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# SETTLEMENT ANALYSIS OF AXIALLY LOADED PILES IN A LAYERED SOIL

# TASSEMENT DE PIEUX VERTICALEMENT CHARGES DANS UN MASSIF DE SOL STRATIFIE

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This paper presents a boundary element algorithm for settlement analysis of axially loaded single piles in stratified soil deposits. The proposed algorithm is based on the use of special Green's functions to evaluate displacements caused by vertical point loads applied in the interior of a multi-layered elastic half-space (generalized Mindlin's problem). One of the main advantages of these displacement functions is that they are determined explicitly in the spatial domain, thus not requiring any additional numerical transformations from a counterpart space. Results from some numerical examples, when compared to those obtained through other available numerical solutions, indicate that the proposed method is feasible, simple yet quite general, and it can be applied to foundation problems involving any type of soil layering.

## INTRODUCTION

In the past three decades, pile design procedures evolved from empirical techniques based on simple correlations towards sounder theoretical methods based on principles of continuum mechanics and sophisticated constitutive relationships. Among the major factors responsible for this change are the rapid development of numerical methods, such as the finite element method (FEM) and the boundary element method (BEM), as well as the now commonplace availability of micro-computers and other computational resources.

The analysis of single pile problems by means of a boundary element algorithm requires the integration of an appropriate elementary point force solution for the soil medium over the discretized elements of the pile-soil interface. Usually, this solution assumes that the soil mass can be represented as a homogeneous, isotropic elastic half-space (Mindlin's solution, 1936), despite the fact that in many instances natural soil deposits exhibit stratified geotechnical profiles. For settlement analysis of axially loaded single piles in multi-layered soil deposits, several authors, among them Poulos (1979) and Yamashita et al. (1987), proposed the use of a simplified version of the boundary element method based on Mindlin's equations. However, since Mindlin's formulation was originally derived for a homogeneous isotropic elastic half-space, these approximate proposals simply attempt to adapt Mindlin's solution by assuming some equivalent values of the soil modulus obtained from empirical correlations (some sort of averaging process).

The purpose of this paper is to analyze the settlement behavior of axially loaded single piles in a stratified soil deposit through the boundary element algorithm presented by Mattes and Poulos (1969), but using specific Green's functions for the case of a multi-layered elastic half-space. These functions were mathematically derived by Seale (1985) and their great advantage, when compared to other available numerical solutions for this problem, is that they are obtained explicitly in the spatial domain, thus not requiring any additional numerical transformations from a counterpart space (Fourier or Hankel transform spaces). Furthermore, for problems involving only static loadings the Green's functions can be obtained as closed form solutions. The accuracy and applicability of several methods for settlement analysis are also investigated in this research, considering some multi-layered soil problems.

## ANALYSIS BASED ON MINDLIN'S PROBLEM

An elastic solution for the settlement analysis of axially loaded single piles in a homogeneous, isotropic elastic half-space was presented by Mattes and Poulos (1969). It involves division of the pile into n shaft elements, over each one acts an unknown interaction stress. Through Mindlin's equations, the soil vertical displacement at the midpoint of all elements along the pile-soil interface can be expressed as

$$\{s_s\} = \frac{d}{E_s}[I_s]\{p\} \tag{1}$$

where $\{s_s\}$ is the soil displacement vector, d is the shaft diameter, $E_s$ is the Young's modulus of the soil, $[I_s]$ is a (n+1) square matrix of soil displacement factors evaluated by integration of Mindlin's equations over the element surfaces and $\{p\}$ is the (n+1) interaction stress vector.

Given the simplicity of the algorithm implied by equation (1), several approximate techniques have been proposed in the literature aiming to adapt Mindlin's equations to the case of a multi-layered half-space. Among these proposals, all based on some choice of the equivalent soil modulus $E_s$, the following are briefly reviewed:

### Equivalent Uniform Soil

In cases where the soil modulus variation between successive layers is not large, the simplest approach is to use an equivalent soil modulus $E_s$ found by the weighted average,

$$E_s = \frac{1}{L}\sum_{m=1}^{M} E_{sm}h_m \tag{2}$$

where L is the pile length, m is the number of soil layers along the pile length, $E_{sm}$ and $h_m$ are the Young's modulus and the thickness, respectively, of a generic layer m.

**Poulos' Method (1979)**

In this proposal, the equivalent value for Es is given by,

$$E_s = 0.5\,(E_{si} + E_{sj}) \tag{3}$$

where Esi, Esj are the soil modulus at the influenced element i and the influencing element j, respectively. This equation becomes inaccurate if large differences in soil modulus exist between adjacent elements or if a soil layer, in the geotechnical profile, is overlain by a much stiffer layer.

**Yamashita, Tomona and Kakurai's Method (1987)**

Equation (3) ignores the soil modulus of layers other than those corresponding to the i-th and the j-th elements. Yamashita et al. (1987) developed a more general approximation in which the values of Esi and Esj are calculated as a weighted average considering the values of soil modulus at all layers along as well as beneath the pile.

$$E_{si} = \frac{\beta_{-(i-1)} E_{s1} + \cdots \beta_{-1} E_{s(i-1)} + E_{si} + \beta_1 E_{s(i+1)} + \cdots + \beta_{n+r-i} E_{s(n+r)}}{\beta_{-(i-1)} + \cdots \beta_{-1} + 1 + \beta_1 + \cdots \beta_{n+r-i}} \tag{4a}$$

$$E_{sj} = \frac{\beta_{-(j-1)} E_{s1} + \cdots \beta_{-1} E_{s(j-1)} + E_{sj} + \beta_1 E_{s(j+1)} + \cdots + \beta_{n+r-j} E_{s(n+r)}}{\beta_{-(j-1)} + \cdots \beta_{-1} + 1 + \beta_1 + \cdots \beta_{n+r-j}} \tag{4b}$$

where n is the number of shaft elements and r is the number of layers below the pile tip. It is also suggested that the weighting coefficients can be expressed as $\beta_{-k} = \beta_k = a^k$ where a = 0.5 and k < 5.

## NORMALIZED STRAIN DIAGRAM

In this proposal (Rocha Filho e Silva, 1990), the settlement of a single pile in a multi-layered half-space is given by

$$s = \frac{Q}{d} \sum_{m=1}^{2M} \frac{I_{Fm}}{E_{sm}} \left(\frac{\Delta z}{L}\right)_m \tag{5}$$

where s is the pile head settlement due to the vertical load Q; IFm represents strain influence factors obtained from the distribution of the soil vertical strains along the shaft and below the pile tip ($0 \le z/L \le 2$); Esm is the soil modulus at a generic layer m and $(\Delta z / L)_m$ is the ratio between the layer thickness and the pile length. The number of soil layers between the surface and the depth corresponding to twice the pile length L is indicated by 2M.

Values of the strain influence factors IFm can be obtained directly from figure 1 for two special cases of rigidity index K = 1000 and K = 5000 (K = Ep/ Es where Ep is the Young's modulus of the pile material) and slenderness ratio L / d = 25 and L / d = 50. Values of the strain influence factors for other situations involving different degrees of non-homogeneity (including the case of a multi-layered soil), different slenderness ratios and rigidity indices can be obtained by interpolation from the presented distributions. The graphical solutions shown in figure 1 were obtained from finite element analyses and field measurements of instrumented pile load tests. By means of simple graphical procedures, similar to those involved in the Schmertmann's method (1970), this set of results can be easily applied to any layered soil problem.

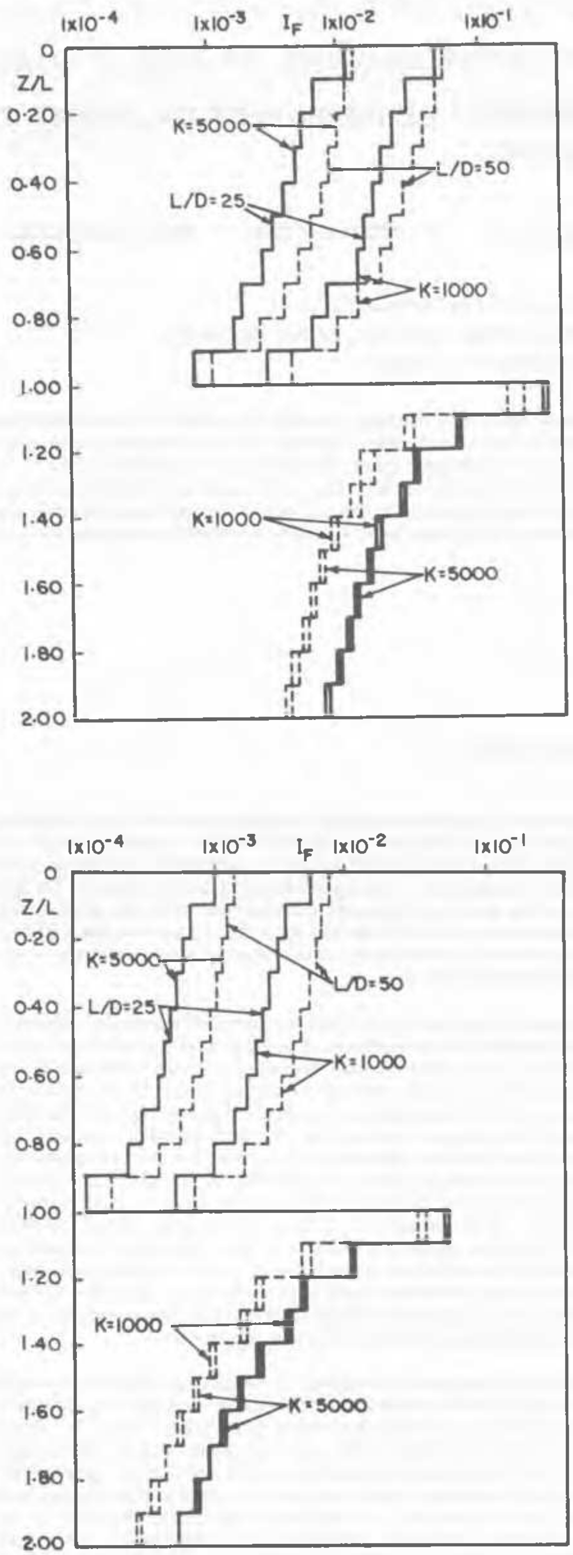


Figure 1 - Vertical strain influence factors for a homogeneous half-space (top) and a highly non-homogeneous (Gibson soil) half-space (bottom).

## GENERALIZED MINDLIN'S PROBLEM

A solution for the problem of point loads applied in the interior of a multi-layered elastic half-space (generalized Mindlin's problem) was presented by Seale (1985) and Kausel & Seale (1987).

In the stiffness matrix method the relationship between loads and displacements in the frequency-wavenumber domain is given by an expression of the form

$$[K]\{\overline{U}\} = \{\overline{Q}\} \tag{6}$$

with $\{\overline{Q}\}$, $\{\overline{U}\}$ being the load and displacement vectors, respectively, and [K] being the stiffness matrix for a multi-layered half-space. For static problems, when using the Thin Layer Method (Kausel and Peek, 1982), the global stiffness matrix can be expressed as

$$[K] = [A]k^2 + [B]k + [C] = [K(k)] \tag{7}$$

where k is the wavenumber and [A], [B] and [C] are symmetric matrices of dimension 2N x 2N, being N the number of layers, including the half-space. The reader is referred to Seale (1985) for a complete description of the matrix expressions.

Inversion of equation (6) can be performed with a spectral decomposition of the stiffness matrix [K] in terms of the eigenvalues $k_L$ obtained from the following linear eigenvalue problem of dimension 4N,

$$k_L \begin{bmatrix} 0 & A \\ A & B \end{bmatrix} \begin{Bmatrix} k_L\phi_L \\ \phi_L \end{Bmatrix} + \begin{bmatrix} -A & 0 \\ 0 & C \end{bmatrix} \begin{Bmatrix} k_L\phi_L \\ \phi_L \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{8}$$

in which $[\phi_L]$ is a 2N x 4N matrix of associated eigenvectors. Of the 4N roots involved in the in-plane case (equation 8), there are two null roots (rigid body modes), two real, negative roots and 2N - 2 complex conjugate pairs.

Solved the eigenvalue problem, it is then possible to obtain an explicit representation of the flexibility matrix [F], whose coefficients at the i-th elevation due to unit loads at the j-th elevation are:

$$f_{xx}^{ij} = \frac{1}{2kG_N(1-\alpha_N^2)} + \sum_{L=3}^{4N} \phi_x^{iL}\phi_x^{jL} a_L^R \tag{9a}$$

$$f_{xz}^{ij} = \frac{\alpha_N^2}{2kG_N(1-\alpha_N^2)} + \sum_{L=3}^{4N} \phi_x^{iL}\phi_z^{jL} a_L^R \tag{9b}$$

$$f_{zz}^{ij} = \frac{1}{2kG_N(1-\alpha_N^2)} + \sum_{L=3}^{4N} \phi_z^{iL}\phi_z^{jL} a_L^R \tag{9c}$$

$$f_{zx}^{ij} = \frac{\alpha_N^2}{2kG_N(1-\alpha_N^2)} + \sum_{L=3}^{4N} \phi_z^{iL}\phi_x^{jL} a_L^R \tag{9d}$$

where

$$a_L^R = \frac{1}{2k_L(k-k_L)} \quad \text{and} \quad \alpha_N = \sqrt{\frac{G_N}{\lambda_N + 2G_N}} \tag{9e}$$

with $\lambda_N$ and $G_N$ being the Lamé constants of the half-space. The eigenvectors in equation (9) were rearranged according to the degrees of freedom along axes x, z. Combining the Hankel transformed loads with the flexibilities given by equations (9), followed by an inverse Hankel transformation, the required displacements are then obtained. For an unit vertical load the displacement $s_z^{ij}$ (settlement) is given by

$$s_z^{ij} = \frac{1}{2\pi}\left[\frac{1}{2\rho\, G_N(1-\alpha_N^2)} + \sum_{L=3}^{4N} \phi_z^{iL}\phi_z^{jL} I_{1L}^R\right] \tag{10a}$$

with

$$I_{1L}^R = \frac{1}{2}\left\{\frac{1}{\rho k_L} + \frac{\pi}{2}[H_0(-\rho k_L) - Y_0(-\rho k_L)]\right\} \tag{10b}$$

where $H_0$ and $Y_0$ stand for the Struve and Neumann functions, respectively, and $\rho$ is the cylindrical coordinate that express the horizontal distance from the vertical load.

The matrix of soil displacement factors [Is] referred in equation (1) can now be evaluated through numerical integration of equation (10a) over the element surfaces.

## NUMERICAL RESULTS

Tables 1 and 2 show settlement results for the problem of an axially loaded single pile in several stratified soil profiles. The method based on the generalized Mindlin's problem yields numerical results that agree quite well with those obtained by finite element analysis using quadratic isoparametric elements. Results obtained with the approximate techniques based on the modified Mindlin's solution (Poulos, 1979, and Yamashita et al.,1987) exhibit some discrepancies which tend to be more severe for the case when a pile penetrates through stiffer strata into much more softer material (as in case 2, table 1). The equivalent uniform soil approach, as pointed out by Poulos (1979), seems to be useful only in the early stages of design when a rough estimate of pile settlement may be adequate.

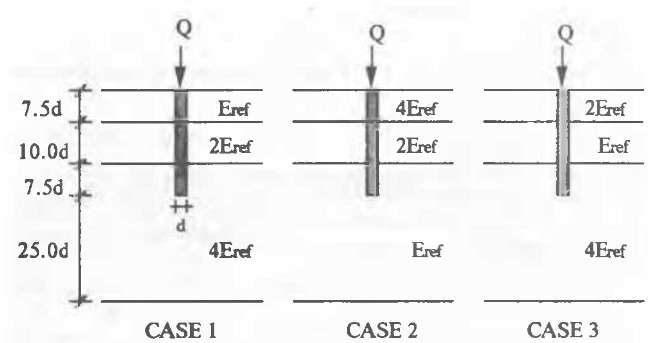


Figure 2 - Layered soil problems investigated in table 1 ($E_p$ = 1000 Eref) (Apud Poulos, 1979)

**Table 1 - Values of the settlement factor $I_s$ = sdEref / Q (Apud Poulos, 1979)**

| REFERENCES | | CASE 1 | CASE 2 | CASE 3 |
|---|---|---|---|---|
| POULOS'S | METHOD (1979) | 0.0386 | 0.0330 | 0.0366 |
| FEM | (POULOS, 1979) | 0.0377 | 0.0430 | 0.0382 |
| EQUIVALENT | UNIFORM SOIL | 0.0381 | 0.0706 | 0.0391 |
| NORMALIZED STRAIN DIAGRAM | HIGHLY NON-HOMOGENEOUS | 0.0399 | 0.0936 | 0.0386 |
| | HOMOGENEOUS | 0.0288 | 0.0603 | 0.0266 |
| YAMASHITA'S | METHOD | 0.0404 | 0.0364 | 0.0402 |
| GENERALIZED PROBLEM | MINDLIN'S | 0.0380 | 0.0419 | 0.0377 |

Settlement results from the normalized strain diagrams indicate that this graphical procedure can be interesting for practical applications, although its use depends on experience and engineering judgement for selection of an adequate value of the degree of non-homogeneity. Furthermore, the normalized strain diagram is an useful design chart to estimate the influence of important factors on the pile behavior such as the rigidity index K, the stratigraphic pattern, etc.

## CONCLUSION

The proposed method based on the generalized Mindlin's problem represents a significant improvement over other approximate methods suggested in the literature for settlement analysis of axially loaded piles in stratified half-spaces. It is feasible, efficient, computationally simple and accurate results can be obtained with a small number of discretized elements along the pile-soil interface.

On the other hand, the normalized strain diagram may also be used with confidence in those practical engineering problems where specific numerical results are not readily available or when it is important to rapidly assess the effects of several factors on the overall pile behavior.

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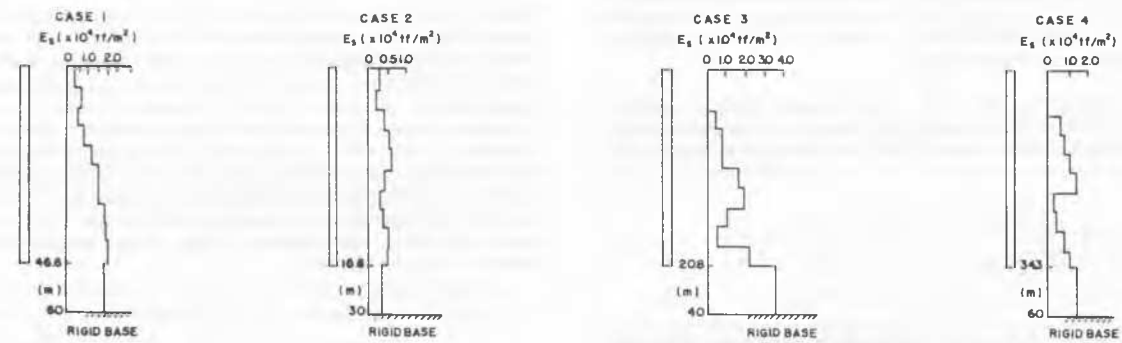


Figure 3 - Layered soil problems investigated in table 2 (Apud Yamashita et al., 1987 - 1 tf/m2 = 9.8 kN/m2).

Table 2 - Pile head settlement (expressed in mm) obtained by several methods.

| CASE | NORMALIZED STRAIN DIAGRAM | | | | Yamashita's Method (1987) | Equivalent Uniform Soil | Poulos' Method (1979) | Finite Element Method | Present Paper |
|---|---|---|---|---|---|---|---|---|---|
| | K | Homogeneous | Moderately Non-homogeneous | Highly Non-homogeneous | | | | | |
| 1 | 893.72 | 2.73<br>81.0% | 3.10<br>92.0% | 3.42<br>101.5% | 3.51<br>104.2% | 2.46<br>73.0% | 3.60<br>106.8% | 3.32<br>98.5% | 3.37<br>100.00% |
| | 127.68 | 3.00<br>89.0% | 3.42<br>101.5% | 3.78<br>112.2% | | | | | |
| | 223.43 | 2.97<br>88.1% | 3.38<br>100.3% | 3.73<br>110.7% | | | | | |
| 2 | 1300.00 | 2.00<br>94.8% | 2.16<br>102.4% | 2.36<br>111.8% | 2.15<br>101.9% | 1.81<br>85.8% | 2.05<br>97.2% | 2.09<br>99.1% | 2.11<br>100.00% |
| | 433.33 | 2.09<br>99.1% | 2.21<br>104.7% | 2.38<br>112.8% | | | | | |
| | 650.00 | 2.03<br>96.2% | 2.24<br>106.2% | 2.42<br>114.7% | | | | | |
| 3 | 650.00 | 2.00<br>89.3% | 2.18<br>97.3% | 2.73<br>121.9% | 2.31<br>103.1% | 3.09<br>137.9% | 2.27<br>101.3% | 2.24<br>100.0% | 2.24<br>100.00% |
| | 123.81 | 2.09<br>93.3% | 2.28<br>101.8% | 2.91<br>129.9% | | | | | |
| | 208.00 | 2.07<br>92.4% | 2.27<br>101.3% | 2.88<br>128.6% | | | | | |
| 4 | 1083.33 | 3.01<br>90.6% | 3.44<br>103.9% | 3.87<br>116.9% | 3.46<br>104.5% | 2.40<br>72.5% | 3.35<br>101.2% | 3.28<br>99.1% | 3.31<br>100.00% |
| | 240.74 | 3.17<br>95.8% | 3.46<br>104.5% | 4.14<br>125.1% | | | | | |
| | 393.94 | 3.14<br>94.9% | 3.45<br>104.2% | 4.09<br>123.6% | | | | | |