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# UN-STEADY SETTLEMENT OF CLAYS AND CONSOLIDATION

## TRANSFERTION NON-STATIONARE DE LA CONTRAINTE ET LA CONSOLIDATION

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**SYNOPSIS:** This article deals with the theory of un-steady stress and its application to the theory of consolidation of clayey soils beneath constructions and to the theory of un-steady bearing capacity of clays. It provides the practical charts for computation of rate of settlement with different conditions of layers. There diagrams for the effect of the growth time of load are developed here, too.

### STATIONARY (FINAL) STATE OF STRESS ( $c_v t = \infty$ )

is given by well-known equation Fröhlich (1934)

$$y = \frac{\sigma_z}{q} = 1 - \cos^{\nu_F} \alpha = 1 - \frac{1}{(\sqrt{1 + v^2})^{\nu_F}} \quad (1)$$

where  $v = r/z$ ;  $\nu_F$  is Fröhlich's coefficient, for isotropical soil  $\nu_F = 3$ . For the medium of  $\nu_F = 4$  modulus increases with the depth according to  $E = E_0 z^b$ , for  $b=1$  is  $\nu_F = 4$ . For  $b=0$  is  $\nu_F = 3$ . Hruban (1945) gives for growing of modulus with depth  $E = E_0 \sqrt{z}$  equation  $y = 1 - \cos^3 \alpha$ .

There are steady values for various  $\nu_F$  in Fig.1 given. For transfer of heat  $y = (T - T_z) / (T_0 - T_z)$ , where  $T_0$  = temperature of half-space,  $T_z$  is temperature of foundation.

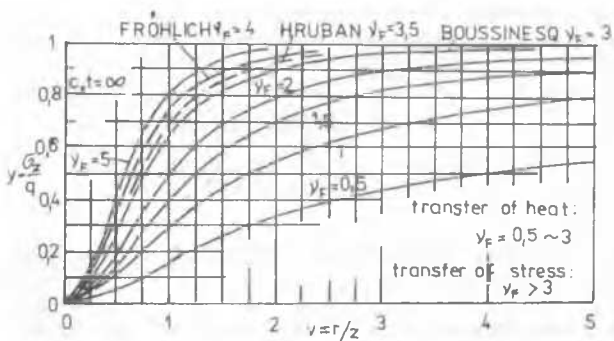


Fig.1. Steady state of stress

For UN-STEADY STRESS TRANSFER  $\infty > c_v t > 0$   
the authors developed these equations

$$\frac{\sigma_z}{q} = \exp - \frac{r}{2v\sqrt{c_v t}} - \cos^{\nu_F} \alpha \left( \exp - \frac{r}{2v\sqrt{c_v t}} \cos^{-\nu_F} \alpha \right) \quad (2)$$

$$\frac{\sigma_z}{q} = 1 - \text{tghyp} \frac{r}{2v\sqrt{c_v t}} - \cos^{\nu_F} \alpha \left( 1 - \text{tghyp} \frac{r}{2v\sqrt{c_v t}} \cos^{-\nu_F} \alpha \right) \quad (2)$$

$$\frac{\sigma_z}{q} = \text{erfc} \frac{r}{2v\sqrt{c_v t}} - \cos^{\nu_F} \alpha \left( \text{erfc} \frac{r}{2v\sqrt{c_v t}} \cos^{-\nu_F} \alpha \right) \quad (2)$$

where  $v = r/z$ ,  $c_v$  is coefficient of consolidation,  $\nu_F$  is Fröhlich's concentration factor,  $t$  is time interval,  $\text{erfc}(x) = 1 - \text{erf}(x)$  is Gauss-Laplace integral. These equations (2) were derived for foundation with radius  $r = 3$  (loaded in a computer). If  $r \geq 3$  is the time factor  $T_v = c_v t / m$ , where  $m = (3/r)^2$ . The depth of layer is assumed to be infinite and for isotropical soil  $\nu_F = 3$ . Equations (2) transform for  $c_v t = \infty$  in equation (1) for steady state.

The curves of the functions  $\text{tghyp}(x)$ ,  $\exp(-x)$  and  $\text{erfc}(x)$  are drawn in Fig.2.

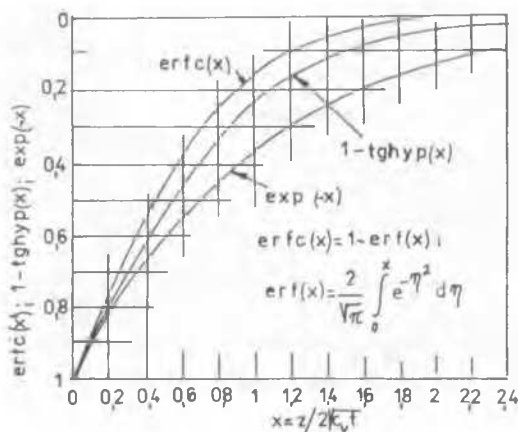


Fig.2. Functions of stress transfer

Fig.3. shows unsteady transfer of stress for  $\nu_r=3$  and for the function  $\exp(-x)$ (dashed lines) and the curves for un-steady stress transfer for function  $1-\text{tghyp}(x)$ (full lines). There are the curves for error function  $\text{erfc}(x)$  here too (dashed and dotted lines). The upper curves are for steady state ( $c_v t = \infty$ ). The chart refers to radius  $r = 3$ . If  $r_0 \geq 3$ , then we multiply  $c_v$  by coefficient  $m=(3/r_0)^2$ .

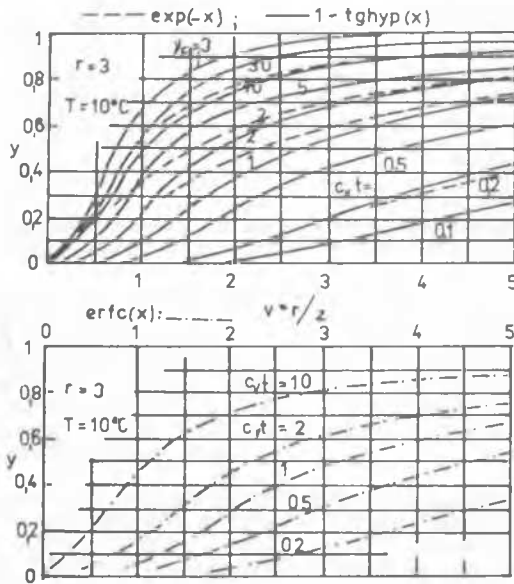


Fig.3. Un-steady stress transfer

#### DEGREE OF CONSOLIDATION U (%)

is given by the areas of the certain product  $c_v t$  of un-steady state to the area of steady state  $c_v t = \infty$ , see Fig. 4. So we obtain Fig.4. for various functions for the counting of consolidation. On its horizontal axis there is the time factor  $T_v = c_v t / r^2$  versus the degree of consolidation U, on vertical axis. Then the time settlement is equal

$$s_t = U \cdot s_{t=\infty} \quad (3)$$

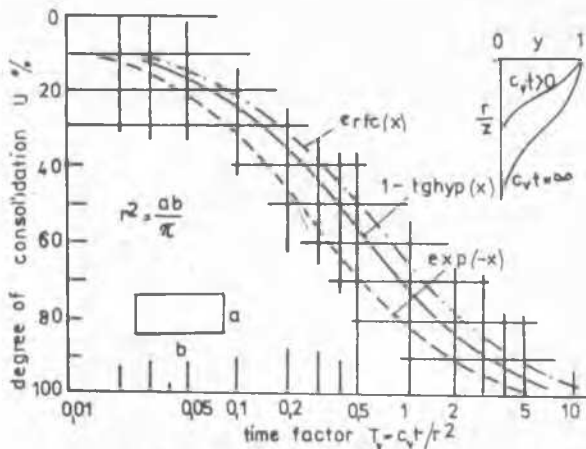


Fig.4. Chart of rate of consolidation

On the basis of given theory of un-steady stress transfer it is possible to prepare diagram enabling the consolidation for different conditions, Fig.5. On its horizontal axis there is the ratio of a:b of different rectangular footings. The curves show the time factor  $T_v = c_v t / r^2$  and on vertical axis there is degree of consolidation U. The depth of clayey subsoil is endless,  $h = \infty$ . For rectangular footing (the sides a,b) is equivalent radius  $r = \sqrt{ab/\pi}$  valid.

Fig.5. presents the curves of consolidation for various changes of moduli

$$E = E_0 \cdot z^b \quad (4)$$

where the exponent b is equal  $\pm 0,5$   $\pm 1$  or  $\pm 2$ . For the change of modulus by Hruban (1945)

$$E = E_0 \cos \alpha \quad (5)$$

the dashed curve H. is valid.

For the change of modulus according to  $E = E_0 \sqrt{z}$  by Hruban is  $\nu_r = 3,5$  valid.

#### INFLUENCE OF THE DIFFERENT PERMEABILITY

In case of different horizontal permeability  $k_h \neq k_v$ , or different moduli  $E_v \neq E_h$  these curves are given, Fig.5. :

curve 1 is for  $k_h = k_v$  or for  $E_v = E_h$ ,  
curve a is for  $k_h = 1,5 k_v$  or  $E_v = 1,5 E_h$ ,  
curve b is for  $k_h = 2 k_v$  or  $E_v = 2 E_h$  = curve 7,  
curve c is for  $k_h = 5 k_v$  or  $E_v = 5 E_h$ ,  
curve d is for  $k_h = 10 k_v$  or  $E_v = 10 E_h$ .

The different permeability  $k_h \neq k_v$  is caused due to the geological origin of clayey sediments.

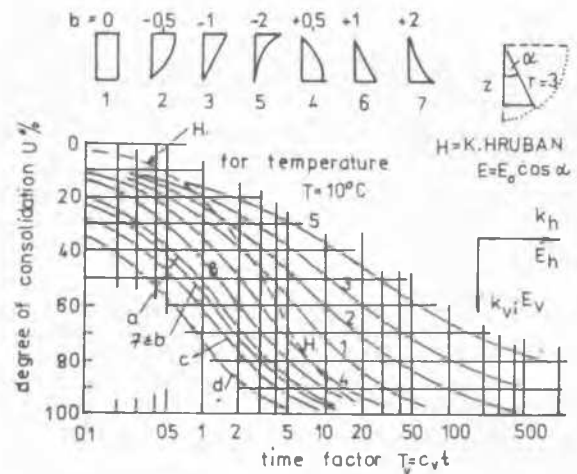


Fig.5. Chart of consolidation for different permeability and moduli of soil

For these cases we can show Fig.6. where are the ratios between the different values of Poisson's ratio  $\nu = 1/m$ , Poisson's constant m and coefficient  $\nu_r = m+1$ . There is so-called Hruban's solution  $m = b + 1$ ; it is valid for  $\nu_r = b + 2$  and Ohde's solution  $m = b + 2$ ;  $\nu_r = b + 3$ . On vertical axis there are exponents b of  $E = E_0 z^b$  and ratio  $n = E_h/E_v$ . The coefficient at rest

$$K_r = \frac{1}{m-1} = \frac{\nu}{1-\nu}$$

According to O.Fröhlich  $\gamma_F = 4$  for  $b = 1$ .

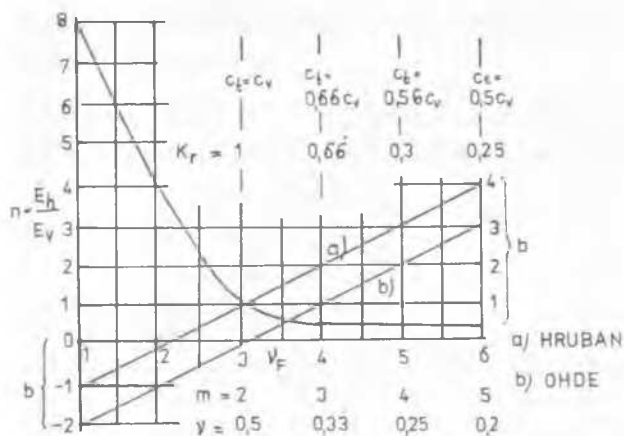


Fig.6. Effect of different parameters

Fig.7. shows the EFFECT OF THE GROWTH TIME OF LOAD on the consolidation. This chart was derived for the function of stress transfer  $1-tghyp(x)$  and for different time factor  $T_s$

$$T_s = c_v t_s \quad (6)$$

where  $t_s$  is time building of construction, see Fig.7.

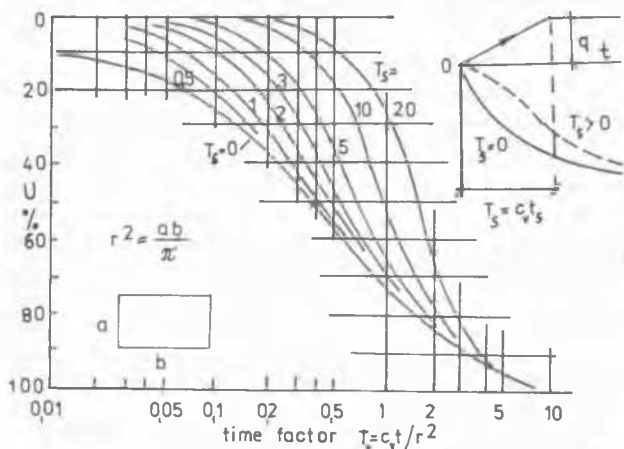


Fig.7. Influence of increasing of pressure on the consolidation

#### CONSOLIDATION OF PERVIOUS AND IMPERVIOUS LAYER

When under the foundation there is a layer of clays and on the top of it the layer is pervious and on the bottom it is pervious too, the Fig.8. is valid. This figure is valid for different ratios of rectangular foundations (on the horizontal axis).

The layers shown in Fig.9. are of transversal permeability when  $k_h$  is not equal  $k_v$ . This figure is valid for ratio  $h:a = 2$ .

Fig.10. shows the influence of sand-gravel cushion on the consolidation of clays under them. The curves are, for  $a:b = 1:1$  to  $1:10$  and for ratio  $h:a = 0,5$  and  $1$ .

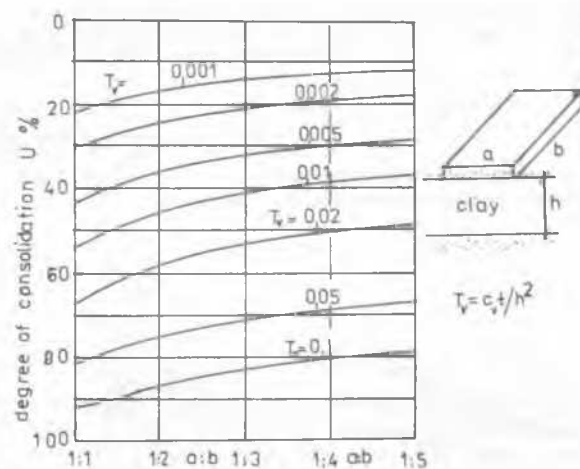


Fig.8. Consolidation of clayey layer

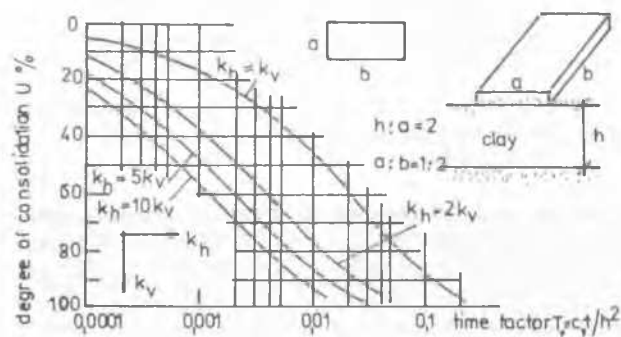


Fig.9. Consolidation of clayey layer of transversal permeability

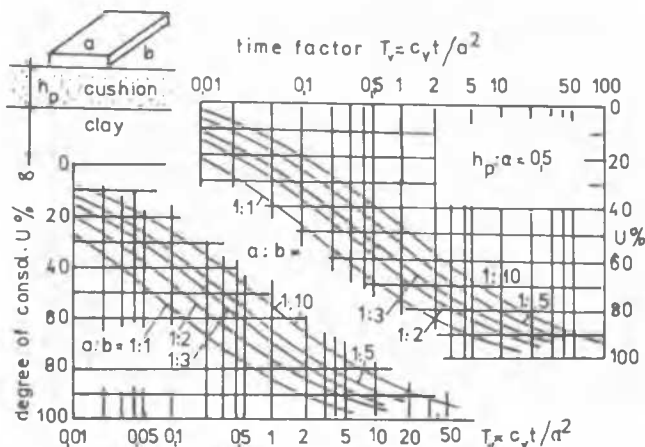


Fig.10. Consolidation of clay under sand-gravel cushion

Fig.11 shows so-called un-steady bearing capacity. We can see that the coefficient of bearing capacity (e.g.  $N_y$ ) of clays increases with time. For non linear strength we can use

$$\tau = c \left( 1 + \frac{U \sigma \operatorname{tg} \varphi}{c} \right)^\lambda \quad (7)$$

e.g.  $\lambda = 0,8$ ; for linear strength  $\lambda = 1$  is valid; (for function of un-steady transfer of stress  $\exp(-x)$ ).

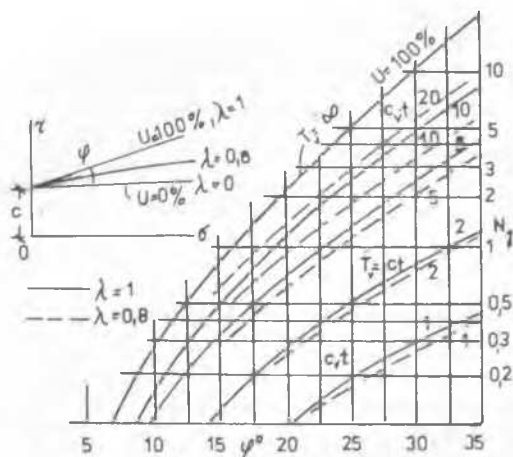


Fig.11. Un-steady coefficient of bearing capacity of clays  $N_y$

On the basis of THE COMBINATION OF STRESS AND HEAT TRANSFER we can develop the so-called thermoconsolidation of clays under constructions which are sources of heat (e.g. tunnel kilns, brickyards, chimneys etc.). The time behaviour of stress is also affected by temperature, since it influences the viscosity of pore water and consequently the rate of water filtration. Then the temperature also influences the consolidation of clays. The time factor  $T$  and coefficient of permeability  $k$  are in following relation to viscosity  $\eta$

$$k_{T^{\circ}\text{C}} = \frac{k_{10^{\circ}\text{C}} \cdot \eta_{10^{\circ}\text{C}}}{\eta_{T^{\circ}\text{C}}} \frac{\gamma_{10^{\circ}\text{C}}}{\gamma_{T^{\circ}\text{C}}} \quad (8)$$

If the temperature of soil is equal to the contact temperature  $T_c$ , we can speak of isothermic consolidation (Fig.12a). If  $T_c = 70^{\circ}\text{C} > T_s = 10^{\circ}\text{C}$  (temperature of subsoil) we get the real thermoconsolidation. When we use for the transfer of stress e.g. the function  $1 - \operatorname{tghyp}(x)$  and for the heat transfer  $\operatorname{erfc}(x)$ , we obtain so-called hybrid thermoconsolidation. As an example we can see Fig. 12b. In the case of the heat transfer the coefficient of heat conductivity grows with the depth  $a = a_0 z$  (full lines). The modulus of soil is constant  $E = E_0$ . If the stress transfer the moduli growing with the depth non-linear  $E = E_0 \sqrt{z}$  are drawn with dashed lines. The coefficient of heat conductivity, horizontal and vertical, are different  $a_h : a_v = 2 : 1$  and  $5 : 1$ .

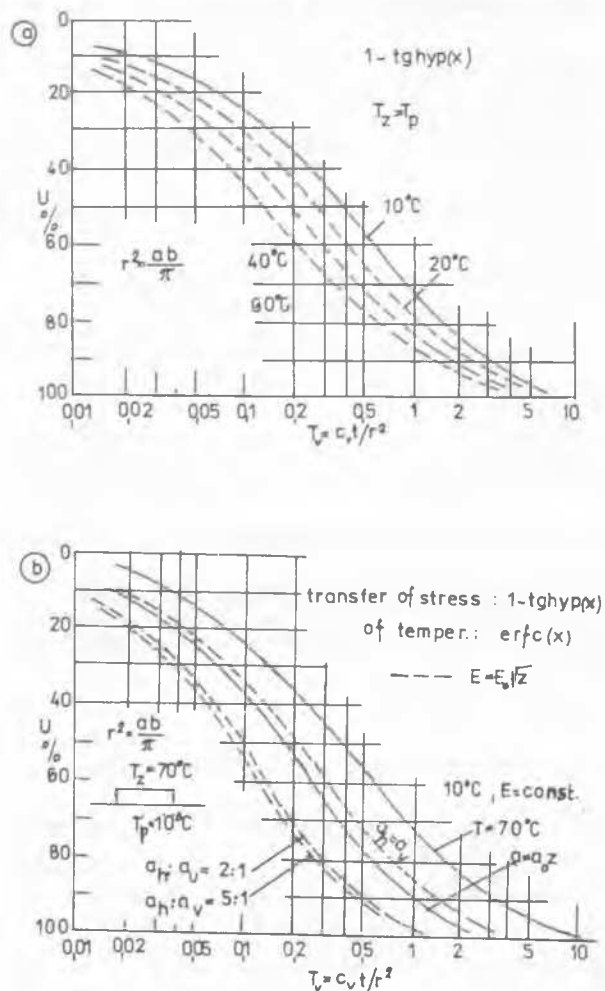


Fig.12. Thermoconsolidation of clays

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