is directly dependent on the difference between the minimum and maximum values of a and c. The above equations (7) and (8) implicitly assume that a and c are 3 standard deviations lower or higher, respectively than the mean value x_m. This means that approximately 100% of the possible data should be enclosed by the range of a to c. Measured values do usually not represent 100% of the statistically possible data, but often only about +/- 2 standard deviations, which should be accounted for.

Table 1 presents typical values for the coefficient of variation V_x. They are generally valid and their magnitudes have been confirmed by many researchers worldwide. (e.g. Rethati, 1988; Lumb, 1974).

> It has been found on a worldwide data basis that V_x is approximately constant for a certain soil property

→ Therefore we always have very valuable prior information available for estimating the characteristic soil properties, even in the absence of any test values. Such a priori values are presented in Table 1.

Table 1

Soil Property	Range of typical values of V_x	Recommended average values V_x
Densities	0.01 - 0.10	0
Angle of internal Friction	0.05 - 0.15	0.1
Cohesion	0.3 - 0.5	0.4
Compressibility modulus	0.2 - 0.7	0.4

b) *Test values available without a priori information*

In this case, the 2 required values x_m and V_x are calculated using standard equations from statistics, as shown in Figure 3.

c) *Combination of test values + a priori information*

The concept of Bayes' theorem can be used to combine a priori knowledge (experience, judgment) with measured test values. The method presented here has been adapted from proposals by Tang (1971). The application of Bayes' theorem results in a combined a posterior information, which is (in a statistical sense) superior to the a priori as well as the test results alone.

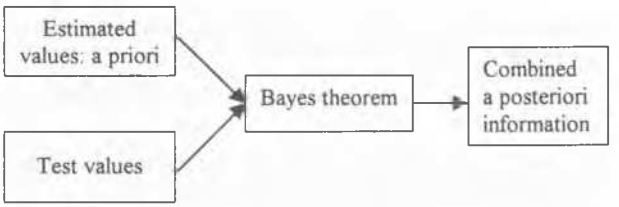

Figure 2

Numerical procedure

- Estimated a priori values: x_{m1}, V_{x1} and $S_{x1} = x_{m1} \cdot V_{x1}$
  As described in 4.2.a)
- Test values: x_{m2}, S_{x2} and V_{x2}

$$x_{m2} = \frac{\sum x_i}{n} \qquad S_{x2} = \sqrt{\frac{\sum (x_i - x_{m2})^2}{n-1}} \qquad V_{x2} = \frac{S_{x2}}{x_{m2}}$$

- Combined a posteriori information: x_{m3}, S_{x3} and V_{x3}

$$x_{m3} = \frac{x_{m2} + \frac{x_{m1}}{n} \cdot \left(\frac{S_{x2}}{S_{x1}}\right)^2}{1 + \frac{1}{n} \cdot \left(\frac{S_{x2}}{S_{x1}}\right)^2} \qquad S_{x3} = S_{x2} \cdot \sqrt{\frac{1}{n + \left(\frac{S_{x2}}{S_{x1}}\right)^2}}$$

$$V_{x3} = \frac{S_{x3}}{x_{m3}}$$

Best estimate of the true mean:

$$\longrightarrow \quad x_{ki} \cong x_{mi} \cdot \left(1 - \frac{V_{xi}}{2}\right)$$

i = 1: judgment/experience
i = 2: test values
i = 3: combined knowledge

5 EXAMPLE

Example with values from vane shear tests (Denver & Ovesen, 1994)

The following results from 5 undrained vane shear tests (n=5) of a clay till were presented:

50 ; 50 ; 60 ; 81 and 89 kN/m^2

All the values are in the influenced volume of a strip footing to be designed on the basis of the characteristic undrained shear strength cu_k. It is assumed that the vane strength values correspond also to the undrained shear strength (Denver & Ovesen, 1994).

The estimated a priori information is given in the paper as:

$$\left.\begin{array}{l} x_{m1} = cu_{m1} = 100 \text{ kN/m}^2 \\ V_{x1} = Vcu_1 = 0.25 \end{array}\right\} \quad Scu_1 = 25 \text{ kN/m}^2$$

a) *Determination of the characteristic undrained shear strength from the a priori information only (see Figure 3)*

$$cu_{k1} = 87.5 \text{ kN/m}^2$$

b) *Determination of the characteristic undrained shear strength from the vane tests only (see Figure 3)*

$$\left.\begin{array}{l} cu_{m2} = 66 \text{ kN/m}^2 \\ S_{cu2} = 18.04 \text{ kN/m}^2 \\ V_{cu2} = 0.27 \end{array}\right\} \quad cu_{k2} = 57.1 \text{ kN/m}^2$$

c) *Determination of the characteristic undrained shear strength using Bayes' theorem to combine the information for obtaining the best estimate of the mean (see Figure 3)*

$$\left.\begin{array}{l} cu_{m3} = 68 \text{ kN/m}^2 \\ S_{cu3} = 7.7 \text{ kN/m}^2 \\ V_{cu3} = 0.11 \end{array}\right\} \quad cu_{k3} = 64.3 \text{ kN/m}^2$$

6 SUMMARY AND CONCLUSIONS

The characteristic values x_k used in today's approach (global factor of safety) as well as in EC7 are for all practical purposes equivalent and should be of the same magnitude.

In today's practice x_k is selected primarily based on experience and personal judgment. This is a well-proven practice and can, or even should, still be employed for EC7. Difficulties however can arise as soon as test values are available in addition to local experience. The question then is: How can they be treated to produce characteristic values or how can they be combined with local experience? These problems mainly arise, because today the desirable safety target is not explicitely formulated, but is more a matter of intuition, experience, "feeling" and sometimes even economy.

EC7 however does go one step further and presents a definition of x_k in qualitative and even quantitative terms. This puts the proposed safety system on a more rational basis. X_k in EC7 constitutes an essential part of the safety chain governing the entire design process. The characteristic value x_k therefore is the fundamental soil property in EC7, from which the corresponding design values x_d are readily derived by means of the list of partial factors of safety as given in EC7, Table 2.1.

The statistical mean value x_m itself is not used in EC7, but will always be replaced by the corresponding characteristic value x_k.

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