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ASSESSMENT OF CHARACTERISTIC VALUES OF SOIL PARAMETERS FOR DESIGN

DETERMINATION DE LA RESISTANCE CARACTERISTIQUE DES SOLS POUR PROJECTS

Hans Denver[1] ***N. Krebs Ovesen***[2]

[1]Sen. Res. Engineer, [2]Managing Director
Danish Geotechnical Institute, Lyngby, Denmark

SYNOPSIS: The geotechnical design process involves a number of elements: Soil parameters, loads, calculation methods and safety factors constitute the most important ones. The selection of soil parameters is today often the weak link in the design process. Engineering judgment, company traditions and personal experience enter into this selection process and thereby contribute to an unnecessary large amount of the inherited uncertainty in the geotechnical design process. Making use of Bayesian statistics the paper proposes a method for selection of soil parameters in a systematic and reproducible way. The method is described and it is demonstrated by means of an example on how to use apriori knowledge in combination with the results of field and laboratory tests to assess the characteristic value of soil properties corresponding to a specific fractile in the statistical density function of the parameter. In addition the paper treats the problems of identifying the soil influence zone and the soil material model for the geotechnical design property.

INTRODUCTION

The geotechnical design process consists normally of two main phases. During the first phase a soil investigation involving soundings, borings and field and laboratory testing is carried out to determine the soil and ground water profile including the soil parameters. During the second phase physical, numerical and mathematical modelling is employed to predict the behaviour of the structure and safety requirements are implemented.

Both phases are important as they contribute to the final design result. In the geotechnical literature there is a tendency to focus mainly on field and laboratory testing and on physical, numerical and mathematical modelling. Rarely, however, the geotechnical literature focus on the very important question of how to select soil parameters on basis of the results of field and laboratory tests for use in the geotechnical design process. The interrelation between the selection of soil parameters and the safety level introduced in the design is often disregarded too.

In the opinion of the authors the selection of soil parameters for geotechnical design purposes is consequently often the week link in the design process. A large amount of personal skills, company traditions and engineering judgment is involved in this process but the choice of soil parameters is indeed rather arbitrary. In addition experience where this selection process has failed is scarce and not systematically collected. Consequently, this process of selecting soil parameters contributes to an unnecessary large amount of the inherited uncertainty in the geotechnical design process.

LIMIT STATE DESIGN

In order to ensure an adequate, technical quality of a geotechnical structure it is required that the structure as a whole and its various parts fulfils certain fundamental requirements of stability, stiffness, etc. during construction and throughout the design life. The fundamental requirements are normally expressed in specific terms as performance criteria.

Whenever a geotechnical structure fails to satisfy one of its performance criteria it is said to have reached a limit state. According to the concept of limit state design each limit state is considered separately in the design and its occurrence is either eliminated or shown to be sufficiently improbable.

In geotechnical design it is general practice to distinguish between two main classes of limit states:

- Ultimate limit states at which:
  - either a failure mechanism is formed in the ground
  - or a failure mechanism is formed in the superstructure or severe structural damage occurs due to movements in the ground

- Serviceability limit states at which deformations in the ground will cause loss of serviceability in the superstructure.

The limit state design procedure is often used in connection with partial factors of safety. According to this format the prescribed characteristic loads are multiplied by certain partial factors to obtain design values of the loads, and the characteristic material properties are divided by other partial factors to obtain the design material properties. The design criterion then simply is to design for equilibrium in the limit state using design values of loads as well as material properties.

The limit state design concept making use of partial factors is today's preferred format for codes of practice. The new European code of practice for geotechnical design, Eurocode 7 (1993), represents an example of such a code. In the code which is part of a set of harmonized codes for structural design values are given for loads, partial factors, etc. However, one of the most difficult problems encountered in this code - and this applies to geotechnical design in general - is how to define the characteristic soil parameters on basis of apriori knowledge, results of field and laboratory testing, etc. The present paper will illustrate some of the problems involved in the selection of soil parameters for geotechnical design and propose a method for assessment of characteristic values of soil parameters.

SOIL INFLUENCE ZONE

The soil conditions may vary substantially on a construction site. It is consequently necessary to identify the specific volume of the soil which governs the behaviour of the structure in the limit state under consideration. In the following this volume is denoted the soil influence zone.

The extent of the soil influence zone depends on

• The structure, its dimensions and the limit state considered
• The loads and their combination
• The soil conditions

It is evident that the type and the size of the structure have a most significant effect on the extent of the soil influence zone. Having identified the type and size of the structure a suitable calculation model is selected corresponding to the limit state considered and the soil volume involved in this state is identified. This soil volume amounts to the soil influence zone.

The soil influence zone furthermore depends on the loads involved. For instance the shape of the failure surface below a strip footing depends on the ratio between the horizontal and the vertical components of the load.

The variation of the soil conditions with depth and place may also influence the extent of the soil influence zone. E.g. a failure surface in sand is dependent on the value of the angle of shearing resistance and a weak soil layer located beneath a firm layer may extend the influence zone to a greater depth.

It should be emphasized, however, that distinction shall be made between the soil influence zone and the extent of the volume of soil which need to be investigated by soundings and borings as part of an appropriate soil investigation prior to the design.

SOIL MATERIAL MODEL

Before discussing the principles for selecting a characteristic value for a soil parameter a soil material model valid for the soil influence zone shall be established. In this context the most important step is to identify the different soil strata which constitute the soil influence zone. This is due to the fact that completely different classes of soil parameters are assigned to different soil types.

A soil material model consists of a description of the variation of one or more parameters within the soil influence zone both in terms of mean values and variances. Otherwise, it will not be possible to derive a characteristic value defined by a specified fractile as demonstrated in the following section.

An advanced soil material model of this kind (denoted type I) can be derived if a large number of measured values of the soil parameter is available. Such a model consists of a complete description of the spatial parameter variation - i.e. $\theta = \theta\ (x,\ y,\ z)$. Reference may be made to the concept of stochastic interpolation where the parameter is assigned the values determined in the measuring points. In other points the parameter varies in a systematic pattern converging to the known values in the measuring points. An important feature is that this method allows for an evaluation of the reliability of the parameter for each point in statistical terms.

A simpler type of soil material model (type II) is based on the assumption that the spatial variation of the soil parameter is solely a function of the depth - i.e. $\theta = \theta\ (z)$. This assumption is convenient in connection with horizontal soil layers or for soil properties dependent on the vertical stress level. Vertical profiling resulting from borehole tests or penetration tests are excellent means to obtain data for a soil model of this kind.

Another type of soil material model (type III) includes no information on the spatial variation of the parameters. It simply consists of the local bulk values of the parameter θ expressed in statistical terms as a probability density function or described as a mean value and a variance.

It should be noted that all models include a measure of reliability of the soil parameters. In a case where only few measurements are available apriori knowledge of parameter variations may be extremely important for estimating the variability. A consistent implementation of such apriori knowledge is to use the concepts of Bayesian statistics in a procedure where the apriori information can be updated when more information as a result of more soil testing becomes available. In table 1 the need for apriori information is shown for the three soil material models mentioned.

CHARACTERISTIC VALUE

In Eurocode 7 the definition of the characteristic value of soil parameters is supported by specifying a fractile of the statistical distribution of the parameter as follows:

> *.... the characteristic value should be derived such that the calculated probability of a worse value governing the occurrence of a limit state is not greater than 5%.*

An argument for keeping a significant part of the reliability margin within the procedure for selection of characteristic soil parameters is that the parameters can be obtained by various testing methods with different inherent uncertainties. Furthermore, the number of tests carried out for the single project may vary substantially. If a mean value definition was adopted an extensive list of partial safety factors would be needed in order to avoid too conservative designs for a majority of geotechnical structures.

It is within the context of the definition given above for the characteristic value to perform a rational reduction of data from the soil model. It should be recalled that soil model I can be used directly in connection with such advanced numerical calculation models as finite elements or where the spatial data information is weighted proportionally with spatial dissipation of energy for an approximate solution for homogeneous soil.

In many cases a soil parameter profile can be used directly as input for a calculation model. It is in this connection appropriate to define a characteristic profile in a similar way as the characteristic value. Here it should be recalled that weak layers in some connections may be compensated for by adjacent stronger layers. This should be kept in mind if different portions of a characteristic profile is used for design of structures of different types and sizes, e.g. if column footings of different sizes are designed by use of the same characteristic profile.

UPDATING PROCEDURE

It is assumed that the soil type is known prior to the time when test results become available allowing an experienced geotechnical engineer to estimate the variation of a certain parameter θ in terms of a mean value and a standard deviation. As negative values are prohibitive for most soil parameters a logarithmic normal distribution of θ for the site is usually assumed.

The concept of updating knowledge has been treated extensively in the statistical literature. This paper is based on a procedure described by Ditlevsen & Madsen (1990, 1993), where the probability density function of the pair (M, Σ) is consecutively updated. M is the mean value (μ) interpreted as a stochastic variable and Σ^2 is the similar interpretation of the variance (σ^2) of the logarithm of the normalized parameter. The theory behind the procedure will not be presented here, but a set of formulas is given which will allow a determination of characteristic values.

The knowledge of the soil type is assumed to form the basis for providing estimates for $E(M_L)$ and $V(M_L)$ where E is the mean value and V is the coefficient of variation defined as $V = D/E$ where D is the standard deviation.

Table 1: Soil models

Soil model type	Parameter variations	Need for Apriori knowledge	Need for Number of measurements
I	θ (x,y,z)	-	≈ 100
II	θ (z)	+	≈ 10
III	θ	+++	≈ 1,2,3

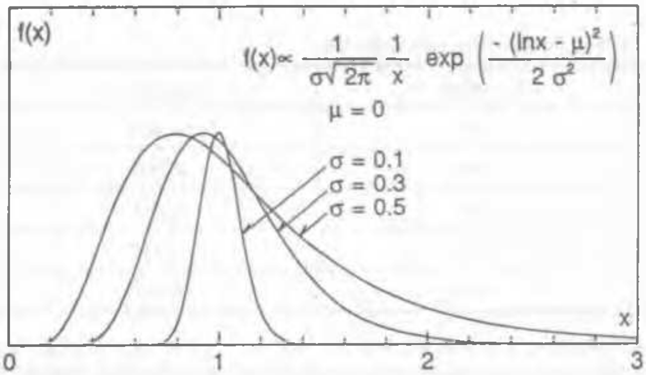

Figure 1. A set of logarithmic normal distributions

The suffix L refers to the logarithmic normal probability function. Furthermore, an estimate of the mean value $E(V_L)$ of the coefficient of variation valid for the influence zone shall also be made.

$E(M_L)$ can be estimated as the mean of the highest and lowest site mean values expected for the actual soil type. The width of this interval is characterized by $V(M_L)$ - examples of lognormal density functions is shown in figure 1. The estimated coefficient of variation with respect to an actual value of M_L for a specific site is thus denoted E (V_L).

This means that a wide interval for the parameter θ for a certain soil type can be anticipated. On a certain site a smaller variation can be assumed - although the mean value of this smaller variation can fluctuate in the above-mentioned wide interval for the entire soil type.

It is then necessary to transform the prior information as well as the results from the testing into a normal distribution. A convenient normalization constant is $k = E(M_L)$ and the following approximations are valid:

$$E(M) \approx \ln(E(M_L)/k) = 0 \tag{1}$$

$$D(M) \approx V(M_L) \tag{2}$$

$$E(\Sigma) \approx E(V_L) \tag{3}$$

where $(M, \Sigma) = (E(\eta), D(\eta))$ and $(M_L, V_L) = (E(\theta), V_\theta)$.

This apriori knowledge can be interpreted as a fictive sample with the parameters for sample size (ν) and sample standard deviation (β) determined by

$$D(M) = \frac{\beta}{\sqrt{\nu - 3}} \tag{4}$$

$$E(\Sigma) = \beta \sqrt{\frac{\nu}{2}} \frac{\Gamma\left(\frac{\nu-2}{2}\right)}{\Gamma\left(\frac{\nu-1}{2}\right)} \tag{5}$$

where Γ is the gamma function and η is the parameter in the transformed space - i.e. $\eta = \ln(\theta/k)$. To facilitate the computations $E(\Sigma)/D(M)$ and $D(M)/\beta$ are shown as functions of ν in figure 2.

The above-mentioned apriori knowledge is supplemented with the results of soil tests on n samples with a mean value of $\bar{\eta}$ and a sample standard deviation of s defined as

$$s = \sqrt{\sum_{i=1}^{n} (\eta_i - \bar{\eta})^2 / n} \tag{6}$$

The total sample has the following parameters for sample size, mean value and sample standard deviation, respectively:

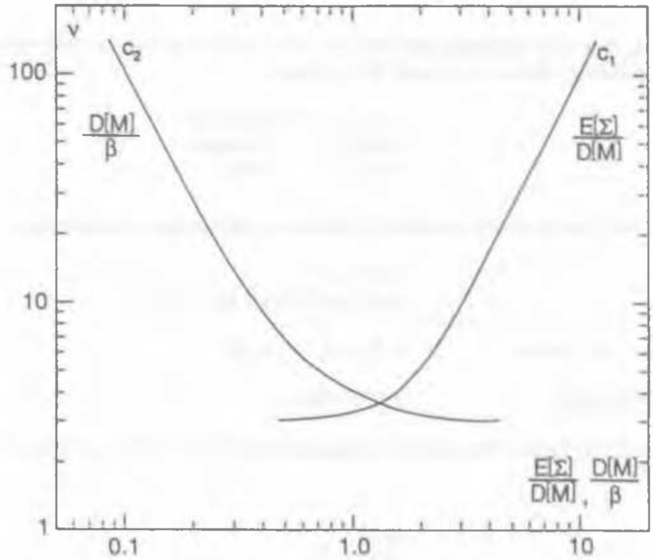

Figure 2. $E(\Sigma)/D(M)$ and $D(M)/\beta$ as functions of ν

$$n^* = n + \nu \tag{7}$$

$$\bar{\eta}^* = \frac{n\bar{\eta}}{n + \nu} \tag{8}$$

$$s^* = \sqrt{\frac{ns^2 + \nu\beta^2}{n + \nu} + n\nu\left(\frac{\bar{\eta}}{n + \nu}\right)^2} \tag{9}$$

From the predictive, posterior density distribution a certain, lower λ-fractile of η can be calculated by

$$\eta^*_\lambda = \bar{\eta}^* - s^* \sqrt{\frac{n^* + 1}{n^* - 1}}\, t_{1-\lambda} \tag{10}$$

where $t_{1-\lambda}$ is found from a table as the (1-λ)-fractile in the t-distribution with $(n^* - 1)$ degrees of freedom.

The posterior density of the expectation $M = E(\eta)$ is also related to the t-distribution and similarly λ-fractile of this distribution is

$$E(\eta)^*_\lambda = \bar{\eta}^* - s^* \frac{1}{\sqrt{n^* - 1}}\, t_{1-\lambda} \tag{11}$$

Finally, the fractiles should be transformed back to the original logarithmic distribution by using $\theta^*_\lambda = k \exp \eta^*_\lambda$ and $E(\theta)^*_\lambda = k \exp E(\eta)^*_\lambda$ for equation (10) and equation (11), respectively. The value θ^*_λ thus means the λ-fractile in the predictive density for θ. Correspondingly, the value $E(\theta)_\lambda$ means the λ-fractile of predictive mean value of θ.

Equation (11) is well-suited to calculate characteristic values defined by a specified fractile if the problem can be related directly to the properties of the mean value of the soil material parameter within the influence zone.

EXAMPLE

In this example the characteristic value of the undrained mean shear strength $E(c_u)$ of clay till should be determined in connection with the design of a strip footing with a width of about 0.5 m. The characteristic value shall be used to determine the bearing resistance of the footing in the ultimate limit state.

Apriori it is estimated that the clay till is weak and has the following statistical values of undrained shear strength

$$E\,(M_L) = k = 100\ \mathrm{kN/m^2}$$
$$D\,(M_L) = 25\ \mathrm{kN/m^2}$$
$$E\,(V_L) = 0.25$$

The corresponding parameters for the normal distribution are approximately

$$E\,(M) = 0$$
$$D\,(M) = D\,(M_L)/E\,(M_L) = 0.25$$

and furthermore $E\,(\Sigma) = E\,(V_L) = 0.25$

This yields $E\,(\Sigma)/D\,(M) = 1$

and from figure 2 the statistical parameters for the fictive sample are obtained

$$v = 3.2$$

$$\frac{D(M)}{\beta} = 2$$

$$\beta = \frac{D(M)}{2} = 0.125$$

The soil testing consists of 5 vane tests performed in the influence volume - i.e. to a depth of d = 0.7 b beneath the base. The results of the tests are given in table 2.

According to Danish experience the vane strength (c_v) may be interpreted directly as a measure for the undrained shear strength (c_u) for clay till without application of any conversion factor.

With $\eta = \ln\,(c_v/k)$ the mean value and sample standard deviation are: $\bar{\eta} = -0.445$ and $s = 0.241$, respectively.

According to equations (7), (8) and (9) the parameters for the total sample are

$$n^* = 8.2$$
$$\bar{\eta}^* = -0.271$$
$$s^* = 0.243$$

and the lower 5% fractile of the mean value is obtained from equation (11) as $E\,(\eta)^*_{5\%} = -0.443$. The characteristic value can now be determined as

$$c_{v,k} = k \exp E(\eta)^*_{5\%} = 64\ \mathrm{kN/m^2}$$

If no apriori information had been applied the following values would have been obtained

$$\bar{\eta} = -0.445 \text{ and } s = 0.241\,\sqrt{n/(n-1)} = 0.269$$

The characteristic value would then have been calculated as $c_{v,k} = k \exp E(\eta)^*_{5\%} = 50\ \mathrm{kN/m^2}$.

Certainly, a lower value than when the apriori assumption with an anticipated mean of $c_v = 100\ \mathrm{kN/m^2}$ was included. It should be added that the present example deals with values from a real site where the apriori knowledge was not based on test results. More than 5 vane tests were actually performed, but the five results presented here were selected randomly.

Table 2. Results from vane shear tests

c_v kN/m²	ln (c_v/k)
50	-0.693
50	-0.693
60	-0.511
81	-0.211
89	-0.117

Calculations performed on basis of more test results gave considerably greater mean value, and the procedure by incorporating the apriori knowledge was thus highly justified in this case.

DISCUSSION AND CONCLUSIONS

It may be worth mentioning that geotechnical structures very rarely fail due to the fact that the calculation procedure used does not precisely model the problem or due to the fact that a too low safety factor has been introduced. Geotechnical structures fail due to such gross errors as the existence of unidentified weak soil layers or unexpected pore water pressures. The adverse effects of such errors cannot be prevented by any rational design process. Only the knowledge and skills of the designer will play a role in avoiding such gross errors.

The limit state design format with the use of partial factors of safety is at present considered the most rational design procedure for geotechnical routine problems. Within this framework the characteristic values of soil properties have to be selected in a well defined way if a specific safety level against failure in the various limit states shall be obtained. The experienced engineer will always attempt to incorporate his apriori knowledge of the soils involved in this selection process. However, the apriori knowledge will have to be quantified in such a way that it can be combined with the results of soil testing when selecting characteristic values. The Bayesian statistical method provides a tool for such a quantification.

It should be noted, however, that not all aspects of the concept of structural reliability are included in the above mentioned procedure. At the present stage of development where only a few partial factors are used design by means of such factors is in itself an approximation when the intention is to obtain a specified level of reliability for a large category of structures.

A great amount of research is being performed in the area of structural and geotechnical reliability and the numerical values of the partial factors which are at present used for geotechnical design will undoubtedly undergo substantial changes in the future both with respect to a more precise definition and their numerical values.

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