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LIMIT ANALYSIS OF ECCENTRICALLY OBLIQUELY LOADED FOOTING

ANALYSE DE LIMITE DE GRADIN DE FONDEMENT SOUS CHARGE OBLIQUE EXCENTRE

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SYNOPSIS : In this paper an attempt has been made to develop limit analysis (upper bound) solution for ultimate bearing capacity of eccentrically obliquely loaded strip footings. One sided rupture failure on the side of the eccentricity and inclination of load was assumed and partial mobilisation was considered on the other side. Results have been presented in the form of non-dimensional parameters N_c, N_q, and N_γ for different values of angle of shearing resistance (Φ), eccentricity-width ratio (e/B) and inclination of the load (i). Comparison of bearing capacity factors obtained by the present approach with limit equilibrium approach has been made. Results were also verified with large number of model tests.

INTRODUCTION :

Footing serving foundations for retaining walls, abutments, stanchions and portal framed buildings may be subjected to moments and shear in addition to vertical load. These forces and moments may be replaced by an eccentric-inclined load on the footing. The bearing capacity of such footing is obtained by analysing the problem in two separate parts : (1). The bearing capacity of footing subjected to eccentric vertical load (Mayerhof 1953; Prak ash and Saran 1971); (2). The bearing capacity of footing subjected to central oblique load (Mayerhof 1953; Hansen 1955; Kezdi 1961; Hansen 1961; Saran 1971)

In this paper, an analytical solution is presented to obtain the bearing capacity of eccentrically obliquely loaded strip footing using limit analysis (upper bound) approach and compared the results with the earlier solution based on limit equilibrium approach (Saran and Agrawal, 1991).

LIMIT ANALYSIS :

Assumptions

The following assumptions have been made in the limit analysis
1. Soil is ideally plastic with no restriction to dilation;
2. Plane strain conditions with intermediate principal component of plastic strain $\dot{\varepsilon}_2 = 0$;
3. Coulomb yield criterion is valid;
4. The associated flow rule is observed;
5. Failure mechanism is kinematically admissible with no geometric changes during plastic flow;
6. A constant degree of shear stress mobilisation occurs through out failure mechanism;
7. The failure mechanism is taken exactly the same as adopted in limit equilibrium (lower bound) solution (Saran and Agrawal, 1991);
8. The soil above the base of foundation has been replaced by a uniform surcharge; and
9. There is unrestricted plastic flow on the side of eccentricity at the instant of impending plastic flow. On the other side the nominal deformation takes place which also follows the associated flow rule (Karal,1977).

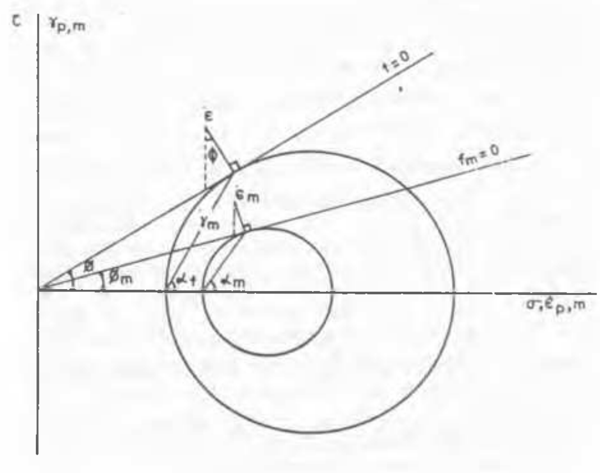

Fig 1. Coulomb criterion and mobilisation factor defination.

The vector $\dot{\varepsilon}_m$ (Fig 1.) is considered as the vector of normal nominal plastic flow. Under equilibrium conditions in C-Φ soils, the vectors $\dot{\varepsilon}_m$ and $\dot{\varepsilon}$ have different directions with respect to σ-axis, where ε is the normal plastic strain rate. This implies that the ratio $\dot{\varepsilon}_m/\dot{\gamma}_m$ depends on m, $\dot{\gamma}_m$ being nominal plastic shear strain rate. The normal strain rate $\dot{\varepsilon}_m$ becomes less and so does the nominal volume expansion. The direct consequence of nominal yield is that the development of nominal displacements and failure mechanism depends on m, where

$$m = \frac{C_m + \sigma \tan \Phi_m}{C + \sigma \tan \Phi} \quad .. (1)$$

Analytical Solutions

$$V_r = \frac{V_p \sin\alpha_1}{\text{Cos } \Phi} \quad .. (3)$$

The logarithmic spiral shear zone BED is considered to be composed of a sequence of rigid triangles (Fig 2). All the small triangles and zone BDC moves as a rigid bodies in the direction with an angle Φ, with the discontinuity lines ED and DC. The velocity of each small triangles are determined by the condition that relative velocity between triangles in contact must have direction with an angle Φ to the contact surface. In the logarithmic spiral zone, the velocity increase exponentially as

$$V_2 = V_1 \, e^{(\theta_1 \tan\Phi)} = \frac{V_p \cos(\Phi-\alpha_1)}{\cos\Phi} \, e^{(\theta_1 \tan\Phi)} \quad .. (4)$$

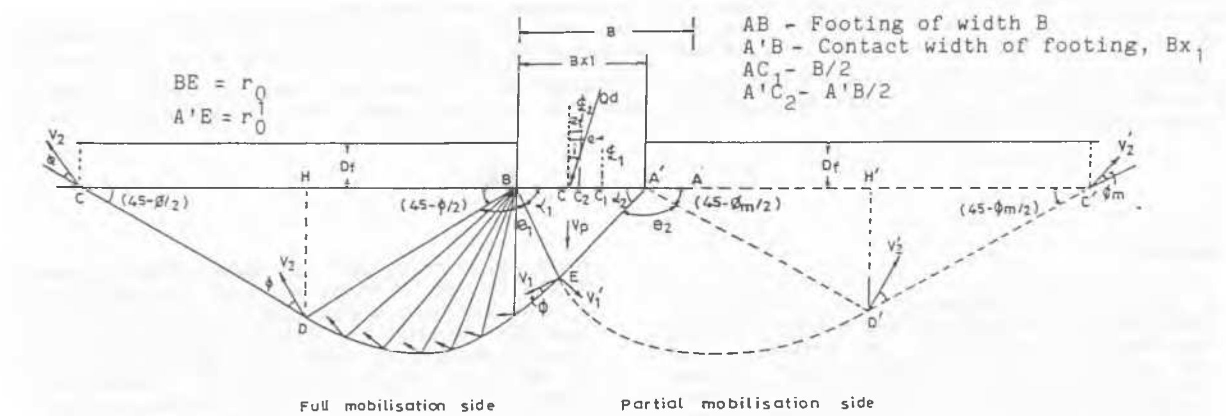

Fig 2. Failure mechanism adopted in the study

The footing is considered rigid with rough base. It is assumed to lose contact with the increase in eccentricity. The failure mechanism considered in this analysis consists of triangular wedge A'BE with the base angles α_1 and α_2 moving downward as a rigid body with the velocity of footing V_p, a log-spiral zone ED of central angle θ_1 at B and a rigid wedge BDC (Fig 2). The lines BE and A'E in addition to ED and DC are lines of velocity discontinuity. The soil below the failure line ED and DC remains at rest, so that every where the velocity along this line is inclined at an angle Φ to this line. The velocity of the soil V_1, just to the left of discontinuity line BE is perpendicular to BE and its magnitude must be such that change in velocity V_r, across BE is inclined at an angle Φ to BE. Referring to compatibility velocity diagram (Fig 3), the velocity V_1 will have the magnitude

$$V_1 = \frac{V_p \cos(\Phi-\alpha_1)}{\text{Cos } \Phi} \quad .. (2)$$

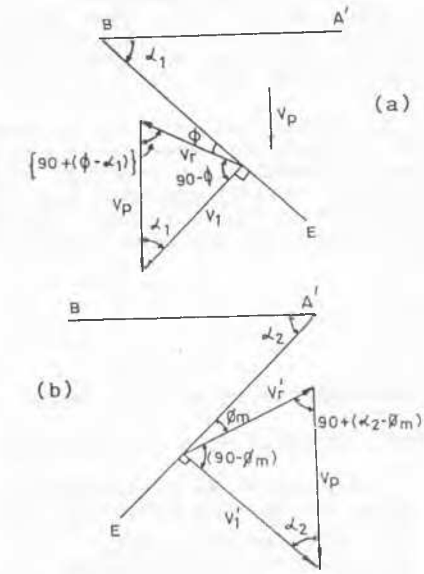

Fig 3. Velocity diagram (a) Full mobilisa- side, (b) Partial mobilisation side

Since BDC zone translates as a rigid body, the zone has the velocity equal to V_2, perpendicular to the radial line BD.

Bearing Capacity Equation

The bearing capacity equation in limit analysis is obtained by equating the total rate of energy dissipated to the total rate of work done. The total rate of energy dissipated (Fig 2) equals the rate of energy dissipated along the (line BE + radial shear zone BED + spiral ED + line DC + line A'E + radial shear zone A'ED' + spiral ED' + line D'C'), which equals

$$C V_r r_o \cos\Phi + C V_1 r_o \left[\frac{e^{(2\theta_1 \tan\Phi)} - 1}{\tan\Phi}\right] + C V_1 e^{(2\theta_1 \tan\Phi)} r_o + C V_r' r_o^1 \cos\Phi_m + C V_1' r_o^1 \left[\frac{e^{(2\theta_2 \tan\Phi_m)} - 1}{\tan\Phi_m}\right] + C V_1' r_o^1 e^{(2\theta_2 \tan\Phi_m)} \cos\Phi_m \qquad ..\ (5)$$

Total rate of work done (Fig 2) is the rate of work done by the soil mass (A'BE + BED + BDC + A'E D'+ A'D'C') plus the rate of work done by surcharge on (BC + A'C') plus work done by the footing load, which equals

$$q_u \cos i B x_1 V_p + \tfrac{1}{2} B x_1 r_o \sin\alpha_1 V_p - \tfrac{1}{2} \gamma r_o^2 V_p \frac{\cos(\Phi - \alpha_1)}{\cos\Phi\,(1+9\tan^2\Phi)} \left\{ e^{3\theta_1 \tan\Phi} [3\tan\Phi \cos(\theta_1 + \alpha_1) + \sin(\theta_1 + \alpha_1)] - (3\tan\Phi \cos\alpha_1 + \sin\alpha_1) \right\}$$
$$- \gamma V_1 r_o^2 e^{3\theta_1 \tan\Phi} \cos^2(45-\Phi/2) \sin(45-\Phi/2) - 2\gamma D_f r_o e^{2\theta_1 \tan\Phi} \cos^2(45-\Phi/2) V_1 - \tfrac{1}{2} \gamma (r_o^1)^2 \frac{V_1'}{(1 + 9\tan^2\Phi_m)} \left\{ e^{3\theta_2 \tan\Phi_m} [3\tan\Phi_m \cos(\theta_2 + \alpha_2) + \sin(\theta_2 + \alpha_2)] - (3\tan\Phi_m \cos\alpha_2 + \sin\alpha_2) \right\}$$
$$- \gamma (r_o^1)^2 V_1' e^{3\theta_2 \tan\Phi_m} \cos^2(45-\Phi_m/2) \sin(45-\Phi_m/2) - 2\gamma D_f r_o^1 e^{2\theta_2 \tan\Phi_m} \cos^2(45-\Phi_m/2) V_1' \qquad ..\ (6)$$

Equating the total rate of work done by the force on the foundation and the soil weight in motion to the total rate of energy dissipation along the lines of velocity discontinuity, the upper bound bearing capacity can be expressed

$$Q_u = B \left[\tfrac{1}{2} \gamma B N_\gamma + \gamma D_f N_q + C N_c \right] \qquad ..\ (7)$$

in which quantities N_γ, N_q and N_c are called the bearing capacity factors, where

$$N_\gamma = \frac{1}{\cos i} \left[\frac{x_1 \sin^2\alpha_2}{\sin^2(\alpha_1 + \alpha_2)} \frac{1}{(1 + 9\tan^2\Phi)} \left\{ e^{3\theta_1 \tan\Phi} [3\tan\Phi \cos(\theta_1 + \alpha_1) + \sin(\theta_1 + \alpha_1)] - (3\tan\Phi \cos\alpha_1 + \sin\alpha_1) \right\} \frac{\cos(\Phi - \alpha_1)}{\cos\Phi} - \frac{x_1 \sin\alpha_1 \sin\alpha_2}{\sin(\alpha_1 + \alpha_2)} \right.$$
$$+ \frac{2x_1 \sin^2\alpha_2}{\sin^2(\alpha_1 + \alpha_2)} e^{3\theta_1 \tan\Phi} \cos^2(45-\Phi/2) \sin(45-\Phi/2) \frac{\cos(\Phi - \alpha_1)}{\cos\Phi} + \frac{x_1 \sin^2\alpha_1}{\sin^2(\alpha_1 + \alpha_2)} \frac{\cos(\Phi_m - \alpha_2)}{\cos\Phi_m} \frac{1}{(1 + 9\tan^2\Phi_m)} \left\{ e^{3\theta_2 \tan\Phi_m} [3\tan\Phi_m \cos(\theta_2 + \alpha_2) + \sin(\theta_2 + \alpha_2)] - (3\tan\Phi_m \cos\alpha_2 + \sin\alpha_2) \right\}$$
$$\left. + \frac{2x_1 \sin^2\alpha_1}{\sin^2(\alpha_1 + \alpha_2)} e^{3\theta_2 \tan\Phi_m} \cos^2(45-\Phi_m/2) \sin(45-\Phi_m/2) \frac{\cos(\Phi_m - \alpha_2)}{\cos\Phi_m} \right] \qquad ..\ (8)$$

$$N_q = \left[\frac{2\sin\alpha_2}{\sin(\alpha_1 + \alpha_2)} e^{2\theta_1 \tan\Phi} \cos^2(45-\Phi/2) \frac{\cos(\Phi - \alpha_1)}{\cos\Phi} + \frac{2\sin\alpha_1}{\sin(\alpha_1 + \alpha_2)} e^{2\theta_2 \tan\Phi_m} \cos^2(45-\Phi_m/2) \frac{\cos(\Phi_m - \alpha_2)}{\cos\Phi_m} \right] \frac{1}{\cos i} \qquad ..\ (9)$$

$$N_c = \frac{1}{\cos i} \left[\frac{\sin\alpha_1 \sin\alpha_2}{\sin(\alpha_1 + \alpha_2)} + \frac{\sin\alpha_2 \cos(\Phi - \alpha_1)}{\sin(\alpha_1 + \alpha_2) \cos\Phi} \left\{ \frac{e^{2\theta_1 \tan\Phi} - 1}{\tan\Phi} \right\} + \frac{\sin\alpha_2 \cos(\Phi - \alpha_1)}{\sin(\alpha_1 + \alpha_2)} e^{2\theta_1 \tan\Phi} + \frac{\sin\alpha_1 \sin\alpha_2}{\sin(\alpha_1 + \alpha_2)} + \frac{\sin\alpha_1 \cos(\Phi_m - \alpha_2)}{\sin(\alpha_1 + \alpha_2) \cos\Phi_m} \frac{e^{2\theta_2 \tan\Phi_m} - 1}{\tan\Phi_m} + \frac{\sin\alpha_1 \cos(\Phi_m - \alpha_2)}{\sin(\alpha_1 + \alpha_2)} e^{2\theta_2 \tan\Phi_m} \right] \qquad ..\ (10)$$

For computation of N_γ, N_q and N_c factors equations 8,9, and 10 has been used respectively. By substituting the values of N_γ, N_q and N_c factors in the equation 7, the values of ultimate bearing capacity can be determined.

The computation of N_γ, N_q and N_c factors were done individually for three different cases (a) $C = q = 0$; (b) $C = \gamma = 0$; and (c) $\gamma = q = 0$; for different values of Φ, e/B and i

TESTS PERFORMED :

Tests were performed in a steel tank of size 1.5m × 1.5m × 1.0m on a footing of sizes (10cm × 60cm, 20cm × 20cm, and 20cm × 40cm) resting on a dry Ranipur sand (D_{10} = 0.17mm, Cu = 1.76) having relative density 84%. The pressure settlement characteristics of footing in each tests were obtained. The failure pressure was obtained with the help of pressure-settlement characteristics of footing. The details of model tests are given elsewhere (Agrawal, 1986)

COMPARISON OF BEARING CAPACITY FACTORS :

Comparison of bearing capacity factors obtained by the present approach and limit equilibrium approach is shown in Table 1., for $\Phi=40^\circ$

Table 1. Comparison of Bearing Capacity Factors ($\Phi=40^\circ$)

		Limit equilibrium analysis			Limit analysis		
i	e/B	N_γ	N_q	N_c	N_γ	N_q	N_c
0°	0.0	166.15	81.56	95.98	166.60	81.56	95.98
	0.1	101.35	55.93	66.79	101.90	55.93	67.81
	0.2	57.67	42.84	48.90	58.15	42.69	50.66
	0.3	25.50	28.10	32.32	26.14	27.92	33.78
10°	0.0	90.94	48.09	53.60	91.37	48.10	55.39
	0.1	46.53	33.99	42.86	47.18	33.90	43.82
	0.2	28.90	26.75	33.00	28.92	26.23	34.66
	0.3	12.70	17.60	20.16	12.98	17.38	22.25
20°	0.0	54.34	28.41	37.86	55.31	28.35	40.50
	0.1	27.27	20.17	29.25	27.85	20.11	30.31
	0.2	14.26	16.25	20.98	14.35	16.14	21.56
	0.3	6.50	10.89	13.98	6.94	10.76	15.12

COMPARISON OF EXPERIMENTAL AND COMPUTED VALUES OF BEARING CAPACITY

Fig 4. shows the comparison between ultimate bearing capacity obtained from the model tests and from the computed ultimate bearing capacity using the present analysis.

CONCLUSION :

It is evident from the Table 1 that the two approaches give almost the same values of bearing capacity factors.

The ultimate bearing capacity observed in the model test and computed by the present analysis are in good agreement (Fig 4).

The values of N_γ, N_q and N_c depends on the values of Φ, e/B and i The non-dimensional charts of evaluating N_γ, N_q and N_c are given elsewhere (Saran and Agrawal, 1991; Agrawal, 1986). As the value of i and e/B increases, the value N_γ, N_q and N_c decreases.

It can be concluded that the rupture surface assumed in the analysis is closer to real one.

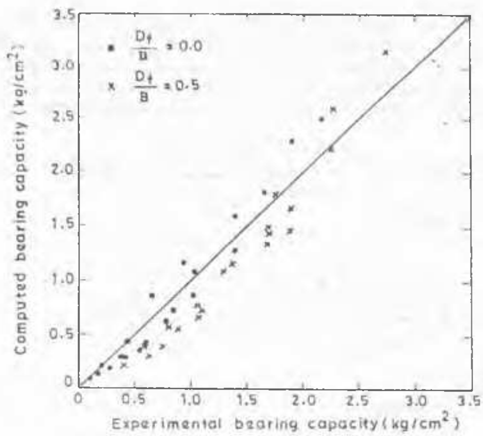

Fig 4. Comparison of computed and experimental Bearing capacity.

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