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1-D STRAIN IN A SAND COMPOSED OF BRITTLE PARTICLES

DEFORMATION EN COMPRESSION MONODIMENSIONNELLE D'UN SABLE A GRAINS FRAGILES

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SYNOPSIS : Grain-crushing caused by stress increases modifies the composition, the fabric and the structure of uncemented sands composed of angular, brittle grains. In these sands the crushing process substantially changes the grain-size distribution and the arrangement of the particles, thus heavily influencing the mechanical behaviour of the material. In the paper, results of one-dimensional compression tests carried out up to stress of 19.5 MPa on a sand composed of minute subangular fragments of marine shells are reported. The relationships among 1-D strain, initial void ratio of the sand, applied effective stress and particle breakage are discussed. The available data point out scale effects concerning grain-crushing and sand compressibility. The compressibility is not uniquely related to initial voids ratio.

INTRODUCTION

The deformation of sands, as a consequence of their particulate nature, is mainly controlled by interactions among individual grains when grains are rounded and strong; if the stress level is not high, strains essentially originate from interparticle sliding and rotation, i.e. from relative motion of grains. If the sand particles are angular, concave - shaped and brittle, however, additional, and frequently prevailing, sources of strain are the deformation of grain-grain contacts and the crushing of particles.

In carbonate sands particle crushing take place even at low effective pressure. Particle crushing dramatically changes the grain-size distribution, the fabric and the structural arrangement. Mechanical behaviour of the material undergoes marked modifications, both during crushing and after the "state" of the soil has been changed by the crushing process itself, in consequence of the increase in the number of particles and in the specific surface area, of the co-ordination number and as a result of the changes in grain angularity. Grain-crushing greatly adds to the irreversibility of the deformation of these sands, and is the main cause of their peculiar undrained behaviour upon reloading, characterized by values of B and C pore pressure parameters much less than unity (Lambe and Whitman, 1969; Veyera et al., 1992; Coop, 1990; Black and Lee, 1973; Chaney et al 1979; Eigenbrod and Burak, 1990; Fragaszy and Voss, 1986).

The knowledge of the behaviour of these sands is not only relevant per se, but also for understanding the role of grain breakage in the response of other granular materials.

In order to study particle breakage and its effects at low and high pressures, a laboratory research programme has been initiated on a natural sand composed of brittle grains.

In the paper, results of oedometer tests are reported, carried out to investigate the link between one - dimensional strain, initial porosity of the sand, effective stress and particle breakage.

PROPERTIES OF THE SAND AND TESTING PROCEDURES

The material used is a natural sand from an extensive deposit bordering the seaside of a small town near Palermo. The sand is almost entirely composed of minute fragments of whitish marine shells, worked out by the action of sea-waves, which incessantly supply new material to the beach. Two samples, A and B, were retrieved from the beach, from two zones 50 m apart, both a few meters from the shoreline. The sand will be termed shortly "Shell sand".

The particles are from angular to subrounded, frequently elongated, and concave. Shell fragments larger than 1 mm, amounting to less than 1 percent, were sieved out.

The sand was preliminarily immersed in trap flowing water, at least for 12 hours, in order to wash away salts from the surface of the particles.

Samples A and B are identical as to the type of component particles but have

slightly different grain-size distribution, Fig. 1. Shell sand is finer than Dogs Bay sand tested by Coop (1990); its calcium carbonate content is 90%. Some other index properties are summarized in Table 1. The sand is uncemented.

The sand was placed dry in the oedometer and then gently vibrated by hand. Immediately after the first load was applied the specimen was saturated with distilled water, except for specimen 8 which was tested dry.

The incremental loading procedure was adopted; each load increment was maintained for 24h before applying the next one; only for samples A1, B1, C1, D1, the duration of load increments was doubled (48h).

Some tests were performed on specimens with artificial grading.

After testing the specimen were removed from the apparatus and sieved.

The height of the sample ranges from 3.26 to 40 mm, the diameter of the oedometer varies between 50.5 and 100 mm. The maximum applied effective vertical ranges from 100 to 19500 kPa. In some tests unloading and reloading cycles were also carried out.

All testing activity was conducted at temperature between 18 and 25°C.

The initial values of voids ratio e_0 are reported in Table 2.

RESULTS

Table 2 gives relevant data on oedometer one-dimensional compression tests for Shell sands A and B. Some other data concerning the crushing process are summarized in Table 3. Typical results are plotted in Fig. 3.

The mean normal effective stress p' has been calculated assuming a constant K_0 value of 0.5, according to Coop; however a value of 0.33 does not appreciably alter the curves. Clearly, each specimen could be tested only once, since the material grading markedly changed during test. The degree of particle crushing was evaluated at the end of the test, by comparing values of characteristic diameters (D_{10} , D_{25} , D_{50} , D_{80}) of the tested sand with their corresponding initial values. Some SEM photos of tested material were taken; they show that the angularity of sand particles may increase after load application at relatively low stress levels.

MECHANICAL BEHAVIOUR

Compression

The initial part of the compression curve of specimens with height equal to 20 or 40 mm in the $v: \ln p'$ space, is almost straight, line a in Fig. 2 with slope k_a , up to values p'_1 of 400 kPa, then steadily - but not abruptly - bends until at a mean pressure p'_2 from 1600 to 3200 kPa the curve merges into a straight line - b in Fig. 2 with slope k_b up to the maximum applied stress. A similar trend is seldom noted for specimens with $h_0 \leq 10$ mm, since initial and final straight trend cannot be clearly identified. The settlement-log t curves are straight up to 2400 kPa; a definite sigmoidal trend is

TABLE 1 - Index properties of Shell Sand

| | Sample A | Sample B |
|---------------------------------------|----------|----------|
| Calcium carbonate content | 90% | 90% |
| Specific gravity [kN/m ³] | 27.6 | 27.6 |
| Maximum particle diameter | 0.85 | 0.85 |
| Minimum particle diameter | 0.18 | 0.074 |
| Percent finer than 0.85 mm | 98.4 | 99.4 |
| Percent finer than 0.074 mm | 0 | 0 |
| D80 particle size | 0.54 | 0.43 |
| D50 particle size | 0.37 | 0.32 |
| D25 particle size | 0.31 | 0.28 |
| D10 particle size | 0.26 | 0.23 |
| Uniformity coefficient (D60/D10) | 1.58 | 1.48 |

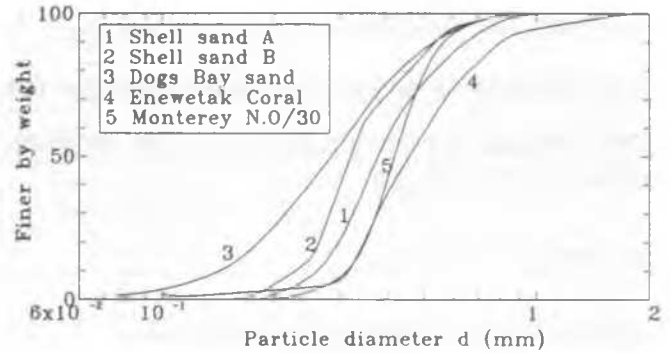


Fig. 1 Grading curves of untested Shell sand A and B (1, 2). Curves of other carbonate (3,4) and quartz (5) sands are also shown.

TABLE 2 - Summary of specimens properties and oedometer results. Specimens SH14, SH15, SH16, SH17, SH18, SH20 from sample B; all other specimens from sample A

| Specimen | h ₀ [mm] | 0 [mm] | eo | σ'vmax [kPa] | p'1 [kPa] | p'2 [kPa] | p'3 [kPa] | σ'* [kPa] | ka | kb | kc | kb/kc | D10 [mm] | D25 [mm] | D50 [mm] | D80 [mm] |
|----------|---------------------|--------|-------|--------------|-----------|-----------|-----------|-----------|------|------|------|-------|----------|----------|----------|----------|
| 1 | 20 | 56 | 1.111 | 6000 | 200 | 1600 | 968 | 6000 | .012 | .212 | .004 | 53 | 0.20 | 0.26 | 0.36 | 0.49 |
| 2 | 20 | 56 | 1.008 | 6000 | 400 | 1600 | 1176 | 4800 | .012 | .154 | .005 | 31 | / | / | / | / |
| 3 | 20 | 56 | 0.805 | 6900 | 400 | 1600 | 1176 | 6000 | .012 | .137 | .005 | 27 | 0.20 | 0.25 | 0.33 | 0.46 |
| 4 | 20 | 56 | 0.850 | 6900 | 400 | 1600 | 1176 | 6000 | .009 | .133 | .004 | 33 | 0.19 | 0.25 | 0.32 | 0.45 |
| 5 | 20 | 56 | 0.920 | 5700 | 400 | 3200 | 2385 | 5700 | .015 | / | .006 | / | 0.21 | 0.26 | 0.34 | 0.47 |
| 6 | 20 | 56 | 0.940 | 6000 | 400 | 1600 | 1176 | 6000 | .016 | .159 | .005 | 32 | 0.20 | 0.24 | 0.32 | 0.44 |
| 7 | 20 | 56 | 1.000 | 6000 | 400 | 1600 | 1176 | 6000 | .023 | .156 | .007 | 22 | 0.20 | 0.25 | 0.32 | 0.44 |
| 8 | 20 | 56 | 0.887 | 6900 | 400 | 3200 | 1693 | 6000 | .008 | .181 | .005 | 36 | / | / | / | / |
| 9 | 20 | 56 | 0.920 | 6000 | 400 | 1600 | 1097 | 6000 | .013 | .126 | .006 | 21 | 0.21 | 0.26 | 0.34 | 0.47 |
| SH2 | 20 | 56 | 1.080 | 9700 | 400 | 3200 | 1753 | / | .009 | .207 | / | / | 0.073 | 0.16 | 0.26 | 0.35 |
| SH5 | 20 | 56 | 1.094 | 9700 | 400 | 3200 | 1790 | / | .008 | .199 | / | / | 0.075 | 0.12 | 0.25 | 0.37 |
| SH6 | 20 | 56 | 0.980 | 19500 | 400 | 3267 | 1728 | / | .014 | .219 | .002 | 109 | 0.062 | 0.12 | 0.23 | 0.35 |
| SH11 | 20 | 56 | 1.262 | 19500 | 400 | 3200 | 1735 | / | .014 | .231 | .004 | 58 | 0.058 | 0.13 | 0.22 | 0.34 |
| SH14 | 20 | 56 | 0.835 | 1200 | / | / | / | / | .010 | / | / | / | 0.21 | 0.26 | 0.32 | 0.42 |
| SH15 | 20 | 56 | 0.748 | 1190 | / | / | / | / | / | / | / | / | 0.21 | 0.26 | 0.32 | 0.42 |
| 10 | 10 | 56 | 1.200 | 6000 | 400 | 1600 | 1176 | 6000 | .026 | .200 | .010 | 20 | 0.20 | 0.24 | 0.31 | 0.44 |
| 12 | 10 | 56 | 1.080 | 6000 | 200 | 1600 | 871 | 6000 | .022 | .236 | .009 | 26 | / | / | / | / |
| SH16 | 10.2 | 56 | 0.842 | 1200 | / | / | / | / | .016 | / | / | / | 0.20 | 0.25 | 0.31 | 0.40 |
| SH17 | 10 | 56 | 0.760 | 600 | / | / | / | / | .019 | / | / | / | 0.21 | 0.26 | 0.32 | 0.41 |
| SH18 | 7.1 | 56 | 1.226 | 3000 | / | / | / | / | .017 | / | / | / | 0.19 | 0.24 | 0.30 | 0.39 |
| 11 | 5.0 | 56 | 1.410 | 6000 | 400 | 1600 | 948 | 6000 | .066 | .279 | .021 | 13 | 0.19 | 0.26 | 0.37 | 0.45 |
| 13 | 5.0 | 56 | 1.220 | 5000 | 400 | 1600 | 968 | / | .085 | .239 | .021 | 11 | / | / | / | / |
| SH3 ** | 5.0 | 56 | 1.550 | 4900 | 200 | 1600 | 812 | / | .053 | .287 | .021 | 14 | / | / | / | / |
| SH4 ** | 3.26 | 50.5 | 1.547 | 6200 | 247 | 2000 | 841 | / | .088 | .306 | .036 | 9 | / | / | / | / |
| SH12 | 40 | 100 | 1.247 | 2640 | 417 | 1320 | 871 | 2640 | .009 | .118 | .002 | 59 | 0.21 | 0.28 | 0.35 | 0.53 |
| SH20 | 40 | 100 | 0.821 | 3140 | / | / | / | / | .009 | / | / | / | 0.22 | 0.28 | 0.33 | 0.42 |
| B | 20 | 56 | 0.962 | 300 | / | / | / | / | .007 | / | / | / | 0.25 | 0.31 | 0.38 | 0.49 |
| C | 20 | 56 | 0.985 | 600 | / | / | / | / | .006 | / | / | / | 0.24 | 0.29 | 0.37 | 0.48 |
| D | 20 | 56 | 1.020 | 1200 | / | / | / | / | .006 | / | / | / | 0.24 | 0.29 | 0.37 | 0.47 |
| A' | 20 | 56 | 0.920 | 100 | / | / | / | / | .004 | / | / | / | 0.23 | 0.29 | 0.37 | 0.47 |
| B' | 20 | 56 | 0.960 | 300 | / | / | / | / | .001 | / | / | / | 0.24 | 0.30 | 0.37 | 0.48 |
| C' | 20 | 56 | 0.913 | 600 | / | / | / | / | .006 | / | / | / | 0.24 | 0.29 | 0.37 | 0.48 |
| D' | 20 | 56 | 0.941 | 1200 | / | / | / | / | .006 | / | / | / | 0.24 | 0.30 | 0.37 | 0.47 |
| SH7 | 20 | 56 | 0.858 | 19500 | 800 | 6467 | 3328 | / | .007 | .173 | .003 | 58 | 0.12 | 0.20 | 0.27 | 0.37 |
| SH10 | 7 | 50.5 | 0.670 | 7650 | 247 | 3067 | 1400 | 6150 | .024 | .220 | / | / | 0.18 | 0.34 | 0.45 | 0.51 |
| SH8 | 6.3 | 50.5 | 0.750 | 6140 | 247 | 2067 | 1000 | / | .041 | .150 | .009 | 17 | 0.18 | 0.24 | 0.31 | 0.38 |

h₀ initial height;
 eo initial voids ratio;
 σ'vmax maximum applied vertical stress;
 p'1 effective stress for which deviation from linearity of the v-lnp' curve starts (see fig.2);
 p'2 mean effective stress for which the straight compression line merges (see fig. 2);
 p'3 mean effective stress conventionally corresponding to particle breakage (see fig.2);
 σ'* effective vertical pressure for which the onset of sigmoidal trend in the settlement-logt curves is observed;
 ka, kb, kc slopes of lines a, b, c (see fig.2);
 D10, D25, D50, D80 particle diameter corresponding to percent passing by weight of 10, 25, 50, 80 percent respectively;
 σ'v effective vertical stress;
 p' = (1/3)(σ'v + 2σ'h) mean normal effective stress, calculated assuming ko=0.5. p' = 2/3 (σ'v);
 ** loading / unloading / reloading cycle performed.

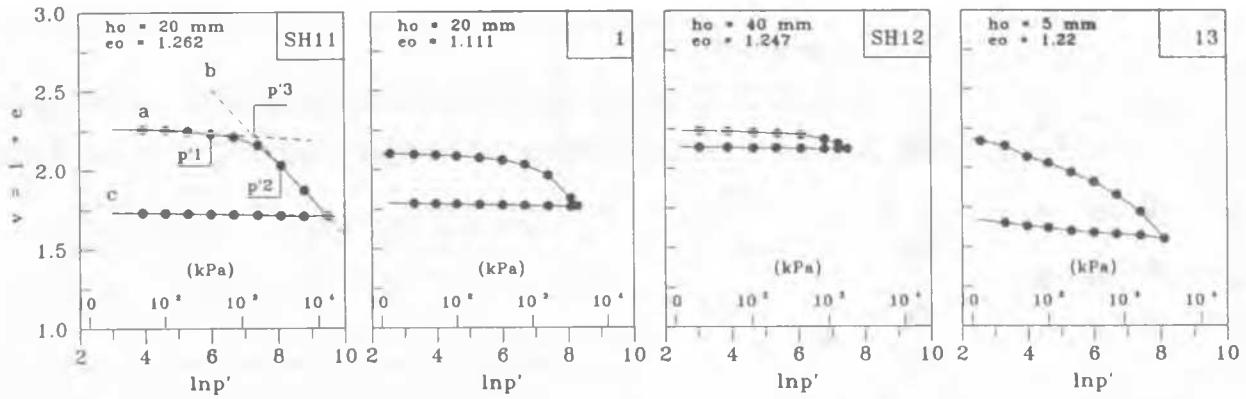


Fig. 2 Typical $v - \ln p'$ curves

not observed until σ'_v reaches about 5000 kPa. The slope k_a ranges from 0.004 to 0.016 when $h_o = 20-40$ mm, and from 0.016 to 0.088 when $h_o \leq 10.2$ mm. No definite dependence of k_a on e_o may be detected in the range of e_o between 0.7 and 1.3. k_a values of "thin" specimens are always greater than those of "thick" ones, irrespective of the initial porosity. k_a varies with the specimen height, Fig. 4. The slope k_b apparently increases with e_o , Fig. 4, regardless of specimen height; data pertaining to sand precrushed prior to testing plot in the upper range. However it is apparent from the diagram a strong fabric effect (Dobry and Petrakis 1990; Youd and Craven, 1972; Youd, 1972 and 1977), since specimen with identical h_o and e_o may exhibit different k_b values. These experimental data seem to support the conclusions of Semple (1988). Further study of the available data, however, reveals that the k_b dependence on e_o holds true only if σ'_{max} does not exceed 6000 kPa; for higher values of σ'_{max} k_b is fairly constant and is not related to e_o ; it may be concluded that k_b rather depends on σ'_{max} . Indeed, at stresses of 9700 or 19500 kPa the original structure and the fabric of the sand are severely altered or fully destroyed. It is, thus, fairly evident that the apparent e_o dependence of k_b possibly conceals an unreliable evaluation of k_b ; a correct determination of k_b requires that the test be carried out up to very high pressures.

Unloading and reloading

Rebound curves upon unloading are straight and present very low slopes, k_c , Table 2 and Fig. 2 and 4. k_c depends on the height of the specimen. There is an appreciable scale effect. Note that the ratio k_b/k_c is very high. The reloading curves are not distinguishable from the unloading ones. This is the essential cause of C pore pressure parameters lower than 1 (Veyera et al., 1992). Shell sand B, initially slightly differs from Shell sand A. This slight difference is sufficient to render it much more resistant to grain crushing.

Progress of grain - crushing

Grain-crushing starts at low applied stresses, and concerns, first, the coarser

TABLE 3 - Variation of different granulometric fractions with applied stress

| fraction X | $f < 0.25$ | $0.25 < f < 0.42$ | $0.42 < f < 0.60$ | $0.60 < f < 0.85$ | untested sand | | | |
|------------|------------|-------------------|-------------------|-------------------|-----------------------|--|--|--|
| | | | | | σ'_{max} [kPa] | | | |
| | 8 | 55 | 19 | 18 | | | | |
| 100 | 8 - 13 | 55 - 56 | 27 - 29 | 5 - 9 | | | | |
| 300 | 10 - 12 | 53 - 54 | 29 - 30 | 5 - 7 | | | | |
| 600 | 12 - 14 | 53 - 54 | 28 - 29 | 5 | | | | |
| 1200 | 11 - 14 | 54 - 56 | 27 - 28 | 5 | | | | |
| 5700 | 23 | 50 | 23 | 5 | | | | |
| 6000 | 21 - 28 | 41 - 51 | 19 - 30 | 4 - 5 | | | | |
| 6900 | 27 | 49 | 20 | 5 | | | | |
| 9700 | 45 - 50 | 37 - 43 | 10 | 2 - 3 | | | | |
| 19500 | 53 - 56 | 35 - 36 | 7 - 8 | 2 - 4 | | | | |

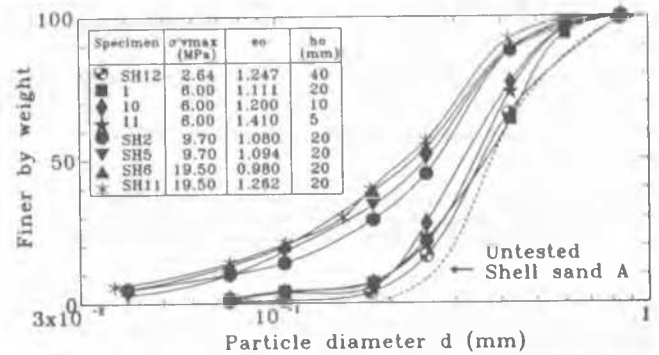


Fig. 3 Variation of the grain - size distribution of Shell sand A after 1-D compression loading versus maximum applied effective stress - Typical results

fraction. Some fractions are more stable than others during the crushing process, Table 3. Maximum applied stress greatly affects grain-crushing, Fig. 3.

In Fig. 5 changes of characteristic diameters are plotted against applied stress; in Fig. 6 the percents finer than 0.18 mm or than 0.075 mm are plotted.

In spite of some dispersion due to different e_o values of the various specimens, Fig. 5 clearly shows that the characteristic diameters abruptly decrease when stresses of 9700 kPa are applied, subsequently the rate of change with σ'_v definitely slows down. The observed trend points out the existence of a critical breaking pressure σ'_{crit} at which the crushing process abruptly propagates. The observed trend is far different from that reported by Coop (1988, 1990); it could be detected only by means of high pressure tests. The above considerations also apply to data plotted in Fig. 6, which are fully consistent with those in Fig. 5. It must be noted that thin specimens undergo much more breakage at low stresses (Fig. 6). From Fig. 6 it appears that $p(0.18)$ is a better index of crushing degree than $p(0.075)$ (percentage of fines).

The critical pressure σ'_{crit} coincides with the pressure σ^* , (Table 2) for which the settlement - log t curves lose straightness and take the sigmoidal shape, and this is far from being fortuitous.

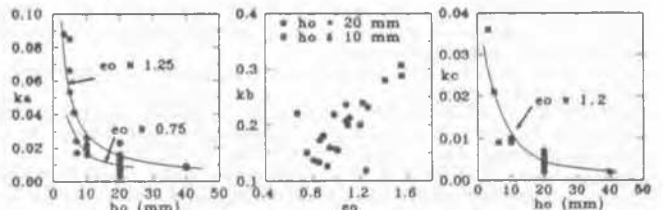


Fig. 4 Variation of k_a , k_b and k_c with initial voids ratio e_o and specimen height

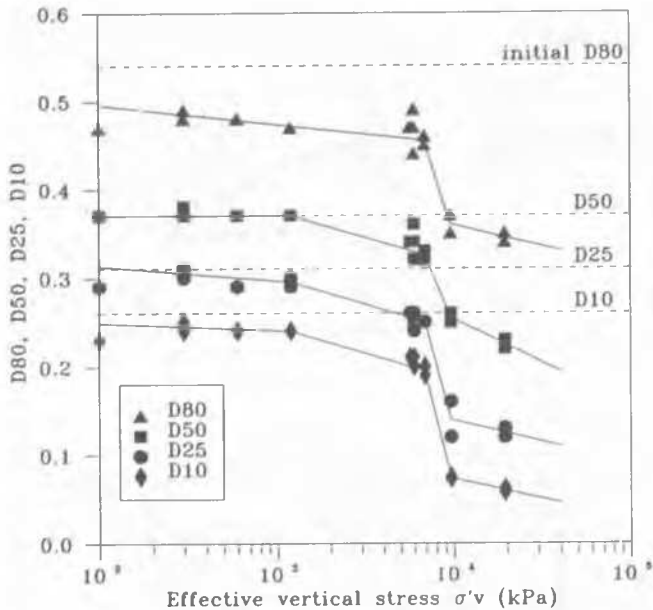


Fig. 5 Changes of characteristic particle diameter versus maximum applied vertical effective pressure. Shell sand A, specimen height: 20mm.

CONCLUSIONS

The results of the one-dimensional compression tests carried out up to a pressure of 19.5 MPa on a sand composed of brittle shell fragments suggest the following conclusions.

A large component of 1-D strain originates from grain-crushing that take place even at applied stresses lower than 100 kPa. At first the coarser grains break.

Grain crushing regularly progresses as the applied pressure increases until a critical effective stress σ'_{crit} is reached, which causes considerable, abrupt changes in the grain-size distribution of the material; subsequent pressure increases produce only small variation of the grading of the sand.

The trend of characteristic diameters D10, D25, D50, D80 with $\lg \sigma'v$ are quite similar, and may be used for the quantitative description of the crushing process.

The value of σ'_{crit} closely corresponds to the effective vertical pressure for which the settlement versus $\lg t$ curve loses its straightness and becomes distinctly sigmoidal. The curves again become straight at pressures larger than σ'_{crit} .

REFERENCES

- BLACK, D. K., and LEE, L. L. (1973) - Saturating Laboratory Samples by Back Pressures. *Jl. of the Soil Mech. and Found. Div.* Vol. 99, No. SM1, 75-93.
- BURLAND J.B. (1988) - Calcareous Sediments Conference -*Engineering for calcareous sediments, Jewell & Khorshid (eds)*, Balkema, Rotterdam.
- COOP M.R. (1988) - Particle crushing of carbonate sands - *Calcareous Sediments Conference -Engineering for calcareous sediments, Jewell & Khorshid (eds)*, Balkema, Rotterdam.
- COOP M.R. (1990) - The mechanics of uncemented carbonate sands. *Geotechnique* N. 4, 607-626.
- CHANEY, R. C., STEVENS, E. and SHETH, N. (1979) - Suggested Test Method for Determination of Degree of Saturation of Soil Samples by B Value Measurement. *Geot. Testing Jl.* Vol. 2, No. 3, 158-162.
- DOBRY, R. and PETRAKIS E. (1990) - Micromechanical model to predict sand densification by cyclic straining - *Jl. of Eng. Mech.* Vol. 116, No. 2., 228 - 308.
- EIGENBROD, K. D. and BURAK, J. P. (1990) - Measurement of B Value Less Than Unity for Thinly Interbedded Varved Clay. *Geot. Testing Jl.*

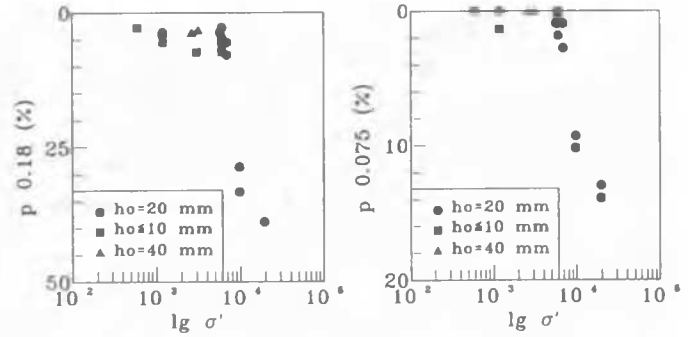


Fig. 6 Variation of the percent by weight finer than 0.18mm $p(0.18\%)$ or than 0.075mm - $p(0.075\%)$ versus applied vertical effective stress. (Note: different ordinate scale)

The degree of crushing may best be described by the percent finer than 0.18mm, which also exhibits a definite trend with applied pressure.

The intensity of crushing is enhanced when the height of specimen is less than 10 mm and lessens when they are thicker than 20 mm. This scale effect, which can have engineering implication especially at interfaces, (e.g. pile surface-sand), is more important at low stress levels.

Compressibility is essentially controlled by grain crushing. At low stress levels, the initial voids ratio e_0 seems to control compressibility of the sand; whilst at moderately high stresses, e_0 does not result to play a significant role on compressibility. These results can be explained considering that basically two deformation mechanisms are at play: particle crushing and particle rearrangement by sliding and rotation. The "share" of particle breakage is larger for higher applied pressures and for lower e_0 .

The conclusion (Semple, 1988; Burland, 1988) that compressibility characteristics of bioclastic sands are related to initial void ratio and that their mechanical behaviour can be anticipated based on initial voids ratio does not apply for the Shell sand at hand. The e_0 value, in fact, is not uniquely related to the angularity of the sand particles; the mechanical behaviour, moreover, is also influenced by the "fabric" of the sand. Results reported in the paper prove that the compressibility is not linked to e_0 at high stresses. Appreciable scale effects on compressibility are also pointed out by test results.

The rebound and reloading curves are straight and their gradient is extremely low. This result also originates from grain-breakage and consequently from the plastic, irrecoverable, nature of the volumetric strain. Correspondingly, the stiffness in unloading and reloading is extremely high; hence, the response of the sand is highly stress- path dependent (Coop, 1990), and the value of the C pore pressure parameter in reloading may never attain unity (Veyera et al., 1992)

Vol. 13, No. 4, 370-374.

FRAGASZY, R. J. and VOSS, M. E., (1986) - Undrained Compression Behaviour of Sand. *Jl. of Geotech. Eng.* Vol. 112, No. 3, March, 334-347.

LAMBE T.W. & WHITMAN R.V. (1969) - *Soils Mechanics*. SI Version John Wiley & Sons, New York.

SEMPLER R. (1988) - State of the art report on engineering properties of carbonate soils -*Calcareous Sediments Conference -Engineering for calcareous sediments, Jewell & Khorshid (eds)*, Balkema, Rotterdam, 807-836.

VEYERA, G. E., CHARLIE, W. A. DOEHRING, D. O., and HUBERT, M.E. (1992) - Measurement of the Pore Pressure Parameter C Less Than Unity in Saturated Sands. *Geot. Testing Jl.* Vol. 15, No. 3, 223-230.

YOUD, T. L. (1972) - Compaction of sands by repeated shear straining. *Jl. Soil Mech. and Found. Eng. Div.*, ASCE, 98(7), 709-724.

YOUD, T. L. and CRAVEN, T.N. (1972)- Discussion *Jl. Soil Mech. and Found. Eng. ASCE* 98 (12) 1423-1425.

YOUD, T. L. (1977) - Packing changes and liquefaction susceptibility. *Jl. Geotech. Eng. Div.*, 103(8), 918-922.