MECHANISM FOR BRITTLE AND DUCTILE BEHAVIOR OF CEMENTED SANDS

MECANISMES POUR LE COMPORTEMENT AIGRE ET DUCTILE DU SABLE CIMÊNTE

Ching S. Chang[1] *M. Golam Kabir*[2]

[1]University of Massachusetts, Amherst, Massachusetts, U.S.A.
[2]Converse Consultants East, Parsippany, New Jersey, U.S.A.

SYNOPSIS: Cementation plays an important role in the stress-strain behavior of both natural and artificial cemented sands which are often encountered in geotechnical engineering practice. The role of cement in sands is to create bonds at inter-particle contacts which, observed from experimental results, have a significant influence on the mechanical behavior of cemented sands. However, very few analytical models for cemented sands can be found in the literature. This paper presents a stress-strain model that considers the particulate nature of sands and the effect of cement bonding. Mechanisms of load carrying through resistance at inter-particle contacts are considered in the analytical model. The strength behavior of ductile and brittle types is discussed.

INTRODUCTION

Cementation plays an important role in the stress-strain behavior of both natural and artificial cemented sands which are often encountered in geotechnical engineering practice. Naturally occurring cemented sands are formed generally by small amount of cementing agents, such as silica, hydrous silicates, hydrous iron oxides, and carbonates deposited at the contacts between sand grains. Artificially cemented sands are produced mostly for the purpose of soil stabilization, employing admixtures of Portland cement, lime, fly ash and bituminous materials.

The role of cement in sand is to create bonds at the inter-particle contacts which significantly influence the mechanical behavior of cemented sands. These materials are characterized as 'contact bound' material. A number of experimental results are available in the literature on the moduli, damping ratio and the strength for cemented sands (Clough et al., 1981; Acar & El-Tahir, 1986; Chang & Woods, 1987; Saxena & Lastrico, 1978; Dupas & Pecker, 1979). However, very few analytical models are available in literature. There is a gap between the knowledge accumulated from experimental studies and the ability to understand the behavior of cemented sands through analytical modeling.

Owing to the particulate nature, sands carry the applied load through resistance at inter-particle contacts. To account for this load carrying mechanism in the stress-strain modeling, it is necessary to consider the contact behavior between particles. Along this line, constitutive relationship for random packings of spheres have been studied by Chang (1988); Chang et al. (1992b,c). The approach has been found to give reasonable agreement with the experimental measurements for both uncemented and cemented sands at small strain level (Chang et al., 1989; Chang et al., 1990). The approach has been recently extended to account for particle separation and sliding (Chang et al., 1992a,b,c). In this paper, the recent theory is applied to evaluate the ductile and brittle behavior of cemented sand at large strain.

PACKING STRUCTURE

Packing structure for sands is very complicated. From micromechanics derivation (Chang et al., 1989), a parameter is found to be useful to define the packing structure of granular material, given by

$$\xi = \frac{4\pi(1+e)}{\bar{n}} \tag{1}$$

where e is the void ratio, $\bar{n}$ is the coordination number, defined as the average number of contacts per particle. The value of ξ ranges from 2 for dense to 6 for loose samples. The relationship between void ratio and coordination number has been studied for assemblies of lead shot (Smith et al., 1929), glass balls (Oda, 1977), rockfill material (Marsal, 1973), and gravel (Yanagisawa, 1983). These experimentally observed relationships are shown in Fig. 1. Using a micromechanics model (Velez, 1992), the backcalculated relationship based on experimental results on shear moduli of sands by Hardin (1972) is given in Fig. 1.

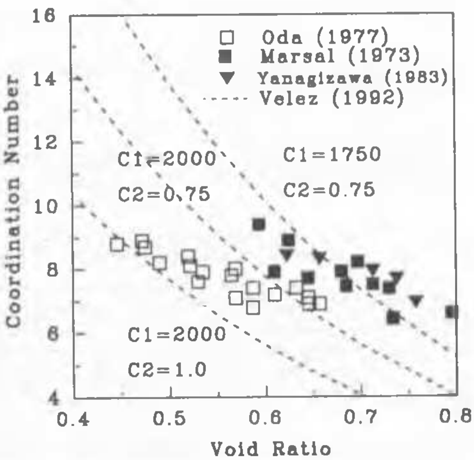

Fig. 1. Relationship between coordination number and void ratio

CONTACT BEHAVIOR

The normal stiffness of two idealized topographically smooth non-conforming elastic bodies in contact was studied by Hertz (Johnson, 1985b). For natural sands, we express the normal stiffness in the following form

$$k_n = c_1 r P_a \left(\frac{f_n}{P_a r^2}\right)^{\alpha} \tag{2}$$

where r is the mean particle radius; f_n is the contact normal force; P_a is the atmospheric pressure; c_1 and α are two dimensionless constants. Typical values of α and c_1 for sands are given in Table.1. As $\alpha = 1/3$ and

$$C_1 = 3^{\alpha}\left(\frac{G}{(1-\nu)P_a}\right)^{2\alpha} \tag{3}$$

Eq. 2 reduces to the ideal Hertzian contact.

Table 1. Typical values of α and c_1 .

Sand Type	α	c_1
Ottawa Sand	0.3-0.42	1600-2060
Reid Bedford Sand	0.29	1500
Hostun Sand	0.33	1300
Atlanta Sand	0.48	918

The tangential contact stiffness was studied by Mindlin and Deresiewicz (1953). Partial slip develops at the contact as tangential force is applied and sliding occurs when the tangential force exceeds the frictional strength. The relationship for tangential stiffness is given by

$$k_r = c_2 k_n \left(1 - \frac{f_r}{f_n \tan\phi_\mu}\right)^{\alpha} \tag{4}$$

where c_2 is a dimensionless constant; ϕ_μ is the inter-particle friction angle; and f_r is the resultant tangential force at contact. The parameter c_2 for sands varies from 0.5 to 1.0. The value of ϕ_μ varies from 14 to 22 degree.

Cement, added to sands, coats the particle to form bonds at the particle contacts. The bond area between two particles depends on the amount of cement at the contact. The area of bond will affect the strength at the contact. The bonds may be broken in the cement or at the cement-particle interface. The tensile force required to separate two bonded particles depends on the tensile strength of cement, s_t and the area of bond, A, given by

$$f_{yn} = s_t A \tag{5}$$

Similarly, the tangential force required to break the bond depends on the shear strength of cement, s_r , given by

$$f_{yr} = s_r A \tag{6}$$

The tensile strength of Portland cement is approximately 2-4 N/mm^2. The shear strength of cement is typically 4 to 7 times of tensile strength.

The bond area between two particles increases with the degree of cementation. At low degree of cementation, the bond area is relatively small as the cement coats the particle to form weak bonds at the particle contacts. With addition of cement, the bond area increases and the bonds become stronger at inter-particle contacts. At higher degree of cementation, the additional cement merely fill up the void space without contributing much to the area of bond, thus has insignificant effect on bonding strength (Chang et al., 1990).

The strength envelope of the contact is shown in Fig. 2. The normal strength of a bonded contact is controlled by the tensile bonding strength f_{yn}. The shear strength is controlled by the shear bonding strength f_{yr} at low confining pressure (i.e., $f_n < f_{nc}$ at the contact) whereas it is

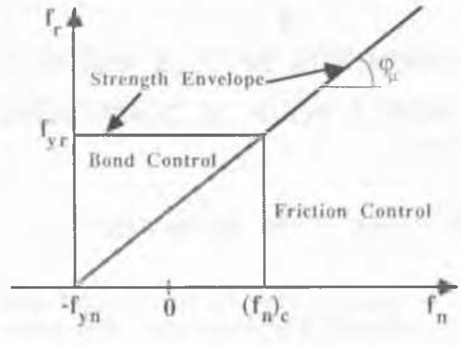

Fig. 2. Strength envelope of an inter-particle contact.

controlled by the friction at high confining pressure, as shown in Fig. 2. However, after a bond is broken, the strength due to bond is lost. Thereafter, the strength is governed by Coulomb's friction law. Therefore, at low confining pressure, after the cement bond is broken by shearing, the shear force that can be carried at the contact is reduced to the friction capacity, and a large portion of the shear force will be released upon breakage of bond.

Normal and shear stiffness of a contact are also affected by the cementation. Effect of adhesive force on the contact normal stiffness was derived by Johnson (1985a) and has been employed by Chang et al. (1990) in the study of cemented sand. A simplified form of the stiffness considering the adhesive force can be given by

$$k_n = c_1 r P_a \left(\frac{f_n^*}{P_a r^2}\right)^{\alpha} \tag{7}$$

$$k_s = c_2 k_n \left(\frac{f_r}{f_n^* \tan\phi_\mu}\right)^{\alpha} \tag{8}$$

where $f_n^* = f_n + f_{yn}$

CONSTITUTIVE MODEL

A micromechanics based constitutive relationship has been derived for random aggregates of particles with frictional contact (Chang, 1988; Chang et al., 1992). The model accounts for the particulate nature of sands that carry the applied load through resistance at inter-particle contacts. The approach has been found to give reasonable agreement with the experimental measurements for both uncemented and cemented sands at small strain level (Chang et al., 1989; Chang et al., 1990).

The approach has been recently extended to account for particle separation and sliding (Chang et al., 1992a,b,c). In this paper, the strength of bonds due to cementation are considered in the model for analysis of large strain. Parameters required for this model are given in Table 2. The values in the last column of Table 2 are used for the prediction example in the next section.

EXAMPLE OF PREDICTION

Due to the limitation of space only two typical predicted stress strain curves are shown here. The predictions are made for an idealized packing of spheres with mean radius of 0.2 mm. Void ratio of the packing is 0.6 with assumed coordination number of 7, which is representative of a medium dense sand. The inter-particle friction angle is 14 degree for all particles. The area of cementation bond is assumed to be 0.005 mm^2 for

2% cementation and 0.01 mm² for 4% cementation. Two confining pressures are used to show the effects of low and high confining stress. The predicted stress-strain curves are plotted in Figs. 3 and 4. The results are discussed in the following sections.

Table 2. Parameters required for the model.

	Parameters		
Packing Geometry	mean particle size, r		0.2mm
	void ratio, e		0.6
	coordination number, n		7
Particle Property	inter-particle friction, ϕ_u		14
	particle stiffness constants	c_1	2000
		c_2	0.75
		α	0.3
Cement Property	tensile strength, s_t		4 MPa
	shear strength, s_r		20 MPa
Degree of cementation	area of bond, A		0.005 mm² for 2% cement 0.01 mm² for 4% cement

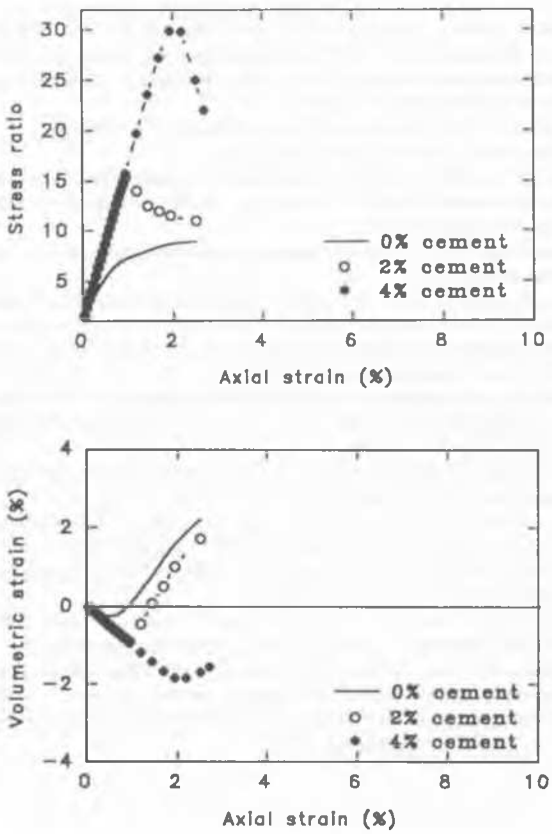

Fig. 3. Computed stress-strain curves at low confining pressure (σ_c =100 kPa)

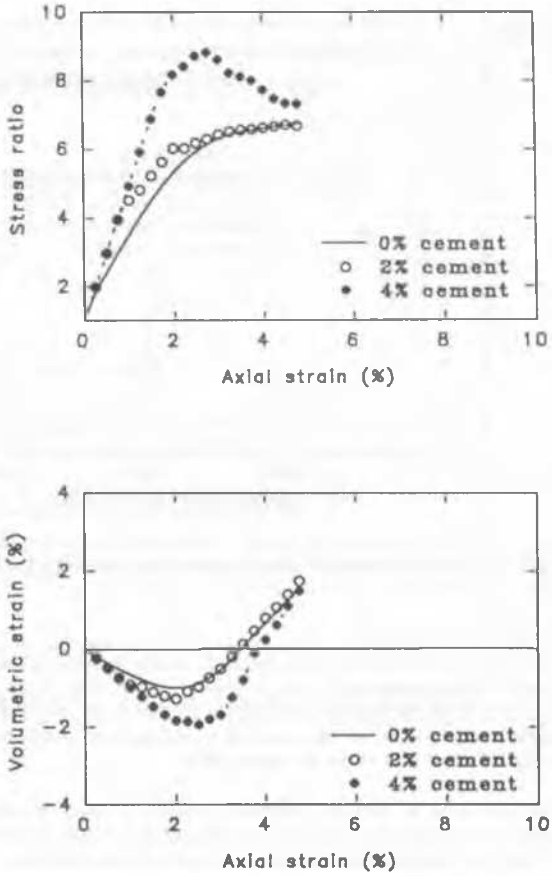

Fig. 4. Computed stress-strain curves at high confining pressure (σ_c = 400 kPa)

Peak Strength

The results show the frictional nature of cemented sands in that the strength increases with confining pressure. Also, the cemented sands show an increase of peak strength with degree of cementation. The residual strength, however, is unaffected by cementation. The reason is that at large strain, the cementation bonds have already been destroyed and friction dominates the strength.

For low confining pressure, the peak stress ratio for cemented sands is 1.6 to 3 times of uncemented sand. Whereas at high confining pressure, the peak stress ratio for cemented sands is 1 to 1.4 of uncemented sand. This behavior compares fairly well with the range of experimental results obtained from Clough et al. (1981), Lade & Overton (1989) and Ismael (1990), as shown in Fig. 5.

Brittle and Ductile Failure Modes

It is noted from the predicted results that the failure mode for weakly cemented sand is brittle at lower confining pressures and ductile at higher confining pressures. It is seen from Fig. 2 that at low confining pressures, brittle behavior is caused by the strength loss after breakage of cement bonds. In contrast, at high confining pressures, the friction dominates the strength. Therefore bond breakage does not cause a strength loss, as a result the behavior is ductile.

The average contact force can be related to the confining stress by (Chang et. al., 1989),

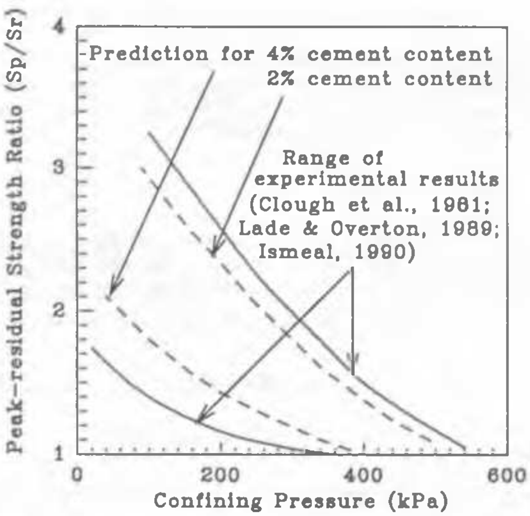

Fig. 5. Peak-residual strength ratio as a function of confining pressure.

$$f_n = \xi r^2 \sigma_c \tag{9}$$

From Fig. 2, the transition from brittle to ductile behavior at any inter-particle contact occurs at $f_n = f_{yr} \tan \phi_u$. From Eq. 9, the average contact force in the prediction example is 0.01 N for σ_c = 100 kPa, and 0.04 N for σ_c = 400 kPa. The contact force at transition is 0.025 N for 2% cementation and 0.05 N for 4% cementation.

The magnitude of confining pressure required to cause the transition depends on particle size, void ratio, cementation and friction property. For a given confining pressure, the strength behavior becomes increasingly brittle with increasing amount of cement content. This behavior agrees with the experimental observations by Clough et al. (1981) and Lade & Overton (1989).

Dilatancy

The predicted results show that as the cementation increases, the volume change due to shear stress becomes less dilative. Dilation is usually associated with particle sliding. Due to cementation, the number of sliding between particles reduces. As a result, the behavior becomes contractive. This behavior is substantiated by the experimental observations of Clough et al. (1981) and Lade & Overton (1989). On the other hand, at large strain when the cement bonds are destroyed, the dilatancy behavior is similar to the uncemented sands.

CONCLUSION

Predicted stress strain behavior for cemented sands using a micromechanics based constitutive model is presented. The model accounts for the discrete nature of particles and the properties of cement bonds at inter-particle contacts. The brittle and ductile behavior of cemented sands predicted by the model compares reasonably well with the experimental results. This is encouraging even though the predictions are calculated for idealized packings of spheres. The approach based on micromechanics with consideration of inter-particle interaction is potentially useful for modeling the stress-strain relationship of 'contact bound' material.

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