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KINEMATICALLY AND STATICALLY PLAUSIBLE CALCULATIONS
CALCULS DE PLAUSIBILITE CINEMATIQUE ET STATIQUE

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SYNOPSIS: The terms kinematically plausible (**KP**) and statically plausible (SP) are defined. Geotechnical calculation methods are usually either KP or SP, but very rarely KP as well as SP. However, this may be achieved by a successive procedure, alternating between KP- and SP-calculations. The result is named a **KSP**-solution being KP as well as SP.

For a **raft foundation** the procedure demonstrates, how decreased unit bearing resistance of the soil and/or decreased bending resistance of the raft increase the max settlements, which finally become infinitely great at Ultimate Failure. This means that the procedure is able to describe a continuous transition from the Serviceability Limit State with characteristic parameters to the Ultimate Limit State with design parameters.

For an **embedded wall** it is shown how the vertical bearing resistance is influenced by
- the stress history of the soil prior to and after the installation,
- the relative compressibility of the wall
- the installation and the loading history of the wall.
- the strength of the soil, which may exert less influence than some of the other factors.

DEFINITIONS

KP-analyses

Analyses of earth pressure problems, bearing capacity problems or stability problems are considered *kinematically admissible*

- if the strains respect assumed stress-strain relationships for soil and structure and
- if the contact areas of the structure and of the soil conform.

In reality the assumed stress-strain relationship reflects the true relationship only to a certain degree. Furthermore some of the methods of analysis applied in practice do not investigate the strains in every detail and may correspondingly lead to different results.

To allow for these approximations it seems more adequate to use the term *kinematically plausible (KP-)* and relate the plausibility to the assumptions and the method of analysis applied (theory of elasticity or plasticity, coefficient of subgrade reaction etc.).

SP-analyses

Substituting the terms "strains" by "stresses" and "stress-strain relationships" by "failure conditions" the previous paragraph also defines *statically admissible* analyses and explains why it is more adequate to use the term *statically plausible (SP-)* and relate the plausibility to the failure conditions and the methods of analysis applied (Mohr-Coulomb condition, Finite-difference-calculations, Extreme methods etc.).

It is implied that the conditions of equilibrium - at least in principle - should be fulfilled for both kinds of analyses.

PROPOSED SUCCESSIVE PROCEDURE.

In principle the proposed successive procedure consists of 4 stages:

KP-calculation
SP-correction
Successive calculations
KSP-calculation

Applying modern computer technique, however, all 4 stages may usually be carried out simultaneously.

KP-calculation

The deflections and movements of the structure and the soil subject to the contact pressure are determined corresponding to traditional stress-strain relationships for soil and structure. Conformity of the two contact areas (soil and structure) determine a KP-distribution of the contact pressure *(KP-CP)* corresponding to a state of equilibrium.

SP-correction

This KP-CP is made SP by adding or subtracting contact pressures where necessary to respect the failure conditions in the soil and/or the structure, generally by assuming local yielding of the soil and/or the structure. The resulting SP-distribution of the contact pressure *(SP-CP)*, however, generally does not represent a state of equilibrium, whereas it may often be considered KP-, because the yielding locally cancels the stress-strain conditions assumed in the KP-calculation.

Successive Calculations

If the SP-CP is not (sufficient) KP-, i.e. if a new KP-analysis indicate, that the two contact areas subject to SP-CP do not coincide, this may be achieved by a KP-correction leading to KP-CP', which again may be SP-corrected to SP-CP' etc., i.e. by a successive procedure alternating between KP- and SP-calculations.

KSP-calculation

The final result of the procedure described above with or without successive calculations is a CP, which in soil and structure

- respects the assumed stress-strain relationships outside yielding elements
- respects the failure conditions
- makes the contact areas coincide,

whereas generally it does not fulfill the conditions of equilibrium.

This may be obtained by disregarding in the first KP-calculation the conditions of equilibrium in such a way that they are fulfilled after the final KP- or SP-correction. The final result will then be KP- as well as SP- and is consequently named a *kinematically, statically plausible (KSP-) solution.*

Below the proposed successive procedure has been applied to a raft foundation and an embedded wall.

RAFT FOUNDATION

A raft foundation having the width B and an average load p_m is considered.

To simplify the description it has been generally assumed that bending moments are not transferred from the raft to the superstructure, that no water pressures occur and - except in fig. 5. - that the major part of the total load acts as two identical line-loads along the edges of the raft, i.e. the weight of the raft and the load acting directly on the free surface of the raft constitute a minor part of the total load p_m*B.

As long as yielding does not occur the relative stiffness of the raft may be defined as $ST = (E_s * I_s)/(E_g * B^3)$, where s and g refer to structure and ground, respectively, E are moduli of elasticity and I moment of intertia.

Stiff, Unyielding Raft

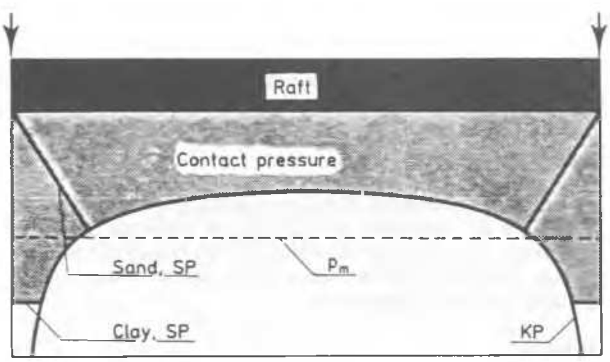

Fig. 1. Stiff, unyielding raft. KP- and SP-contact pressures.

Fig. 1 shows a raft foundation, completely rigid compared to the compressibility of the soil (1/ST=0).

KP-calculation: By the theory of elasticity the well known contact pressure (CP), marked KP in fig 1, is found with $\sigma_o = p_m*2/\pi$ in the center and $\sigma_o = \infty$ along the edges. The settlement is theoretically $\delta_o = \infty$

A KP-calculation assuming a *coefficient of subgrade reaction*

$$k = \sigma_o/\delta_m, \qquad (1)$$

where δ_m is a more reasonably estimated settlement, will result in the same KP-CP.

SP-correction: Contact pressures exceeding the bearing resistance of the soil are cut off. If as an example the soil is clay with an undrained bearing capacity= 1,5*p_m the SP-CP will be as shown shaded in fig.1, marked "Clay,SP". If the raft rests on sand without any q-contribution, the SP-CP will in principle be as marked "Sand,SP". With a q-contribution the cut-off line will represent a correspondingly greater bearing resistance.
However, in any case the KP-CP is reduced and the SP-CP is therefore insufficient for vertical equilibrium.

KSP-calculation: Assuming the coefficient of subgrade reaction (1) to be unchanged outside the areas, where the soil yields, a new KP-CP may be found by multiplying the KP-values by a factor >1, determined by the condition of vertical equilibrium after the SP-correction. The corresponding KSP-CPs are shown in fig. 2.

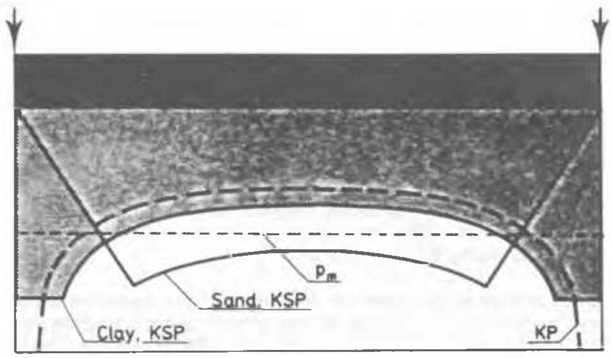

Fig. 2. Stiff, unyielding raft and yielding soil. KSP-contact pressures σ for different bearing capacities of the soil.

Greater bearing capacity of the clay would in the elastic range result in a CP closer to the KP-values, whereas smaller bearing capacities will have the opposite effect. The settlements may be determined as $\sigma/\sigma_o*\delta_m$ for the unyielding areas and are seen to increase with decreasing bearing resistance.

The smallest bearing capacity capable of ensuring vertical equilibrium is p_m corresponding to a uniformly distributed CP and the raft acting as a block foundation.

From the variation in the CPs it follows that the bending moments in the unyielding raft decrease, if the bearing resistance of the soil increases.

Stiff, Yielding Raft On Clay

If the raft in spite of the great stiffness is not able to resist the bending moments corresponding to the CPs from fig.2, the raft starts yielding. Assuming the coefficient of subgrade reaction (1) is unchanged for the not yielding part of the soil, the KSP-CP may be determined by varying the angle of rotation in the yielding hinges of the raft and finding the value resulting in vertical equilibrium in the KSP-state.

The KSP-CPs are for the clay-case shown in fig. 3 for different values of the yielding moments of the raft.

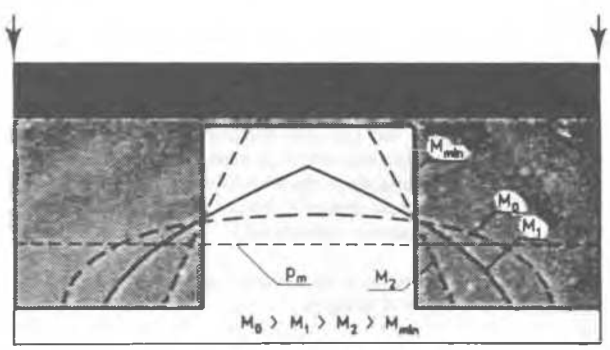

Fig. 3 Stiff, yielding raft and yielding soil. KSP-contact pressures σ for different bending resistance of the raft.

With the CP marked M_o the raft is just about to start yielding, but the settlement is still δ_m for the entire raft. For the smaller bending resistances the settlements may for the not yielding parts of the soil be determined by multiplying δ_m with the ratio σ/σ_o, where σ_o corresponds to M_o in fig.3. It appears, that the settlements at the edges increase with decreasing bending resistance of the raft reaching infinite for $M=M_{min}$, which represents the ULS-investigation prescribed in the Danish Code of Practice for Foundation Structures, Dansk Ingeniørforening (1984).

Limit State Design

If Limit State Design is applied, the Ultimale Limit State (ULS) and the Serviceability Limit State (SLS) are analysed separately.

As examples the two rafts in fig. 4 and 5 have been considered. In the elastic state the rafts are assumed to be completely rigid. In fig. 4 the load acting directly on the raft including the self-weight is negligible, i.e. the line loads represent the total load, whereas in fig. 5 the total load is uniformly distributed. The combined effect of the partial coefficients on the self-weight and the live load is assumed to be a design load = $1{,}2^*p_m$. The remaining partial coefficients are taken as 1,4 on the bending resistance M_k of the raft and 1,5 on the bearing resistance σ_k of the soil. σ_k is assumed to be = 3^*p_m.

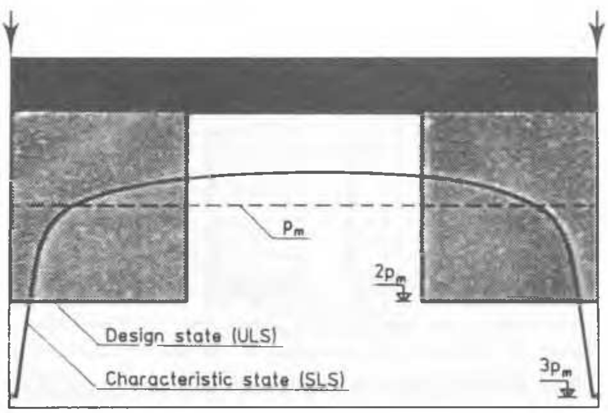

Fig. 4. CP-distribution in the Ultimate Limit State and in the Serviceability Limit State.

Fig. 4 shows the CP-distribution in the ULS and the SLS. The corresponding bending moments $M/(p_m{}^*B^2)$ are 0,090 and 0,097 respectively. ULS determines the necessary charateristic bending resistance $M_k/(p_m{}^*B^2)$ = 1,4*0,09, i.e. the combined effect of the partial coefficients on the load, the bearing resistance and the bending resistance is in this case sufficient to prevent yielding in the characteristic state (SLS). Consequently, the ULS will in many cases become decisive for the design.

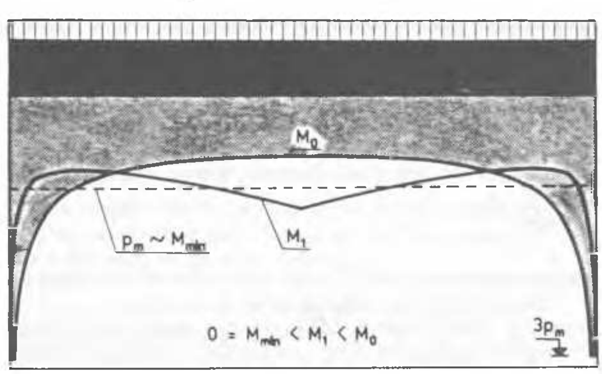

Fig. 5. CP-distribution in the Serviceability Limit State for different values of the characteristic bending resistance M.

In case of conditions as shown in fig. 5 the required bending resistance in the ULS will be 0, i.e. in this case the SLS will become decisive. In fig. 5 CP-distributions for different characteristic bending resistances M are shown. For M < M_1 yielding takes place in several points and the CP-distribution approaches p_m. For M_1 < M < Mo the raft yields in the centre. To avoid yielding (with partial coefficient 1), the requirement is M > M_o = $0{,}026^*p_m{}^*B^2$.

Flexible Rafts

If the elastic deformations of the raft shall be taken into account, the *KP-calculation* may as a fair approximation be carried out by assuming the same coefficient of subgrade reaction (1) as proposed for a stiff raft. The subsequent *SP-correction* and *KSP-calculation* may be carried out as described above. It may however become necessary to take into account the changes in elastic deformations of the raft caused by the transition from KP-CP to SP-CP. If so, *successive calculations* may be required.

BEARING RESISTANCE OF AN EMBEDDED WALL

General

As another example the vertical bearing resistance of an embedded wall installed in unloaded frictional soil with constant unit weight γ and constant angle of internal friction ϕ=30° is considered.

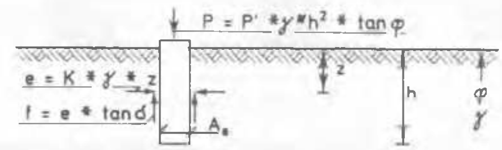

Fig. 6. Notations

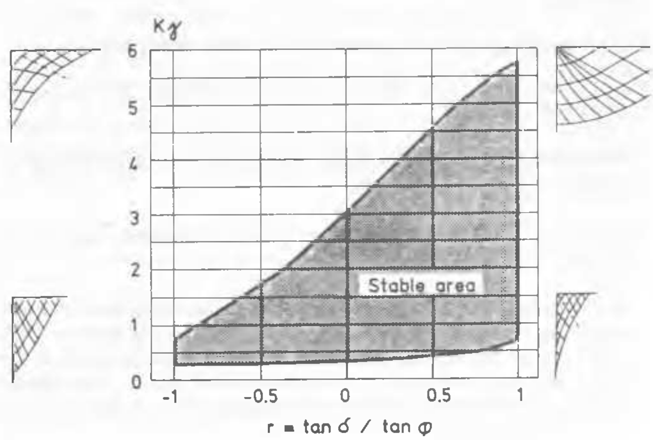

Fig. 7. Extreme earth pressures for unloaded, frictional (ϕ=30°) soil for varying wall friction r=tanδ/tanϕ.

Fig. 6 shows the notations used and fig. 7 the extreme earth pressures for unloaded frictional soil with ϕ=30° and varying direction. The earth pressures are calculated as described in Mortensen (1992). 4 typical failure figures are shown, above the diagram for emax, below for emin, on the right hand side for positive wall friction and on the left hand side for negative friction. Upper left and lower right contain discontinuity lines. The results, however, are only slightly different from the findings according to Coulomb (0,77 and 0,72 against $\cos^2\phi$=0,75), hence K=cos $^2\phi$ is considered sufficiently accurate for practical calculations.

It should be noted, that the connecting vertical lines for r=1 and -1 also represent states of failure (sliding) and that the delimited area contains all SP-earth pressures, whereas points outside this stable area represent statically impossible states.

The stress-strain relationship in the contact area is in the elastic state assumed to be governed by coefficients of subgrade reaction k defined as

$$k=A*\gamma*z/h, \tag{2}$$

where the dimensionless constant A does not necessarily have the same value in horizontal direction governing e and in vertical direction governing f.

Thin, Compressible Wall

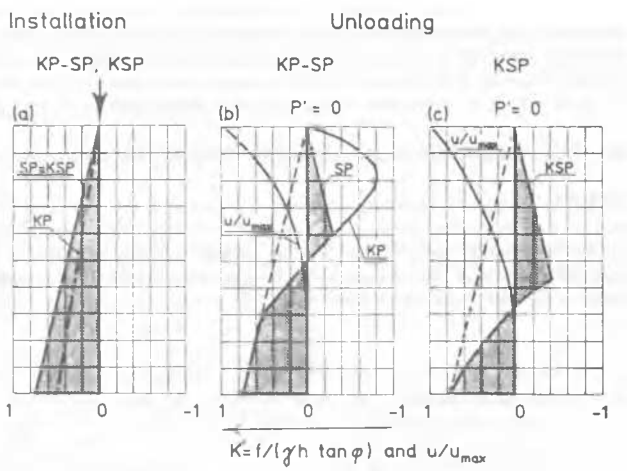

Fig. 8. Wall friction during installation by pressing and subsequent unloading of a thin, compressible wall

Installation

Prior to installation the earth pressure is assumed to correspond to

K_o = 1-sinϕ = 0,5. The wall is so thin that it causes (practically) no displacement of the soil.

KP-calculation: e is unchanged (K=0,5) as marked KP in fig. 8a because no horizontal displacement takes place.

SP-correction: With positive wall friction K must be increased to $\cos^2\phi$=0,75 to become SP as marked SP in fig. 8a.

KSP-calculation: The SP-value becomes KP by considering the horizontal displacement of the soil (=0) as the sum of an elastic movement away from the wall due to the increased horizontal stress and a plastic yield in the opposite direction, because it is an active state of failure. The vertical equilibrium is established simply by increasing the load on the top of the pile, i.e. the installation force (and later on the bearing resistance) corresponds to K=$\cos^2\phi$.

Unloading

If the pile is extremely rigid the shear forces will disappear at unloading and the horizontal stresses remain unchanged.

A compressible pile on the other hand elongates at unloading:

KP-calculation: The result depends on the relative stiffness of the wall, which may be expressed as ST= E_s*A_s/(k*h^2), A_s being the area per unit length of the wall. The necessary differential equations shall not be described, but the KP-result may be as shown in fig. 8b with equal areas between f=0 and the KP-curve on both sides of 0.

SP-correction: With negative wall friction the KP-values for the upper part of the wall must be reduced to K'=-$\cos^2\phi$=-0,75 as shaded and marked SP-in fig. 8b.

The SP-CP corresponds to a remaining vertical load on the wall, i.e. the vertical equilibrium is not satisfied for the unloaded wall.

KSP-calculation. This may be achieved by assuming in the KP-calculation a lower position of the point of no relative movement as shown in fig. 8c. By further loading and pulling, the frictional resistance corresponds to K'= 0,75 and -0,75 with unchanged horizontal stresses.

Thick Walls.

The wall is assumed to be so thick that displacement during installation causes the development of emax.

If the wall is so rigid, that the elongation is negligible (1/ST=0), the shear stresses f will vanish during unloadning and e will after unloadning and during reloading correspond to K=3, whereas pulling and later reloadning will correspond to K=0,75.

If, on the other hand, the wall is relatively compressible so that the vertical defomations of the soil becomes negligible (ST=O), the evolution progresses as shown in fig. 9, where only the final KSP-results are shown.

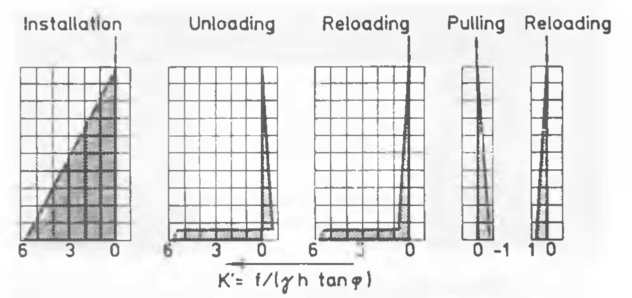

Fig. 9. Wall friction during installation, unloading, reloading, pulling and reloading a compressible, thick wall.

It appears that the wall friction may vary as follows:

Wall	Thin	Thick rigid	Thick compressible
During installation	13%	100%	100%
Reloading after unloading	13%	52%	23%
After tension failure	13%	13%	13%

Althoug the conditions considered may be extreme, the results indicate that the loading history and the stifness of the wall may be more decisive to the bearing capacity than the angle of internal friction.

Similarly the stress conditions prior to the installation as well as later changes in the stress conditions (e.g. by compaction, excavation, boring of neighbouring piles etc.) may also influence the bearing capacity considerably.

References.

Dansk Ingeniørforening (1984). *Danish Code of Practice for Foundation Engineering DS 415*. Teknisk forlag, København.

Mortensen, K. (1992). Statically admissible earth pressure and bearing capacity calculations. *Proc. International Conference on Retaining Structures*, Cambridge.