SOIL DAMPING COMPUTED WITH RAMBERG-OSGOOD-MASING MODEL

AMORTISSEMENT DES SOLS PAR LE MODELE RAMBERG-OSGOOD-MASING

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SYNOPSIS : The validity of the Ramberg-Osgood-Masing (R-O-M) model for computing material damping of soils was evaluated at small to intermediate shearing strains (0.0001% to 0.1%) by comparing computed behavior with measurements in the cyclic torsional shear and resonant column tests. The Ramberg-Osgood parameters were determined for dry sand samples and several cohesive soils. The computed hysteresis loops were compared with measured loops at selected strain amplitudes. Material damping ratios calculated with hysteresis loops from the R-O-M model were also compared with measured values at small to intermediate strains. At small strains below the elastic threshold, the R-O-M model exhibits no damping while all soils exhibit some material damping. At higher strains, the R-O-M model correctly estimates the damping of dry sand on the first cycle of loading but overestimates material damping at larger numbers of cycles and for all cohesive soils tested.

INTRODUCTION

During cyclic loading, the stress-strain behavior of soils is nonlinear and, even at small shearing strains (less than 0.001%), exhibits hysteresis. To develop a constitutive stress-strain relationship under cyclic loading, hysteresis loops are often constructed using a backbone curve described by Ramberg-Osgood (R-O) parameters coupled with an assumption of Masing behavior (Richart and Wylie, 1977; Idriss et al., 1978; and Saada, 1985). The R-O parameters generally fit backbone curves and modulus-strain data quite well at shearing strains less than 0.1%. One of the potential advantages of using a Ramberg-Osgood-Masing (R-O-M) model is that once the R-O parameters are determined, material damping ratio can be computed by assuming Masing behavior as described below.

To evaluate the R-O-M model, R-O parameters were determined for dry sand samples and several cohesive soils using a fixed-free torsional resonant column (RC) device with which torsional shear (TS) tests could also be performed (Kim, 1991). Both TS (at a frequency of 0.5 Hz) and RC tests were performed in a sequential series on the same specimen at small to intermediate strains (0.0001% to 0.1%). Hysteresis loops were computed using the R-O-M criteria. These hysteresis loops were then compared with the measured loops at selected strain amplitudes. Material damping ratios calculated with the hysteresis loops from the R-O-M model were also compared with measured values at small to intermediate strains. One important aspect in this work was that the measured values were compared with the R-O-M model only after the measurements had been corrected for equipment damping (Kim,1991).

The dry sand samples were constructed using the pluviation method. The sand was a medium to fine grained sand which classified as SP in the Unified Soil Classification System. Six samples were used, with void ratios ranging from 0.6 to 0.76 and relative densities ranging from 29% to 87%. The undisturbed cohesive soils were extruded from thin-walled sample tubes and were hand-trimmed to their final dimensions. Nine cohesive soils were tested which ranged in plasticity index from 7% to 52% in natural water contents for 20% to 50%, and in soil classification from ML to CH. All soil samples were typicall) 5.1 cm (2.0 in.) in diameter and about 10.2 cm (4.0 in.) in height at the start of testing and were tested mostly at isotropic confining pressures ranging from 20 to 138 kPa (3 to 20 psi). Further details on the test specimens, methods of sample preparation and test equipment are presented by Kim (1991).

RAMBERG-OSGOOD-MASING (R-O-M) MODEL

An idealized stress-strain loop obtained for a soil specimen subjected to a symmetrical cyclic shearing stresses of $\pm \tau_c$ is shown in Fig. 1. The corresponding shearing strains in a closed hysteresis loop are $\pm \gamma_c$. The curve ACODB, corresponding to the locus of the tips of all possible hysteresis loops, is defined as the backbone curve for the soil specimen. One form of the Ramberg-Osgood stress-strain equation for the backbone curve can be written as:

$$\gamma = (\tau / G_{max}) + C(\tau / G_{max})^R \quad (1)$$

where γ = shearing strain, τ = shearing stress, G_{max} = initial shear modulus, C = dimensionless coefficient, and R = dimensionless exponent.

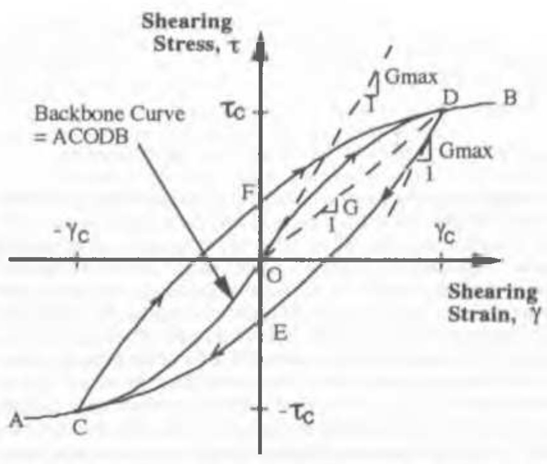

Fig.1 Backbone Curve and Associated Hysteresis Loop

The portion of the backbone curve which is determined with the torsional shear test is shown by ODB in Fig. 1. This portion of the curve is referred to as the initial backbone curve hereafter because it represents the first quarter cycle of TS loading.

The variation of shear modulus with shearing strain can be obtained from both RC and TS tests. By dividing shear modulus by the maximum shear modulus (G_{max}) determined for the particular test, the variation in normalized shear modulus with strain amplitude can be determined. To fit this test data with the Ramberg-Osgood equation, the backbone curve (Eq. 1) is rewritten as:

$$\gamma = G' \bullet \gamma + C\,(G' \bullet \gamma)^{R} \qquad (2)$$

where $G' = G / G_{max}$ = normalized shear modulus.

The most widely accepted assumption made to construct analytical hysteresis loops from the backbone curve is the Masing (1926) criteria. Masing suggested two criteria: i) the shear modulus at each load reversal has a value equal to the initial tangent modulus of the backbone curve (Fig. 1) , and ii) the shape of unloading and reloading curves of the loop are the same as that of the backbone curve with both stress and strain scales expanded by a factor of two and the origin translated to the reversal point. The following expression is then used to construct the unloading and reloading branches of the hysteresis loop:

$$\gamma \pm \gamma_c = \frac{(\tau \pm \tau_c)}{G_{max}} + \frac{C}{2^{R-1}} \left| \frac{\tau \pm \tau_c}{G_{max}} \right|^{R} \qquad (3)$$

Using least-squares curve fitting of either the initial backbone curve or the normalized stiffness versus strain curve determined by either RC or TS tests, the Ramberg-Osgood parameters, C and R, can be determined. Once the R-O parameters are determined, the analytical hysteresis loop can be constructed using Eq. 3.

Hysteretic damping ratio, D, is expressed as :

$$D = \Delta W / (4 \pi W_e) \qquad (4)$$

where ΔW is the amount of energy dissipated per one complete load cycle (i.e. area of the hysteresis loop) and W_e is the peak strain energy in a given load cycle. Jennings (1964) expressed the energy dissipation per load cycle using the R-O-M model as:

$$\Delta W = 4 \frac{C}{{G_{max}}^{R}} \left(\frac{R-1}{R+1}\right) \tau^{R+1} \qquad (5)$$

By substituting Eq. 5 into Eq. 4, material damping ratio can be written using the R-O-M model as:

$$D = \frac{2\,(R-1)}{\pi\,(R+1)} (1 - G') \qquad (6)$$

EVALUATION OF R-O-M MODEL ON DRY SAND

Comparison of a hysteresis loop calculated with the R-O-M model with a hysteresis loop measured for the second cycle of TS testing at a strain amplitude of 0.001% for dry sand is presented in Fig. 2. This behavior is typical of all the sand samples. The R-O parameters obtained from the backbone curve measured during the initial quarter-cycle of loading were used to calculate the hysteresis loop. The calculated stress-strain behavior is nearly linear at these small strains which results in the computed area of loop being almost zero. (In fact, $D_{computed}$=0.1%.) However, the measured hysteresis loop shows some energy dissipation even at very small strains ($D_{measured}$=0.8%). Therefore, it can be seen that the nearly linear stress-strain behavior calculated with the R-O-M model is misleading and underestimates material damping at small strains.

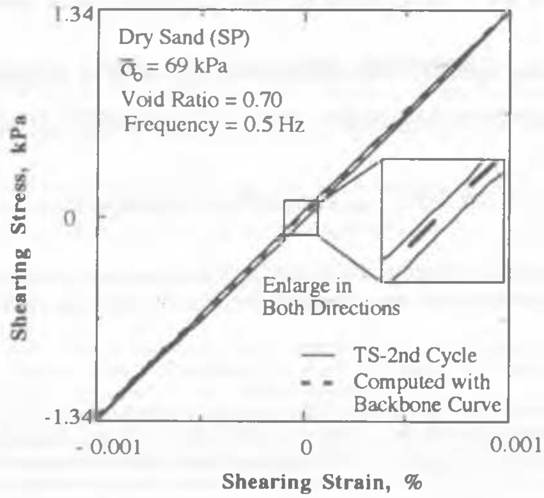

Fig. 2 Typical Hysteresis Loops for Sand at Small Strains

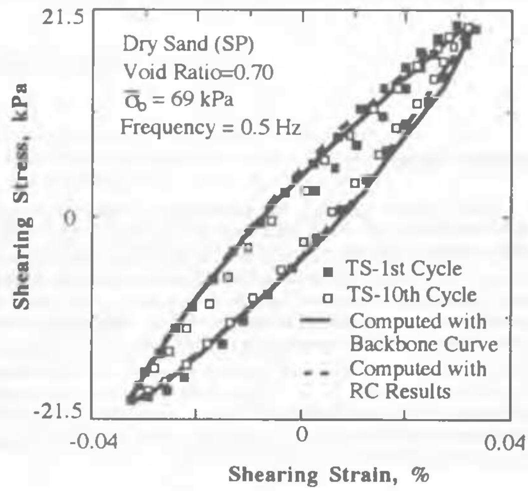

Fig. 3 Typical Hysteresis Loops for Sand at Intermediate Strains

Typical hysteresis loops measured for the first and tenth cycles of TS testing at a peak strain amplitude of 0.032% are presented in Fig. 3. The area enclosed in the first-cycle hysteresis loop (D=10.6%) is bigger than the area of the tenth cycle (D=7.5%) because of cyclic hardening of dry sand. The R-O parameters were calculated two ways i) from the initial loading backbone curve, and ii) from fitting the normalized shear modulus curve from the RC test. Both computed hysteresis loops ($D_{backbone}$ = 9.1% and D_{RC} = 9.7%) are also plotted in Fig. 3. In the RC test, about 1000 loading cycles are applied during the modulus measurement. As seen in the figure, both computed hysteresis loops more closely match the measured loop for the first cycle than the tenth cycle because the area of the loop measured on the tenth cycle is smaller than the computed loops. Therefore, for dry sand, the hysteresis loop of the first loading cycle at intermediate strains (10^{-3}% to 10^{-1}%) is more closely computed with the R-O-M model than the area of loops at larger numbers of cycles because the model does not exhibit the decrease in the area of the hysteresis loop with increasing number of loading cycles. This is true even if the R-O parameters are determined from the normalized shear modulus curve at an equivalent number of loading cycles.

Once the R-O parameters have been determined from the backbone curve or the normalized shear modulus curve, material damping ratio can be calculated using Eq. 6. Damping ratios calculated with the R-O-M model from the initial loading backbone curve and the RC test are plotted together and compared with measured damping ratios in Fig. 4. Obviously, at strains below 0.001%, calculated damping ratios do not match with experimental data. This is typical of all sands. Calculated damping ratios at these strain levels are zero according to the Masing criteria. However, the experimental data clearly show that material damping exists even at very small strains (Kim, 1991). This small-strain damping ratio, which is essentially constant below an elastic threshold strain of about 0.001%, is denoted as D_{min} herein.

To account for small-strain damping, the damping ratio computed by the R-O-M model was modified by adding the measured value of D_{min} to it. Computed damping ratios with this modification are presented in Fig. 5. It is very interesting to see that all modified computed damping ratios match well with damping ratios measured for the first cycle of loading in the TS test. However, these modified damping ratios overestimate measured values for the tenth cycle of TS loading and in the RC test.

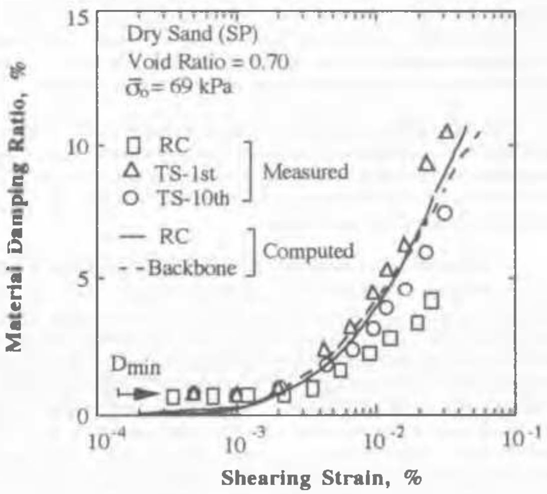

Fig. 4 Typical Damping Ratios of Sand Using R-O-M Model

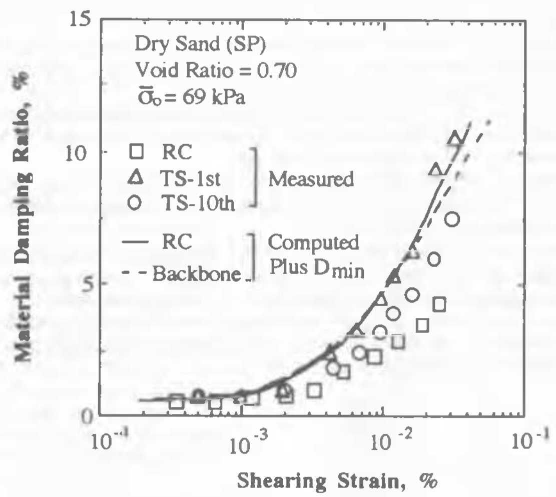

Fig. 5 Typical Damping Ratios of Sand Using R-O-M Model Plus D_{min}

EVALUATION OF R-O-M MODEL ON COHESIVE SOILS

A typical comparison between computed and measured hysteresis loops for a cohesive soil at small strains of 0.004% is presented in Fig. 6 . (The elastic threshold strain is about 0.008% in this case.) The computed loop was constructed using R-O parameters determined from the initial loading backbone curve. At this strain level, the behavior computed by the R-O-M model is nearly linear, and the area of the loop is almost zero. However, the measured loop shows a reasonable amount of energy dissipation (D=2.0%). By comparing this behavior with that of dry sand (Fig. 2), the discrepancy between measured and computed loops at small strains is more severe for cohesive soil. At small strains, nearly linear stress-strain behavior obtained by the R-O-M model can produce a significant error in predicting the cyclic stress-strain behavior of cohesive soils by underestimating the area of the loop.

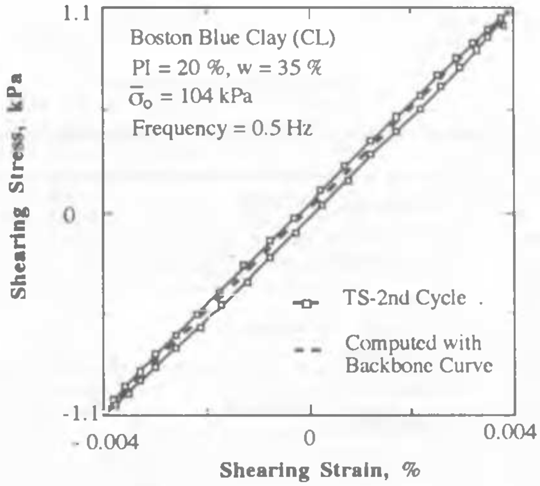

Fig. 6 Typical Hysteresis Loops for Cohesive Soils at Small Strains

Typical stress-strain hysteresis loops measured for cohesive soils for the first and tenth cycles of TS testing at a peak strain amplitude of 0.08% are presented in Fig. 7. In the measured loops, the slope of the tenth cycle is flatter than the slope of first cycle because of cyclic degradation. The area of the two measured loops does not, however, change much with number of cycles (D=8.2% for first cycle and D=7.9% for tenth cycle). The computed loops were constructed using R-O parameters obtained from the backbone curve and the normalized shear modulus curve from RC tests. The slope of the loop computed by the backbone curve is similar to the slope measured for the first cycle. The slope of the loop computed using the RC results is slightly larger than the slope of the tenth cycle measured in TS testing (because of the frequency effect). However, the areas of the computed loops are significantly larger than the measured loops. Therefore, it can be seen that for cohesive soil, the cyclic stress-strain relationship computed by the R-O-M model overestimates the area of the hysteresis loops measured at intermediate strains.

Typical damping ratios of cohesive soils computed by the R-O parameters determined from the backbone curve and the RC test are plotted together and compared with measured values in Fig. 8. Damping ratios computed by the R-O-M model are quite different from those measured in the RC and TS tests over the complete strain range. Obviously, at shearing strains below the elastic threshold (about 0.007%), computed damping ratios are almost zero and do not even come close to the experimental values which, for cohesive soils, can be quite significant. The difference between D_{min} measured in the TS and RC tests occurs from the different testing frequencies (0.5 Hz and 44 Hz, respectively). The R-O-M model can not compute this small-strain damping ratio (D_{min}). At higher strains, computed damping ratio increases rapidly with increasing strain amplitude, and, above a certain strain level, computed damping ratios exceed the measured ones.

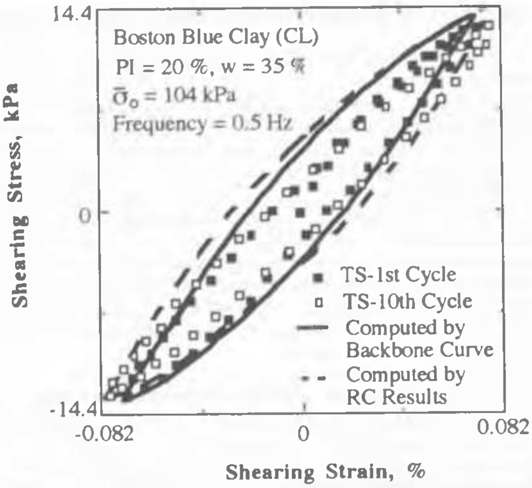

Fig. 7 Hysteresis Loops for Cohesive Soils at Intermediate Strains

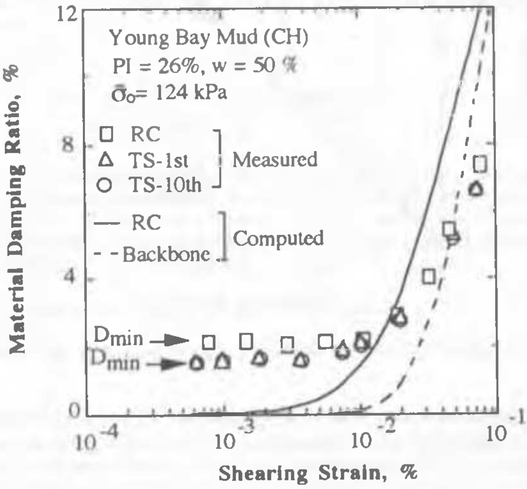

Fig. 8 Typical Damping Ratios of Cohesive Soils Using R-O-M Model

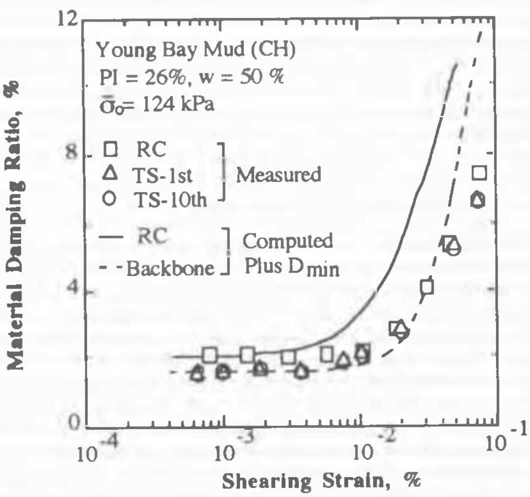

Fig. 9 Damping Ratios of Cohesive Soils Using R-O-M Model Plus D_{min}

To account for small-strain damping (D_{min}), computed damping ratios for each the TS and RC tests were modified by adding the respective value measured for D_{min}. These results are plotted in Fig. 9. With this modification, the R-O-M model can compute damping ratios below the elastic threshold strain. However, above the elastic threshold strain, the modified computed damping ratios are higher than the measured ones, with this difference increasing with increasing strain. Therefore, a reduction in the computed damping ratio is needed if one is to match measured damping ratios in this strain range. The amount of reduction varies from soil to soil. With the modification of adding D_{min} to the R-O-M model measured damping values of nine undisturbed cohesive soils averaged from 70% to 50% of the predicted values at shearing strains ranging from 0.01% to 0.1%, respectively (Kim, 1991). These results indicate that more investigation of material damping is needed and a better predictive model should be developed.

CONCLUSION

Use of the Ramberg-Osgood-Masing model for computing material damping of soils was evaluated by comparing predicted damping with measurements in the cyclic torsional shear and resonant column tests. The Ramberg-Osgood equation generally fits the measured backbone curve or modulus-strain curve quite well at small to intermediate strains. However, use of Masing behavior to construct hysteresis loops is questionable, especially for cohesive soils.

At small strains below the elastic threshold, the R-O-M model can not be used to compute the small-strain damping ratio (D_{min}) which exists in all soils. Damping ratios computed by the R-O-M model need to be modified by adding measured values of D_{min} to obtain a reasonable fit in this region.

At higher strains, computed damping ratios modified by adding D_{min} generally overestimate measured values. For dry sand, predicted damping ratios match quite well with the damping ratio measured for the first cycle in the TS test. However, the R-O-M model does not predict the decrease in damping ratio with increasing number of cycles, even though the R-O parameters are determined at an equivalent number of cycles. For undisturbed cohesive soils, computed damping ratios modified with D_{min} are much higher than measured ones, with a reduction on the order of 50% to 70% needed for the modified R-O-M damping ratios to match measured ones.

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