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# ANISOTROPIC STRENGTH AND DEFORMATION OF SANDS IN PLANE STRAIN COMPRESSION

## RESISTANCE ET DEFORMATION DES SABLES EN COMPRESSION DE DEFORMATION PLANE

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**SYNOPSIS:** Strength and deformation characteristics of isotropically consolidated air-pluviated sands in plane strain compression were studied. Commonly for the six types of sands of world-wide origins tested, the behaviour at strains less than about 0.001 % was recoverable and strain rate-independent, namely elastic. The elastic shear modulus was isotropic with respect to the angle  $\delta$  of  $\sigma_1$  direction relative to the bedding plane between  $0^\circ$  and  $90^\circ$ . However, the sands became gradually more anisotropic as strain increased exceeding the elastic limit. The angle of internal friction  $\phi$  decreased as  $\delta$  decreased from  $90^\circ$  to  $0^\circ$  with the smallest at  $\delta = 0 - 30^\circ$ . At the residual state, the behaviour became isotropic again.

### INTRODUCTION

It has been revealed that the strength and deformation characteristics of sands prepared by pluviating particles through water or air be considerably anisotropic; i.e., by triaxial compression tests (Oda, 1972, Arthur and Menzies, 1972), plane strain compression tests (e.g., Oda, 1981, Kimura et al., 1985, Tatsuoka et al., 1986a), triaxial extension tests (e.g., Lam and Tatsuoka, 1988). It has also been shown that undisturbed sand samples exhibited large anisotropy in triaxial tests (Mura and Toki, 1988) and in PSC (Tatsuoka et al., 1989). Despite the above, many classical theories for earth pressure, slope stability, bearing capacity and so on assume isotropic properties of sands, and they are used widely in current design practice. This situation may be justified, however, when considering extreme difficulties in restoring high-quality undisturbed sand samples. In fact, in most current design practice, conservative strength values are used to cover uncertainties in the strength of sands in-situ.

In contrast, in any research for validating a soil mechanics theory by using the results of element and model tests of a sand, possible anisotropic properties of the sand should be taken into account. However, the anisotropic properties of sand have often been ignored due partly to the lack of the data showing the anisotropy for the sand(s) encountered in each country. In particular, whether anisotropic properties common for sands of world-wide origins exist is not known. In addition, it is not understood how anisotropic (or isotropic) properties of sands change with strain level.

### TEST MATERIALS AND TESTING METHOD

Plane strain compression (PSC) tests were performed by using six types of sands, which have been used extensively for research purposes (Fig. 1 and Table 1). They are all

quartz-rich and poorly graded having sub-round to sub-angular particle shapes. Relatively large specimens (Fig. 2a) were prepared by pluviating air-dried particles through air at different angles  $\delta$  of the direction of the major principal stress  $\sigma_1$  during PSC relative to the bedding plane direction between  $0$  and  $90^\circ$  with the  $\sigma_2$  direction in the bedding plane direction (Step ② in Fig. 3). The specimens were dense ( $D_r = 80 - 90\%$ ), except Silica Sand ( $D_r = 50\%$ ). Except when  $\delta = 90^\circ$ , air-dried specimens were partially saturated to create an apparent cohesion (Step ③) to trim to fit a split mold (Steps ④, ⑤ and ⑥). After made frozen, they were set in a triaxial cell and allowed to thaw at a vacuum of 4.9 kPa. They were made air-dried again by circulating air from the top to the bottom. The effect of a sequence of wetting, freezing, thawing and re-drying was examined for  $\delta = 90^\circ$  specimens of Toyoura Sand (Tatsuoka et al., 1986a) and Silver Leighton Buzzard (SLB) Sand (Park, 1990), and was found negligible.

The axial and lateral stresses were measured with two load cells placed adjacent to a specimen (6 and 13 in Fig. 2b). The rigid  $\sigma_1$  and  $\sigma_2$  boundaries were lubricated by smearing a 0.3 mm-thick latex membrane with a 0.05 mm-thick silicon grease layer. The type of grease was changed depending on grain size and pressure level (Tatsuoka et al., 1984, Goto et al., 1992). The friction angle on the confining platens was typically less than  $1.0^\circ$  as ensured from the readings of the two load cells at the bottom of the confining platens (14 in Fig. 2b), which were used for stress correction. Local axial strains, totally free from the bedding error at the top and bottom ends of specimen, were obtained by measuring axial compression along both  $\sigma_3$  surfaces by means of a pair of Local Deformation Transducer (LDT; Goto et al., 1991). Local lateral strains were obtained by measuring lateral displacements on each  $\sigma_3$  plane with four proximeters (in total eight). PSC tests were performed on isotropically consolidated (vacuumed) specimens at a constant  $\sigma_3 = 14.7$  or 78.5 kPa at an axial

Sand Name (Origin)	$D_{50}$ (mm)	$U_c$	Grain Shape <sup>1</sup>	$\phi$ (°) at $\delta = 90^\circ$	$\sigma_3$ kPa	OCR	$e_{max}/e_{min}$	$e_{0.01}$	$G_c$
Toyouura <sup>1</sup> (Japan)	0.162	1.46	Sub-angular	46.3 48.7	78.5 14.7	1.0 5.3	0.977/ 0.605	0.660	2.64
Silica No.5 (Japan)	0.30	2.11	Sub-angular	40.1	78.5	1.0	0.849/ 0.520	0.685	2.69
Ticino (Italy)	0.502	1.33	Sub-round	47.9	78.5	1.0	0.96/ 0.59	0.660	2.68
Monterey #0 (USA)	0.44	1.74	Sub-round	47.8	78.5	1.0	0.86/ 0.55	0.610	2.64
Silver Leighton Buzzard (UK)	0.62	1.11	Sub-round	45.8 47.7	78.5 14.7	1.0 5.3	0.79/ 0.49	0.520	2.66
Karlsruhe (Germany)	0.45	1.65	Sub-round	43.8	78.5	1.0	0.87/ 0.54	0.620	2.65

1. Listed from the top in the order of grain shape angularity.
2. Void ratio at  $\sigma_3 = 4.9$  kPa.
3. Toyouura Sand No.2 is from a batch different from that used by Tatsuoka et al. (1986, 1986a).

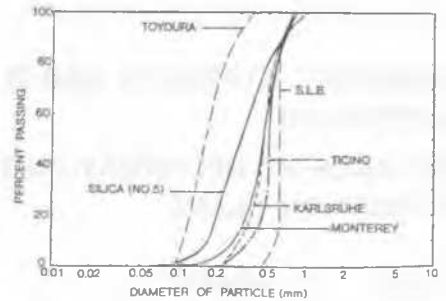


Fig. 1 Grain size distribution curves of the test materials.

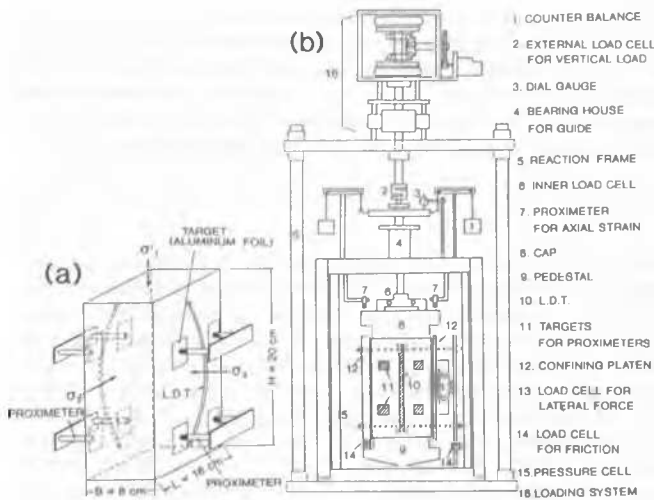


Fig. 2 (a) Local strain measurements and (b) apparatus.

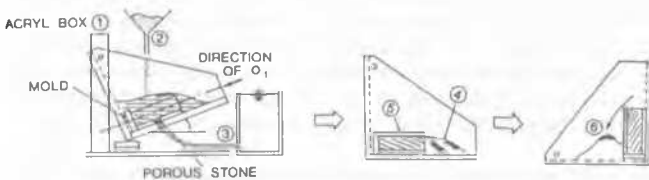


Fig. 3 Specimen preparation method.

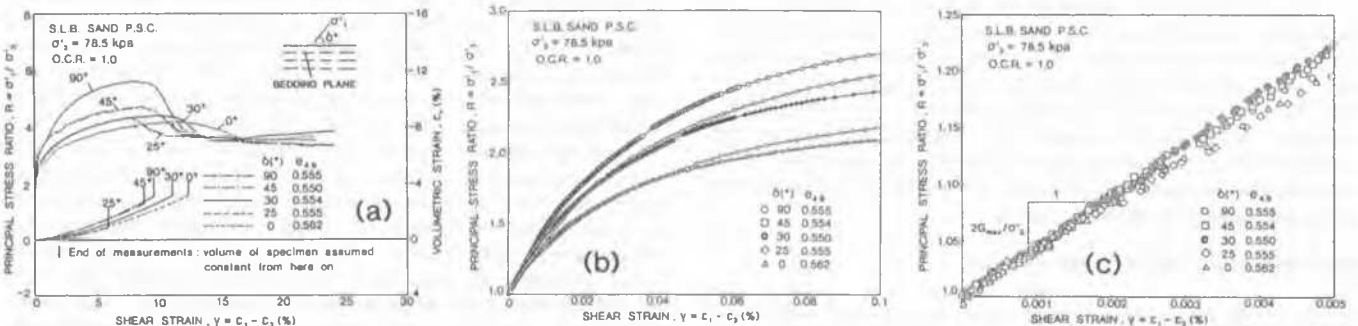


Fig. 4 Typical  $\sigma_1/\sigma_3 - \gamma$  ( $= \epsilon_1 - \epsilon_2$ ) relations of NC SLB Sand: for  $\gamma$  up to (a) 30 %, (b) 0.1 %, and (c) 0.005 %.

strain rate of 0.125 %/min except where otherwise noted.

### TEST RESULTS

**Typical result (Fig. 4):** Only in Fig. 4a, externally measured axial strains were presented, while the volumetric strains presented are equal to the summation of locally measured axial and lateral strains. In the other figures, locally measured axial strains were used. The following may be noted: 1) The initial relations at shear strain  $\gamma <$  about 0.002 % are isotropic (Fig. 4c). 2) As  $\gamma$  increases, the behaviour becomes gradually more anisotropic (Fig. 4b), and the peak strength is noticeably anisotropic (Fig. 4a). 3) The residual strength becomes isotropic again.

**Small strain behaviour:** As seen from Fig. 5, the stiffness for the initial portion is virtually the same with that for the small unload/reload cycle(s); i.e., the initial behavior is recoverable. Further, the effect of strain rate is negligible at small strains (Fig. 6). These two features combined mean elastic behavior, and thus a unique maximum shear modulus  $G_{max}$  can be defined for each test (Figs. 4c, 5 and 6). This elastic behaviour was observed commonly for all the tests in the present study, and also for other types of sands, soft rocks and clays (Tatsuoka and Shibuya, 1992, Shibuya et al., 1992). Fig. 7 shows the values of the parameter  $A = (G_{max}/p_a) / ((\sigma'_3/p_a)^{0.4} \cdot f(e))$  plotted against  $\delta$ , where  $p_a$  is the atmospheric pressure = 98.1 kPa and  $f(e) = (2.17 - e)^2 / (1 + e)$  ( $e$  is void ratio). Iwasaki and Tatsuoka (1977) showed that this value at  $\gamma = 0.0001$  % from resonant-column tests on many Japanese clean sands is about 900. In Fig. 7, these values are around 1,000, in particular about 900 for two Japanese

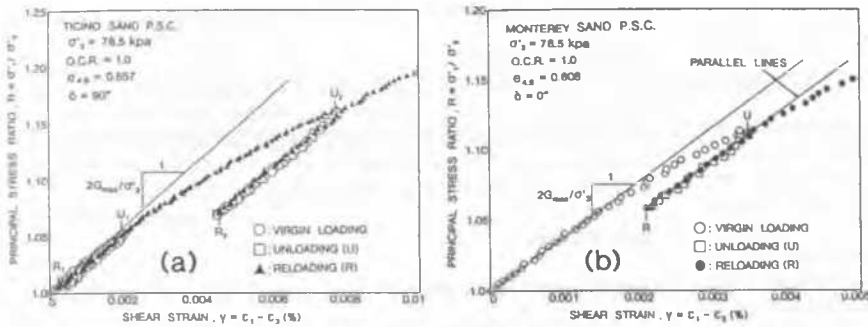


Fig. 5  $\sigma_1/\sigma_3 - \gamma$  relations at small strains; (a) NC Ticino Sand and (b) NC Monterey Sand.

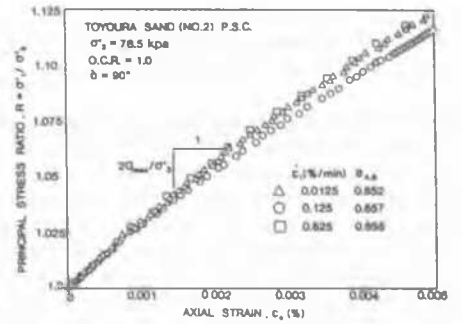


Fig. 6  $\sigma_1/\sigma_3 - \epsilon_a$  relations at three different axial strain rates, NC Toyoura Sand (No. 2).

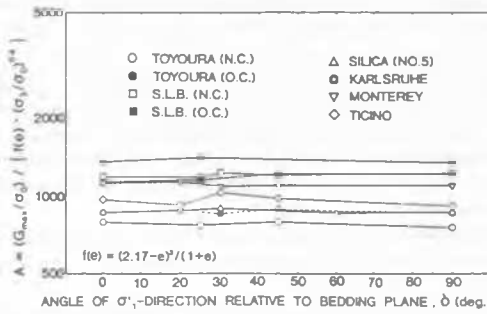


Fig. 7  $G_{max}$  plotted against  $\delta$  for NC and OC Sands.

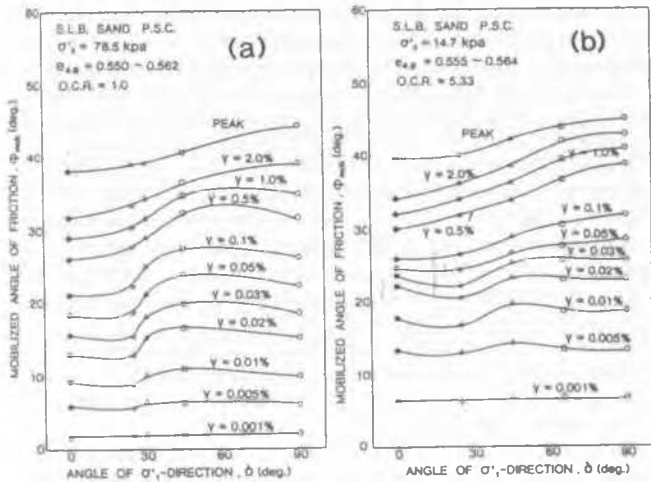


Fig. 8  $\phi_{mob}$  at different  $\gamma$ -values versus  $\delta$  for (a) NC and (b) OC SLB Sand.

sands (Toyourea and Silica Sands). This result also ensures the initial elastic behaviour, which is probably a result of elastic deformations of sand particles. Isotropic elasticity as seen from Fig. 7 may be due to nearly isotropic initial distribution of inter-particle contacts produced during isotropic consolidation. This point may be supported by the fact that the behavior at  $\gamma < 0.1\%$  is more isotropic for over-consolidated (OC) specimens than for NC ones (Fig. 8). This result (Fig. 7) also suggests that possible anisotropy in large strain stiffness and peak strength in the field may not be estimated only from

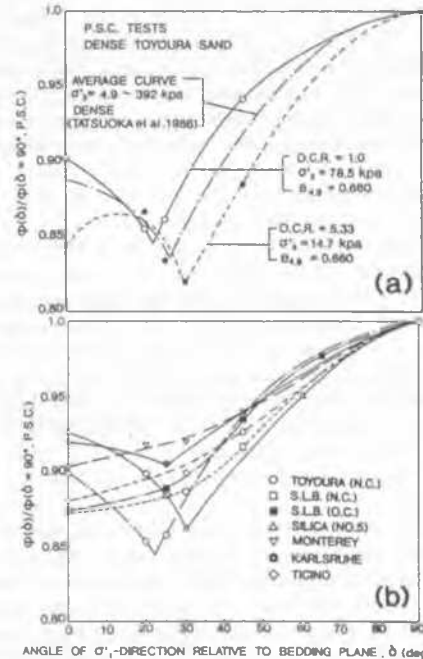


Fig. 9 Summary of  $\phi(\delta)/\phi(\delta=90^\circ) \sim \delta$  relation: (a) NC and OC Toyoura Sand and (b) all the sands tested.

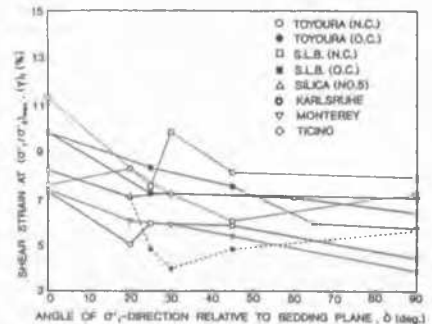


Fig. 10 Summary of  $\gamma_x$  at peak plotted against  $\delta$ .

shear wave velocities measured in a specific direction.

Fig. 7 shows also that the parameter A increases slightly

as the grain size. Further, although the plastic deformation properties of sands at intermediate strain levels are known to be affected by OCR, any discernible effect of OCR on  $G_{max}$  cannot be seen.

**Anisotropic stiffness and peak strength:** As strain increases exceeding the elastic limit, both slip at interparticle contacts and relative rotation of particles become more active, while overall deformation properties become more anisotropic (Fig. 8). The degree of dilatancy also is affected by the angle  $\delta$  in that it becomes less as  $\delta$  decreases (Fig. 4a). For each test series changing only  $\delta$ ,  $\phi$  at a certain  $\delta$  was divided by the corresponding  $\phi$  at  $\delta = 90^\circ$  after being corrected for void ratio variations (Fig. 9). The following points may be seen: 1) For NC Toyoura Sand, the trend is very similar between the present and previous studies (Fig. 9a). 2) For all the sands (Fig. 9b), the trend is similar in that  $\phi$  decreases as  $\delta$  decreases from  $90^\circ$  to  $20^\circ$ - $30^\circ$ . 3) As  $\delta$  decreases further towards zero, for Toyoura, Silica and Karlsruhe Sands,  $\phi$  increases again, while for SLB, Ottawa and Ticino Sands,  $\phi$  still decreases. The former and latter groups have relatively smaller and larger grain sizes, respectively, and have relatively more and less angular grain shapes, respectively. It is not known yet to the present authors which of these two factors or another is the major factor for the above two different trends. 4) The ratios of the smallest to largest values of  $\phi$  are in a small range between 0.82 and 0.9. Note also that in each test series, shear strain at the peak state was the smallest at  $\delta = 90^\circ$  and increased as  $\delta$  decreases (Fig. 10). Some sands exhibited a trend of minimum at  $\delta = 20^\circ$ - $30^\circ$  (see also Fig. 4a), where the planes of the maximum stress obliquity tend to be in parallel with the bedding planes, exhibiting a relatively smaller or smallest strength. 5) Within the limit of OCR examined (OCR= 5.33), both the value of  $\phi$  and the pattern of  $\phi \sim \delta$  relation are not affected by OCR.

A similar  $\phi \sim \delta$  plot as above has been obtained from PSC tests on undisturbed volcanic origin-sand (Shirasu) taken from a layer deposited secondarily under water (Tatsuoka et al., 1989). Although further study will be needed to know whether the trend of strength anisotropy shown in Fig. 9 can be generally applicable to in-situ sands deposited under gravity through air or water, it can be pointed out that most classical isotropic perfectly-plastic soil mechanics theories should be re-evaluated. Indeed, the strength anisotropy and the progressive failure in the ground associated with post-peak strain softening are two major factors which are missing in them, as has been discussed in detail in relation to the bearing capacity problem (Tatsuoka et al., 1991).

## CONCLUSIONS

In PSC tests of six types of isotropically consolidated sands, the deformation characteristics at strains less than about 0.001 % was elastic and isotropic with respect to the angle  $\delta$  of the  $\sigma_1$  direction relative to the bedding plane. As strain increased, the deformation characteristics became gradually more anisotropic. The peak strength was noticeably anisotropic with a similar trend that the angle of internal friction  $\phi$  decreased as  $\delta$  decreased from  $90^\circ$ , and the ratio of the smallest to largest values of  $\phi$  was between 0.82 and 90. Whether  $\phi$  has a minimum at  $\delta = 20^\circ$ - $30^\circ$  depended on the type of sand.

The residual strength became isotropic again.

## ACKNOWLEDGEMENTS

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