STRESS-INDEPENDENT PARAMETERS FOR PRIMARY AND SECONDARY COMPRESSION

PARAMETRES DE LA CONSOLIDATION PRIMAIRE ET SECONDAIRE INDEPENDANTS DE L'ETAT DE CONTRAINTE

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SYNOPSIS: Mesri and Godlewski (1977) have demonstrated that there is a more or less constant relationship between the parameters C_c and C_α in both the overconsolidated and virgin compression range. This holds true for widely different soils. However, they also demonstrate that both parameters are strongly dependent upon stress, especially in soft soils, with C_c and C_α decreasing with increasing stress in the virgin compression range. The wide acceptance of these parameters has led to their use even in cases where it is evidently cumbersome to do so. Thus, a secant compression index C_c' which depends in a complex manner on yield stress, design stress and void ratio, is sometimes resorted to, e.g. when dealing with the sensitive clays of Eastern Canada.

It would clearly be of advantage to describe at least the virgin compression behaviour with single parameters for primary and secondary compression, covering the whole stress range past the yield stress. This paper presents such parameters. Further, a stress - strain - creep-strain-rate constitutive model based on these parameters is outlined.

THE NATURAL COMPRESSION INDEX b

Den Haan (1992) has shown that the virgin compression of a wide range of soils is adequately formulated by (see Fig. 1)

$$v = v_1 (p - p_s)^{-b} \tag{1}$$

which relates the specific volume v (the ratio of total soil volume to solid volume, $v=1+e$) to vertical effective stress p. The power curve expressed by Eq. 1 has a vertical asymptote at $p=p_s$. v_1 is a reference value of specific volume at stress p_s+1 kPa, and b is a dimensionless positive number. The value of p_s is usually near to zero for remoulded clays or clays of which the structure is non-brittle, granular soils, and peats. Eq. 1 then simplifies to

$$v = v_1 \, p^{-b} \tag{2}$$

which can be linearized by writing

$$\ln v = \ln v_1 - b \ln p \tag{3}$$

Natural (Hencky) strain is defined by

$$\varepsilon^H = -\int_{v_o}^{v} \frac{dv}{v} = -\ln(v/v_o) \tag{4}$$

where v_o is the reference value of specific volume from which strain is measured. It differs from the usual linear (Cauchy) strain $\varepsilon^C=-\Delta v/v_o$ in that strain is measured incrementally with respect to the momentary dimension v rather than differentially with respect to the initial dimension v_o. Both strain measures are related through $\varepsilon^H = -\ln(1-\varepsilon^C)$.

Combining Eqs. 3 and 4 results in

$$\varepsilon^H = \ln(v_o/v_1) + b \ln p \tag{5}$$

so that natural strain is linearly dependent on logarithm of stress. Natural strain has been used in the past by Juárez-Badillo, Butterfield and Lefebvre, and its power in linearizing the virgin compression curve has been shown time and again. The parameter b defines the slope of the virgin compression curve, and is termed the *natural compression index*, given by

$$b = \frac{\Delta \varepsilon^H}{\Delta \ln p} = \frac{-\Delta \ln v}{\Delta \ln p} \tag{6}$$

ε^H is linearly related to $\ln v$. Both measures of strain will be used throughout this paper.

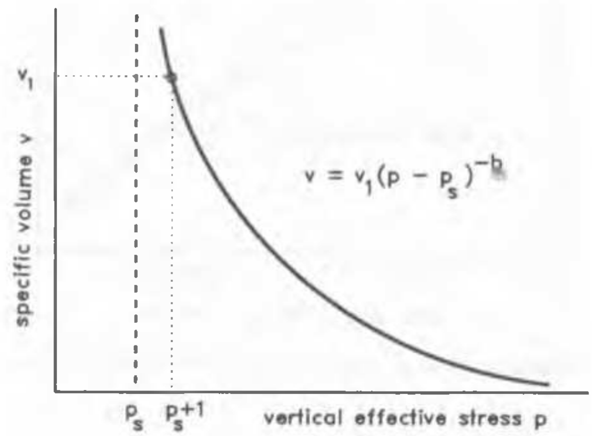

Fig. 1. Generalized power formulation of virgin compression

THE STRUCTURE PARAMETER p_s

The parameter p_s in Eq.1 allows a greatly improved fit to the virgin compression curve of structured, brittle clays which experience breakdown behaviour past the yield stress. Examples of these are the sensitive clays of Eastern Canada, the leached clays of Norway and possibly bonded residual soils. Fig. 2 shows that Eq. 1 can fit the virgin part of a compression curve of sensitive Ottawa Leda clay extremely well, using v_1=4.24, b=0.108, p_s=197 kPa. It would be impossible to characterize this curve with a single value of C_c. Fig. 3 illustrates how this curve is linearized by depicting natural strain against log(p-p_s) rather than against log$p$.

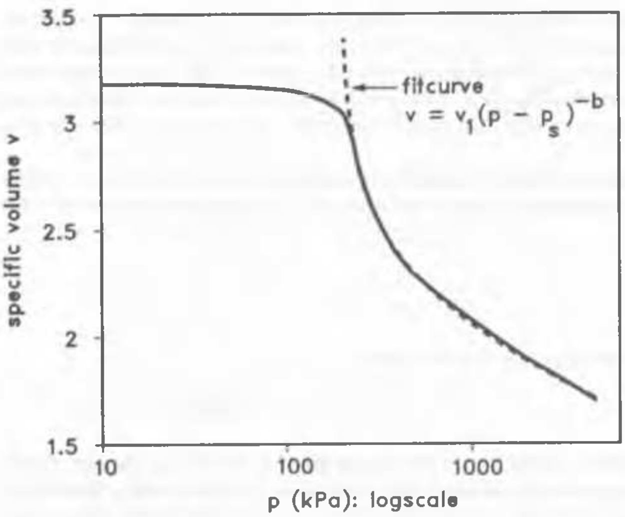

Fig. 2. Curve-fitting of virgin compression of Ottawa Leda clay

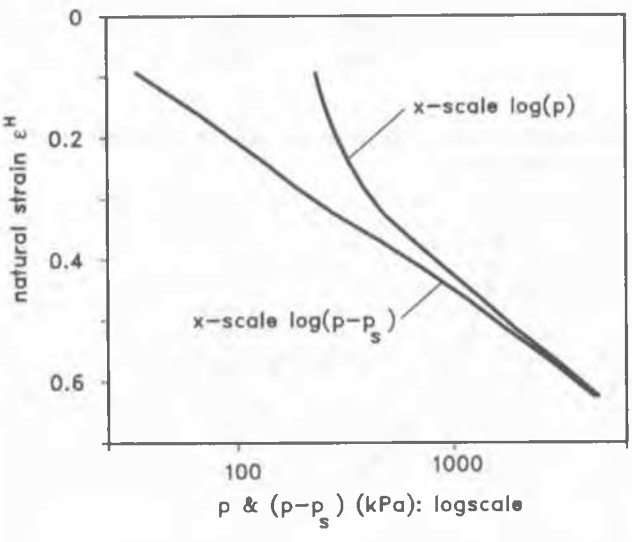

Fig. 3. Linearization of virgin compression curve of Ottawa Leda clay

p_s is a measure of the degree of brittle structure which may result from bonding by cementation or from leaching. As stress increases past the yield value, brittle structure is destroyed in a process called destructuration, and the influence of p_s diminishes. The asymptote at p_s is situated slightly to the left of the yield stress. With increasing destructuration, the asymptote migrates towards the stress origin, but this is a refinement not accounted for in Eq. 1. This could be done by formulating

$$v = v_1[p - (p_s/p)^n p_s]^{-b} \qquad (7)$$

where n is a dimensionless positive number. The asymptote is now at $(p_s/p)^n p_s$, which decreases as p increases. The parameter n expresses how quickly destructuration takes place. A high value of n would correspond to a sensitive clay susceptible to destructuration. However, if only results of stage-loaded oedometer tests are available, it would be difficult to determine n accurately. Better results could be expected from constant-rate-of-strain tests, because of the high resolution with which the stress-strain curve is obtained.

A clay can also exhibit non-brittle structure, and p_s is then zero. This structure can be destroyed by remoulding, and the remoulded curve is lower than the natural curve. Den Haan (1992) dwells more fully on the indications for the existence of brittle and non-brittle structure.
It is remarked that the soft organic clays of Holland, while certainly possessing structure, usually do not need p_s in their formulation. Apparently, the structure of these clays is non-brittle.

STRESS-DEPENDENCY OF C_c

In the manner of Mesri and Godlewski (1977), Fig. 2 can be depicted as C_c against log$p$. Given the excellent fit to the virgin part, C_c can be taken from it by derivation:

$$C_c = \frac{-d\,v}{d\log p} = 2.3bv\frac{p}{p - p_s} \qquad (8)$$

The result is shown in Fig. 4, and as in Mesri and Godlewski, C_c decreases with increasing stress in the virgin compression range. This decrease is in effect formulated by the parameters p_s and b.

THE CONSTANCY OF C_α/C_c

Consensus exists that $C_\alpha/C_c \approx 0.04 \pm 0.01$ for most soils, irrespective of past history and stress state. However, the relationship given by Mesri and Godlewski for sensitive Canadian Leda clay between C_α and C_c is curved, yielding higher extreme values of C_c or lower extreme values of C_α than would follow from this rule. This can be explained if it is assumed that in the presence of a structure parameter p_s, C_α is proportional not so much to C_c as to C_c^* defined as

$$C_c^* = \frac{-d\,v}{d\log(p - p_s)} = 2.3bv = \frac{p - p_s}{p}C_c \qquad (9)$$

That is, the same mechanisms as with non-brittle structure occur, but now at stresses relative to p_s. The value p-p_s can be regarded as a reduced stress, for which the relationship to strain is the same as for absolute stress in a non-brittle soil. Again, the position of the asymptote could be assumed to vary as destructuration increases, thus too influencing the relationship between C_α and C_c.

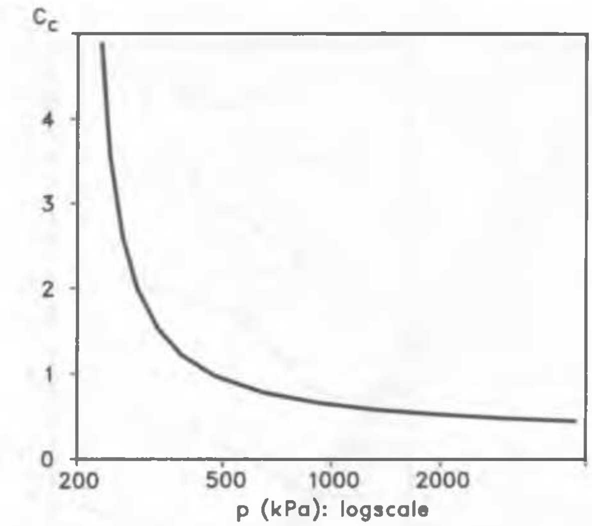

Fig. 4. Stress-dependency of C_c of Ottawa Leda clay

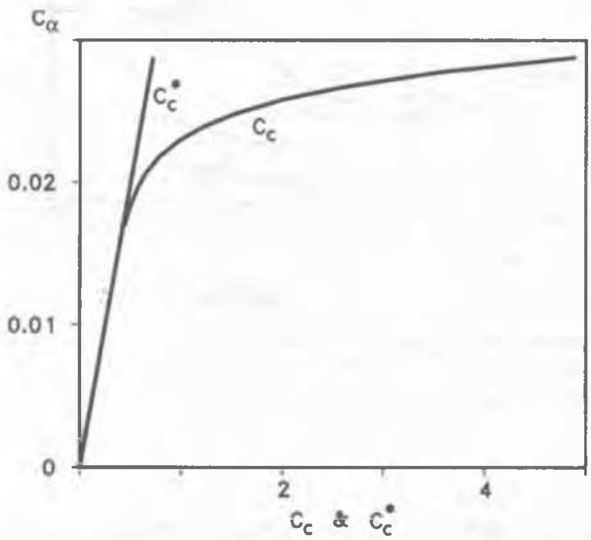

Fig. 5. C_α/C_c and C_α/C_c^* relationships of Ottawa Leda clay

Assuming $C_\alpha/C_c^* = 0.04$, the resulting C_c^* - C_α and C_c - C_α relationships are shown in Fig. 5 for the virgin range of the Ottawa Leda clay. The concave trend of the C_c - C_α relationship is the same as is to be observed in Mesri and Godlewski. Possibly therefore, the C_α/C_c rule should be replaced by a C_α/C_c^* rule. However, more research is needed.

SECONDARY COMPRESSION OR SECULAR COMPRESSION?

The origins of the term secondary compression most probably lie in North America in the 1930's. Gray and Keverling Buisman in the 1st ICSMFE (1936) both refer to the term as being in widespread use there. Although it has always been realized that the secondary phenomenon takes effect from the beginning of loading, there has been a tendency to deal with it separately from primary compression. The established terminology did not stimulate other directions of thought. Keverling Buisman made a slightly different distinction, namely between direct effects and secular effects (saeculum: a long row of years). The term secular effect expresses the intrinsic time-dependency of the soil behaviour, and should therefore be preferred above secondary compression. The primary phenomenon is more correctly expressed by the term hydrodynamic consolidation. However, it is recognized that the terms primary and secondary compression are firmly entrenched in soil mechanics, and therefore this terminology will also be followed here.

SECONDARY COMPRESSION OF SOFT SOILS

Fig. 6a shows settlement-time curves of a Dutch peat on ε^C - log*t* scales. The peat is a well humified wood peat from Zegveld Polder near Woerden, and was taken from 2.50m-G.L. Its bulk density was 10.6 kN/m^3, its loss-on-ignition 48.2% and its specific gravity was 1.825. Initial voids ratio was 6.76; initial watercontent was 370%.
The final slope of the curves in Fig. 6a is proportional to C_α. As stress increases, C_α first increases and then decreases. Although the load increment ratio is constant in the virgin range, the curves are not equidistant as would follow from constant C_c. Fig. 6b shows the same curves on ε^H - log*t* scales. The final slopes are now all practically parallel, indicating that this slope is a suitable secondary compression parameter, and the curves are equidistant, indicating that the natural compression index *b* is indeed constant. Fig. 6c shows the same curves on ε^H - log$\dot\varepsilon^H$ scales ($\dot\varepsilon^H$=dε^H/dt). It is now seen that the final slopes cover a larger part of the curve, allowing a more accurate determination of their magnitude.
The slopes in Figs. 6b and 6c are equal and given by the *natural secondary compression index c*. A stress-strain-strainrate constitutive model for secondary compression can be defined starting from

$$\varepsilon^H - \varepsilon^H_o = c\ln\dot\varepsilon^H_o - c\ln\dot\varepsilon^H \tag{10}$$

which states that strains are linearly related to logarithm of strainrate. The subscript *o* denotes a state from which strains are to be measured.

Rewriting:

$$\exp\left(\frac{\varepsilon^H - \varepsilon^H_o}{c}\right) = \frac{\dot\varepsilon^H_o}{\dot\varepsilon^H} \tag{11}$$

Integrating:

$$\int_{\varepsilon^H_o}^{\varepsilon^H}\exp\left(\frac{\varepsilon^H - \varepsilon^H_o}{c}\right)d\varepsilon^H = \int_{t_o}^{t}\dot\varepsilon^H_o\,dt \tag{12}$$

$$c\left[\exp\left(\frac{\varepsilon^H - \varepsilon^H_o}{c}\right) - 1\right] = \dot\varepsilon^H_o(t-t_o) \tag{13}$$

$$\exp\left(\frac{\varepsilon^H - \varepsilon^H_o}{c}\right) = \frac{\dot\varepsilon^H_o}{c}(t-t_o) + 1 = \frac{t-t_r}{t_o-t_r} \tag{14}$$

if $t_r = t_o - c/\dot\varepsilon^H_o$. Then

$$\varepsilon^H - \varepsilon^H_o = c\ln\left(\frac{t-t_r}{t_o-t_r}\right) = c\ln\frac{\tau}{\tau_o} \tag{15}$$

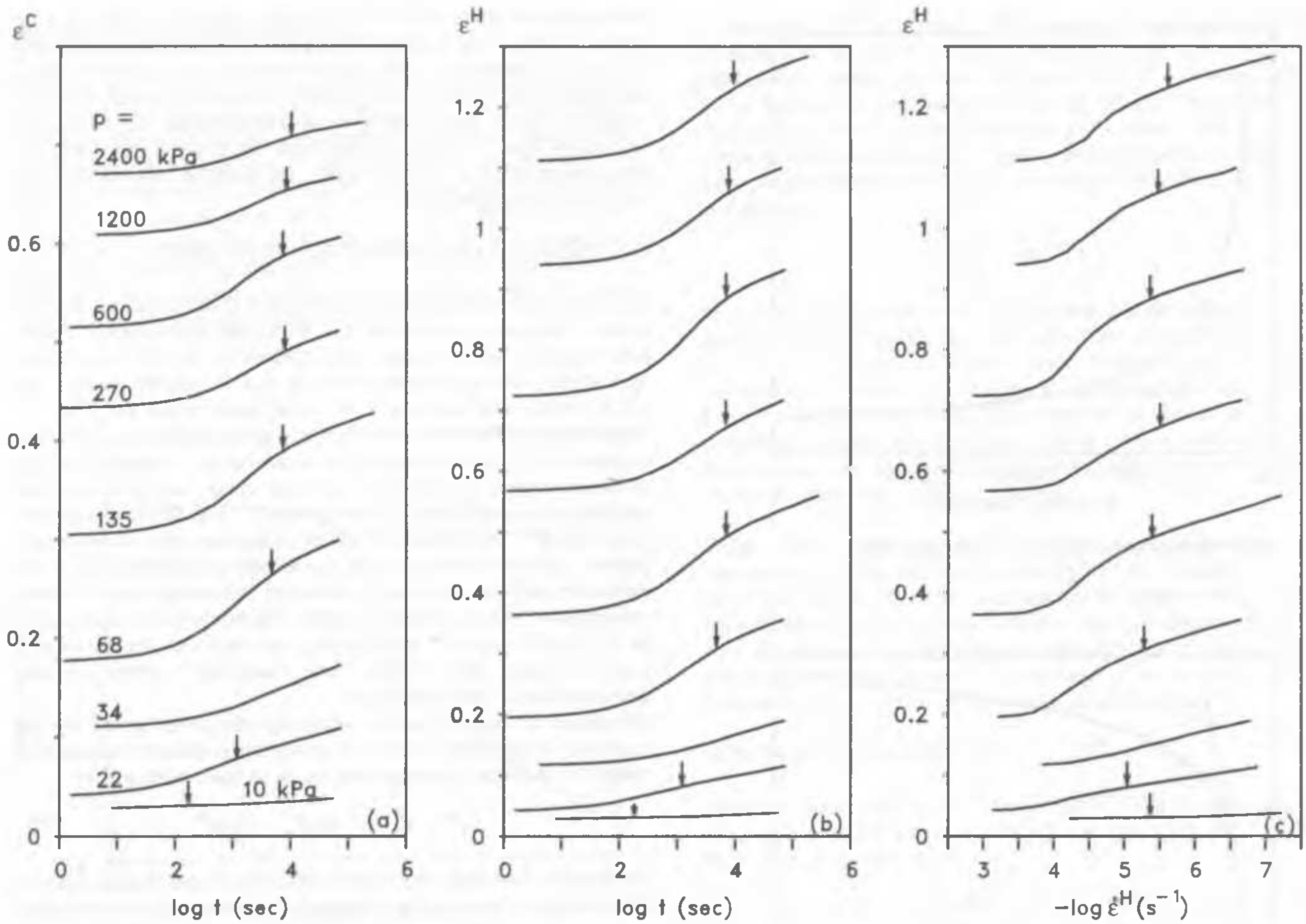

Fig. 6. Time - settlement curves of an oedometer test on Dutch peat on various scales. The arrows signify 99.4% (T=2) consolidation as determined from Taylor's √t method.

where

$$\tau = t - t_r \tag{16}$$

Alternatively,

$$-\ln\frac{v}{v_o} = c\ln\frac{\tau}{\tau_o} \quad or \quad \frac{v}{v_o} = \left(\frac{\tau}{\tau_o}\right)^{-c} \tag{17}$$

Eq. 15 indeed demonstrates that the slopes c in Figs. 6b and 6c are equal.

INTRINSIC TIME

In Eq. 15, the time scale needs to be altered by a quantity t_r, which is called by De Rijk (1978) the "reference time shift". The difference between "test time" t (measured from the beginning of the loading) and reference time shift t_r is given the name *intrinsic time* and is denoted by the symbol τ.

The relationship between natural strain and $\log\tau$ is the constant c, whatever the reference frame of time may be. The time t_o will often represent the primary period, and t_r is then its difference with the intrinsic time τ_o at the beginning of secondary compression.

De Rijk (1978) and Janbu (1969) also arrive at Eq. 14, but here the introduction of natural strain and intrinsic time is new.

THE STRESS - STRAIN - CREEPRATE RELATIONSHIP

The curves of Fig. 6c can be replotted as in Fig. 7. It now appears that the straight, parallel and equidistant tails of Fig. 6c again yield straight, parallel and equidistant lines in Fig. 7. The slope now however is not c but b. In the secondary compression range at creeprates lower than about $10^{-5.5}$ s^{-1} these lines are equidistant at distances controlled by c. This system of lines is formulated by (p_s=0 in these soils)

$$\varepsilon^H = \ln\left(\frac{v_o}{v_1}\right) + b\ln p + c\ln\left(\frac{\tau}{\tau_o}\right) \tag{18}$$

where τ_o now corresponds to the base line through v_1. Combining with Eq. 4:

$$\frac{v}{v_1} = p^{-b}\left(\frac{\tau}{\tau_o}\right)^{-c} \tag{19}$$

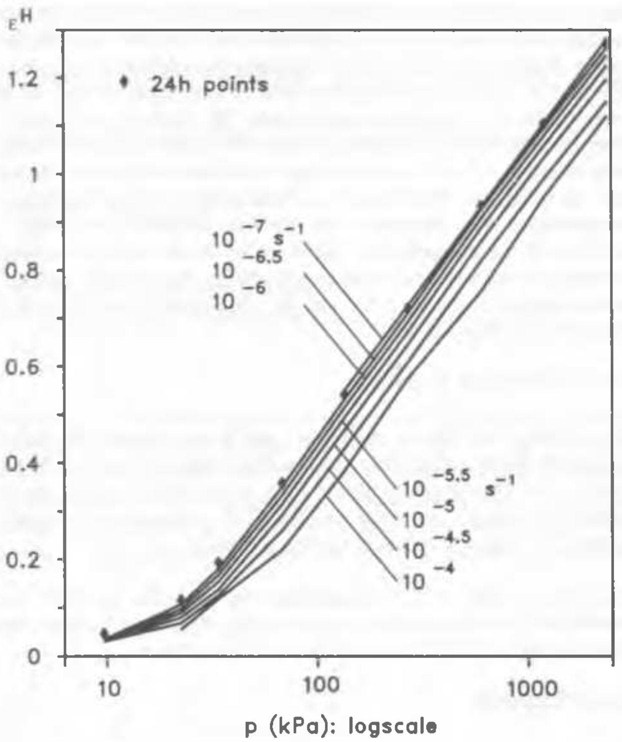

Fig. 7. Stress - strain - creep-strain-rate relationship for Dutch peat

This in effect describes a virgin stress - strain - creep-strain-rate constitutive relationship for the given material. It has proven useful for soft organic clays as well as for peat.

Fig. 8 illustrates the principles of this constitutive relationship. The creep isotaches determine the creeprate depending on stress and strain. On each isotache the creeprate is constant. Each isotache is associated with one value of intrinsic time through

$$\frac{d\varepsilon^H}{dt} = \frac{-d\ln(v/v_o)}{dt} = -\frac{\dot{v}}{v} = \frac{c}{\tau} \qquad (20)$$

The isotaches have slope b. During primary compression from A to B, stress increases and creeprate first increases and then decreases. To the creeprate in this stage must be added the rate of primary deformation due to stress increase. Primary compression rates are not included in the derivations given above.

After B, stress remains practically constant, and deformation follows a creep isobar at slope c. In terms of intrinsic time, a linear relationship (BC) between ε^H and $\log\tau$ emerges, as given by Eq. 15. However, in terms of test time t, the secondary compression curve does not have to be straight. It could be either as B'C or as B"C, depending on whether the primary period t_o is longer, resp. shorter than the intrinsic time τ_o at B. Eventually, at large time, the importance of the time shift diminishes and the slope tends to c in both cases.

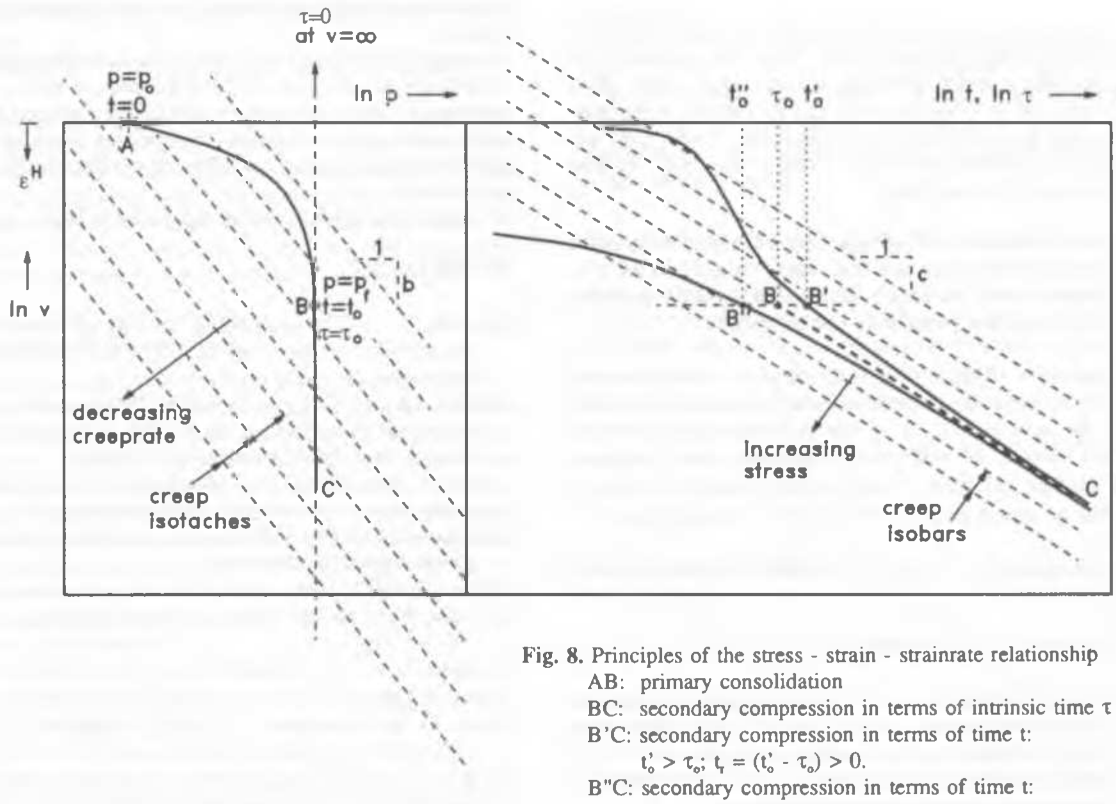

Fig. 8. Principles of the stress - strain - strainrate relationship
AB: primary consolidation
BC: secondary compression in terms of intrinsic time τ
B'C: secondary compression in terms of time t:
$t'_o > \tau_o$; $t_r = (t'_o - \tau_o) > 0$.
B"C: secondary compression in terms of time t:
$t''_o < \tau_o$; $t_r = (t''_o - \tau_o) < 0$.

B'C is representative of large load increments or short previous load increment duration. B"C often occurs with small load increments, and is more pronounced in overconsolidated states. The curves B'C and B"C are reminiscent of the curve types I and III resp. defined by Leonards and Altschaeffl (1964). It now appears that both curve types are fundamentally determined by the same creep law.
Type III curves and curve B"C in Fig. 8 both steepen up to the end of the loading stage. This should be kept in mind when dealing with so-called tertiary compression, where the strain-log$t$ curve also steepens up to the end of the stage. However, tertiary effects due to structural changes after prolonged periods cannot be ruled out, and are not described by the equations presented above.

Creep isobars are straight in terms of ln$v$ versus lnτ. They can be extended backwards to infinite volume and zero intrinsic time. Intrinsic time can be thought of as the time necessary to achieve the present state of volume and stress, if the soil had been loaded immediately by this stress when it was still in its initial, unsedimented state.

RELATIONS WITH EARLIER WORK

The present model incorporates idea's from past work, and it is of interest to indicate the links. The links with De Rijk and with Janbu's time resistance model have already been mentioned. Bjerrum's (1967) stress - strain - time conceptual model, an extension of D.W. Taylor's work, does not distinguish clearly between intrinsic time and time since loading. The present model might be characterized as a stress - strain - intrinsic time model, and the time lines of Taylor and Bjerrum become *intrinsic* time lines (creep isotaches) here.

The creep isobars and isotaches are not limited by instant compression lines as in Bjerrum's model, but cover the full v - p space. This is in accordance with Christie and Tonks (1985). The creep isolines form a background pattern which determine creep rate at any given combination of v and p. This pattern is not dependent on the time since loading, and this is different from Christie and Tonks. They use $t+t_L$, in which t_L is in effect an intrinsic value of time and t is time since loading, where τ is used here.

Szavits-Nossan (1988) refers to "intrinsic time behaviour" in the same sense as is done here, and uses two time scales t and t', where t' is identical to intrinsic time. However, she does not actually introduce the term as an independent measure of soil behaviour.

"Instant" compression (Bjerrum) due to an increase in effective stress occurs at rates dictated by hydrodynamic conditions, and displaces the v,p state to the right in Fig. 8a. If this is described by a natural recompression index a, the same set of parameters as in Garlanger (1972) and Christie and Tonks (1985) is used, namely a, b and c. However, their equations are in terms of e while v is used here.

None of the existing stress - strain - time models incorporate natural strain as is done here.

DETERMINATION OF PARAMETERS b AND c

In Fig. 7 the points corresponding to 24 hours of compression after the beginning of each load increment, are also shown. These points almost coincide with the intrinsic time line having a creep rate of $10^{-6.7}$ s^{-1}. Time lines being parallel, it will therefore often be acceptable to determine the parameter b from a standard 24h curve.

By varying t_r to optimize the regression of the secondary tails in Fig. 6b according to Eq. 15, c can be obtained without resorting to rate plots. t_r varies from 45 to 90 minutes for the Zegveld Polder test. This is sufficiently small to allow determination of c directly from the final slopes of the log$t$ plots. However, primary consolidation (t_p) requires 100 to 150 minutes, and the time shift directly after primary to the intrinsic time τ is therefore considerable. The danger exists that c is overestimated by including the portion of the secondary curve directly after primary. A third method to obtain c is from the slope of the rate plots as in Fig. 6c. This figure was obtained from frequent readings of compression, and regression was used to determine the slope of sections of the compression - time curve. It becomes increasingly difficult to obtain accurate rates as soil stiffens. However, for soft soils under moderate loads, rates can be determined well enough if sufficient readings are taken.

THE CONSTANCY OF c/b

It is evident from Figs. 6 and 7 that c and b are constant. It is logical to inquire whether their ratio has the same value as C_α/C_c. If time is large, $t = \tau$. Then it can be shown $C_\alpha = 2.3cv$ and $C_c = 2.3bv$, so that indeed both ratio's are equal. As the C_α/C_c = constant rule applies generally, c/b = constant also applies generally.

The peat in Fig. 6 is characterized by b=0.256, c=0.0154 and c/b=0.060. This latter value is in the range mentioned by Mesri and Godlewski for C_α/C_c of amorphous and fibrous peats.

CONCLUSION

The constants b and c are a simple alternative for stress-dependent C_c and C_α. Extra formulations of the stress-dependency are not required when using b and c. In soils which possess brittle structure due to e.g. cementation bonding or leaching, an extra structure parameter, p_s is required.
A simple but effective stress - strain - creeprate model (Eq. 19) can be constructed on the basis of the parameters b and c. This model is adequate for the virgin compression of soft clays and peats. Together with a parameter a for "instant" compression, it can be combined with the finite strain theory of consolidation to compute field settlements of soft soils.
It should be possible to extend the model to brittle, sensitive clays.

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