# SPREAD HINGE-TIED ("BLIND") FOUNDATION

# FONDATION EXTENDUE ASSEMBLEE PAR ARTICULATION ("JALOUSIE")

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**SYNOPSIS:** The paper deals with theoretical solutions of the interaction between structures, based on elastic spread hinge-tied foundation construction (called the "blind" foundation) and elastic-viscoplastic soft soils. The numerical solutions are based on FEM enabling large deformations in structures and non-linear time dependent methods in any stress, strain, stability and dissipation's states for soils. A new concept of numerical analysis in consolidation of highly porous soils is presented. Numerical analysis includes rheological and constitutive relationship determined on the basis of laboratory and in situ tests. By yearlong theoretical and experimental investigations, the prototype of the spread "blind" foundation was erected on site. The comparison of numerical and in-situ measurement results indicates that the given solution is suitable in general geotechnical analyses of non-linear and time dependent soil-structure interaction. The practical application on twenty-six terraced family houses resting on soft soils of the Ljubljana marsh, proves the usefulness of the suggested model.

## INTRODUCTION

Compared to conventional systems of foundation laying, the spread hinge-tied foundation constructions, called "blind" constructions, show some fundamental advantages. This is true especially in simultaneous erection of several structures on soft soils, where due to large and inconsistent settlements as a result of substantial distortion deformations, the conventional systems of shallow foundations are not acceptable.
In erection of structures on "blind" foundations, which are generally connected by foundation plates, the whole site is laid with the system of stiff, dilated, hinge-tied foundation elements. Such a system distributes the structural load via the hinge-tied joints to a larger surface, decreases distortional soil deformation and due to flexibility excellently sustains the relative settlements.
Analysing the soil-structure interaction, we have tried to find the simultaneous solution of the soil consolidation problem of the "blind" foundation system and of the structural load.

## THEORETICAL BASIS

The interactive problems in the analysed time increment ($\Delta t_r$) are solved by the bellow described statical procedure. The relationship between the generalised displacements and loads for all three substructures (structure S, foundation F and ground G) is expressed by:

$$\begin{bmatrix} \mathbf{S} & & \\ & \mathbf{F} & \\ & & \mathbf{G} \end{bmatrix} \begin{Bmatrix} \Delta \mathbf{W}_S \\ \Delta \mathbf{W}_F \\ \Delta \mathbf{W}_G \end{Bmatrix} = \begin{Bmatrix} \Delta \mathbf{R}_S \\ \Delta \mathbf{R}_F \\ \Delta \mathbf{R}_G \end{Bmatrix} \qquad (1)$$

where $\mathbf{S}$, $\Delta\mathbf{W}_S$ and $\Delta\mathbf{R}_S$ or $\mathbf{F}$, $\Delta\mathbf{W}_F$ and $\Delta\mathbf{R}_F$ are the stiffness matrix, the vector of the joint displacements and loads of the structure or the "blind" foundation respectively.

$\mathbf{G}$, $\Delta\mathbf{W}_G$ and $\Delta\mathbf{R}_G$ are the consolidation matrix and the vectors of soil displacements and loads respectively.

### "Blind" foundation construction

The static system of "blind" foundation construction is explicitly geometrical problem. At small curvatures it is linear, its deformational state can be expressed by the superposition of deformational states in elastic plates and membranes. At greater curvatures, the superposition of individual linear effects does not give adequate results. Therefore, the effects of the geometrical non-linearity (large deformations) must be taken into consideration.
Analysis of the foundation plates takes into consideration large deformations due to the vertical displacements.
The relationships between the increments of the joint displacements $\Delta\mathbf{W}_F$ and the equivalent joint loads $\Delta\mathbf{R}_F$ can be expressed using the principle of virtual work or the method of the weight residuals, which minimises the errors in numerical calculations.

$$\mathbf{F}\ \Delta\mathbf{W}_F=\Delta\mathbf{R}_F \qquad (2)$$

$\mathbf{F}$ is the stiffness matrix of the "blind" element. In analysis of hinge-tied foundation plates using isoparametric finite elements, the stiffness matrix of "blind" foundation plates, taking into consideration large deformations, can be obtained by:

$$\mathbf{F}=\int_{\mathcal{S}}[F_{ij}]d\mathcal{S} \quad ; \quad F_{ij}=F_{ji} \qquad (3)$$

$$F_{11}=aL_{i',1}L_{j',1}+bL_{i',2}L_{j',2}$$

$$F_{12}=\nu aL_{i',1}L_{j',2}+bL_{i',2}L_{j',1}$$

$$F_{13}=b((v_{3',1}+0.5dv_{3',1})L_{j',2}+(v_{3',2}+0.5dv_{3',2})L_{j',1})L_{i',2}+$$
$$a(v_{3',1}+0.5dv_{3',1})L_{j',1}L_{i',1}+\nu a(v_{3',2}+0.5dv_{3',2})L_{j',2}L_{i',1}$$

$$F_{14}=F_{15}=F_{24}=F_{25}=0$$

$$F_{22}=aL_{i,2}L_{j,2}+bL_{i,1}L_{j,1}$$

$$F_{23}=b((v_{3,1}+0.5dv_{3,1})L_{j,2}+(v_{3,1}+0.5dv_{3,1})L_{j,1})L_{i,1}+ a(v_{3,2}+0.5dv_{3,2})L_{j,2}L_{i,2}+\nu a(v_{3,1}+0.5dv_{3,1})L_{j,1}L_{i,2}$$

$$F_{33}=b((v_{3,1}+0.5dv_{3,1})L_{j,2}+(v_{3,2}+0.5dv_{3,2})L_{j,1})((v_{3,1}+ 0.5dv_{3,1})L_{i,2}+(v_{3,2}+0.5dv_{3,2})L_{i,1})+a((v_{3,1}+0.5dv_{3,1}) L_{j,1}+\nu(v_{3,2}+0.5dv_{3,1})L_{j,2})(v_{3,1}+0.5dv_{3,1})L_{i,1}+ a((v_{3,2}+0.5dv_{3,2})L_{j,2}+\nu(v_{3,1}+0.5dv_{3,1})L_{j,1})(v_{3,2}+ 0.5dv_{3,2})L_{j,2}+d(L_{j,1}L_{i,1}+L_{j,2}L_{i,2})$$

$$F_{34}=-dL_jL_{i,1}$$

$$F_{35}=-L_jL_{i,2}$$

$$F_{44}=dL_iL_j+eL_{i,2}L_{j,2}+cL_{i,1}L_{j,1}$$

$$F_{45}=eL_{j,1}L_{i,2}+\nu cL_{i,1}L_{j,2};$$

$$F_{55}=dL_iL_j+eL_{i,1}L_{j,1}+cL_{i,2}L_{j,2}$$

where i and j denote any optional joint respectively. $L_k$ is a displacement interpolation function. The constants a, b, c, d, e are evaluated by the integration according to the thickness (h) of the plate.

$$a=Eh/(1-\nu^2) \quad ; \quad b=Eh/(2(1+\nu)) \quad ; \quad c=Eh^2/(12(1-\nu))$$

$$d=Eh/(2\alpha(1+\nu)) \quad ; \quad e=Eh^2/(24(1+\nu)) \qquad (4)$$

$\alpha$ is the correction factor and $\nu$ Poisson ratio.
The vectors of the equivalent joint loads $\Delta R_F$ or joint displacement $\Delta W_F$ include five components of the joint forces or displacements at each joint (j) of the finite element.

$$\Delta H_F=\{\Delta R_1,\ \Delta R_2,\ \ldots \Delta R_j, \ldots\ \Delta R_n\}^T_F \qquad (5)$$

$$(\Delta R_j)_F=\{\Delta P_{1j},\ \Delta P_{2j},\ \Delta P_{3j},\ \Delta M_{1j},\ \Delta M_{2j}\}^T_F \qquad (6)$$

$$\Delta W_F=\{\Delta W_1,\ \Delta W_2,\ \ldots \Delta W_j, \ldots\ \Delta W_n\}^T_F \qquad (7)$$

$$(\Delta W_j)_F=\{\Delta V_{1j},\ \Delta V_{2j},\ \Delta V_{3j},\ \Delta\vartheta_{1j},\ \Delta\vartheta_{2j}\}^T_F \qquad (8)$$

n is the number of joints in the finite element, $\Delta P_{kj}$ and $\Delta M_{kj}$ or $\Delta V_{kj}$ and $\Delta\vartheta_{kj}$ are the joint forces and moments or the joint displacements and rotations in the axial directions (k) respectively .
In the incremental analysis of the soil-structure interaction, for approximate calculations, the non-linear parts in the stiffness matrix can be disregarded and the system of equations can be transformed into the system of linear equations.

**Hinge-tied connections**

Regarding the principle of the load distribution among the individual, relatively rigid foundation elements in the "blind" foundation, we declare two types of hinge-tied connections: stiff and flexible.
Line (rod) finite elements can be used for flexible connections, which are explicitly geometrically nonlinear. The stiffness matrix is evaluated for each element on the deformed static system, taking into consideration the deformations at the beginning of the analysed increment.

The relationship between the variations of the joint loads and displacements in the local co-ordinate system is expressed by

$$F^{*}\Delta W^{*}_F=\Delta R^{*}_F \qquad (9)$$

where $F^{*}$, $\Delta W^{*}_F$ and $\Delta R^{*}_F$ represent the connection stiffness matrix, the vector of the connection displacements and loads respectively. They are evaluated on the deformed static system in local co-ordinates $(1^{*},2^{*},3^{*})$, where axis $(1^{*})$ corresponds to the axis of the rod element.
The stiffness matrices of all hinge-tied connections along the discussed edge are evaluated into new innovative final element "HINGE".
In engineering practice, the stiffness matrix of hinge-tied connections is determined experimentally. For approximation analysis, however, laboratory evaluations are also possible, considering the material and geometrical non-linearites.
The finite element "HINGE" for the flexible connections is determined on the assumption that the connections carry axial loads only. The stiffness matrix of the element in the time increment $\Delta t_r$ is defined on the deformed static system, the displacements at the end of the preceding increment is $\Delta t_{r-1}$.

$$F^{*}=\int_{\wp}(E_m\wp_r/\ell^2_r)[F^{*}_{ij}]d\wp \quad ; \quad F^{*}_{ij}=F^{*}_{ji} \qquad (10)$$

$$F^{*}_{11}=(1+v_{1^*,1})^2L_{j,1}L_{i,1} \quad ; \quad F^{*}_{12}=(1+v_{1^*,1})^2v_{2^*,2}L_{j,1}L_{i,2}$$

$$F^{*}_{13}=(1+v_{1^*,1})^2v_{3^*,3}L_{j,1}L_{i,3} \quad ; \quad F^{*}_{22}=v^2_{2^*,2}L_{j,2}L_{i,2}$$

$$F^{*}_{23}=v^2_{2^*,2}v^2_{3^*,3}L_{j,2}L_{i,3} \quad ; \quad F^{*}_{33}=v^2_{2^*,2}v^2_{3^*,3}L_{j,2}L_{i,3}$$

where $E_m$, $\wp_r$ and $\ell_r$ denote the module of elasticity, mean cross section of the hinge-tied joints per length unit of the connection and ratio of the flexible connection lengths in the deformed and undeformed state expressed as follows:

$$\ell_r=1+2v_{1^*,1}+v^2_{1^*,1}+v^2_{2^*,2}+v^2_{3^*,3} \qquad (11)$$

**Rheological relationship for soils**

Rheological characteristics of highly porous coherent soils are given as the relationship between the invariants of stress (J) and strain (I) tensors at time (t). The influence of the loading history in the discussed rheological model is considered by analytical functions and parameters of the greatest values of invariants (J) and (I) of the soil exposure in the recent past.
The normal (p) and shear (q) invariant of the stress tensor (J), or (r) and (s) of the strain tensor (I) respectively, are evaluated:

$$p=\delta_{ij}\sigma'_{ij}/3; \quad q=\sqrt{(S_{ij}S_{ij})/3} \ ; \quad S_{ij}=\sigma'_{ij}-\delta_{ij}p \qquad (12/14)$$

$$r=\delta_{ij}\varepsilon_{ij}/3; \quad s=\sqrt{(4\gamma_{ij}\gamma_{ij})/3} \ ; \quad \gamma_{ij}=\varepsilon_{ij}-\delta_{ij}r \qquad (15/17)$$

$\sigma'_{ij}$ and $S_{ij}$ denote the components of effective stress tensor (J) and its distortional part $(J^d)$, whereas $\varepsilon_{ij}$ and $\gamma_{ij}$ denote the components of strain tensor (I) and its distortional part $(I^d)$ respectively. Normal (r) and shear (s) strains can be expressed as follows:

$$r=(r_o)+r_v=(r_e+r_p)+r_v \qquad (18)$$

$$s=(s_{o})+s_{v}=(s_{e}+s_{p})+s_{v} \tag{19}$$

where the subscripts (o), (v), (e) and (p) denote the elasto-plastic, viscous, elastic and plastic normal (r) or shear (s) strains.

**Consolidation**

Incremental analysis of soils consolidation requires fulfilment of the equilibrium as well as conditions of continuity of fluid (water and air) filtration in all differentially small elements in the analysed space.

$$\dot{\sigma}_{ij',i}-f_{i}=0 \tag{20}$$

$$(k_{ij}u_{w',j})_{,i}-\gamma_{w}(\beta\dot{u}_{w}-\dot{\varepsilon}_{v})=0 \tag{21}$$

where $\sigma_{ij}$ is the tensor of total stress, $f_{i}$ vector of volumetric forces, $k_{ij}$ tensor of fluid permeability of soils, $u_{w}$ pore water pressure, $\gamma_{w}$ unit water weight, $\beta_{w}$ fluid compression coefficient and $\varepsilon_{v}$ volumetric deformation. Stress changes $d\sigma$, which consist of elastoplastic $d\sigma^{ep}$ and viscous $d\sigma^{v}$ components, are the following:

$$d\sigma_{ij}=d\sigma^{ep}_{ij}+d\sigma^{v}_{ij}+\delta_{ij}du_{w}=C_{ijkl}d\varepsilon_{kl}+c_{ij}dt+\delta_{ij}du_{w} \tag{22}$$

$$C_{ijkl}=E^{*}((1/3G-2/9K+(2/3L+1/3F)p/q)\delta_{ij}\delta_{kl}+(2/3K)\delta_{ik}\delta_{jl}-$$
$$-(1/3Fq)\delta_{kl}\delta_{ij}-(2/3Lq)\ \delta_{kl}\delta_{ij}) \tag{23}$$

$$c_{ij}=E^{*}(\dot{r}(S_{ij}/Fq-\delta_{ij}/G)+\dot{s}(\delta_{ij}/L-S_{ij}/3Kq)) \tag{24}$$

$C_{ijkl}$ and $c_{ij}$ are components of constitutive matrix and viscous creep vector, r and s are spheric and distortional deformational rates respectively. Deformation modules K, G, L and F are determined numerically from Eq. (25) to (28) for loading and from Eq. (29) to (32) for unloading stress states.

$$K=r_{o',p};\quad G=s_{o',q};\quad L=r_{o',q};\quad F=s_{o',p} \tag{25/28}$$

$$K=r_{e',p};\quad G=s_{e',q};\quad L=r_{e',q};\quad F=s_{e',p} \tag{29/32}$$

Using weight residuals and finite element method, the incremental equation of the soil consolidation is given as a system of linear equations:

$$G\ \Delta W_{G}=\Delta R_{G} \tag{33}$$

G, $\Delta W_{G}$ and $\Delta R_{G}$ represent the consolidation matrix, the vector of the changes of displacement V and/or pore pressures U and the vector of load variations in finite element nodes for the analysed time increment $\Delta t_{r}$.

$$G=\int_{V}[G_{mn}]dV \tag{34}$$

$$G_{11}=L_{i',k}L_{j',l}C_{1kl1};\quad G_{12}=L_{i',k}L_{j',l}C_{1kl2};\quad G_{13}=L_{i',k}L_{j',l}C_{1kl3}$$

$$G_{14}=-L_{i',1}S_{j};\quad G_{21}=L_{i',k}L_{j',l}C_{2kl1};\quad G_{22}=L_{i',k}L_{j',l}C_{2kl2}$$

$$G_{23}=L_{i',k}L_{j',l}C_{2kl3};\quad G_{24}=-L_{i',2}S_{j};\quad G_{31}=L_{i',k}L_{j',l}C_{3kl1}$$

$$G_{32}=L_{i',k}L_{j',l}C_{3kl2};\quad G_{33}=L_{i',k}L_{j',l}C_{3kl3};\quad G_{34}=-L_{i',3}S_{j}$$

$$G_{41}=-L_{i}S_{j',1};\quad G_{42}=-L_{i}S_{j',2};\quad G_{43}=-L_{i}S_{j',3}$$

$$G_{44}=-\beta S_{i}S_{j}-\Delta t k_{kl}S_{i',k}S_{j',l}/2\gamma_{w}$$

$L_{i}$ and $S_{i}$ denote the interpolation functions of displacements and pore pressures in finite element field.

$$\Delta W_{G}=\{\Delta W_{1},\ \Delta W_{2},\ \Delta W_{3},\dots\ \Delta W_{j},\dots\Delta W_{n}\}^{T}_{G} \tag{35}$$

$$(\Delta W_{j})_{G}=\{\Delta V_{1j},\ \Delta V_{2j},\ \Delta V_{3j},\ \Delta U_{j}\}^{T}_{G} \tag{36}$$

$$\Delta R_{G}=\{\Delta R_{1},\Delta R_{2},\dots\Delta R_{j},\dots\Delta R_{n}\}^{T} \tag{37}$$

$$(\Delta R_{j})_{G}=\int_{V}\begin{bmatrix}(f_{1}L_{j}+L_{j',k}C_{1k})\\(f_{2}L_{j}+L_{j',k}C_{2k})\\(f_{3}L_{j}+L_{j',k}C_{3k})\\(1/\gamma_{w})S_{j',k}S_{i',j}k_{kj}U_{i(r-1)}\end{bmatrix}dV\Delta t+\int_{\mathcal{S}}\begin{bmatrix}p_{1}L_{j}\\p_{2}L_{j}\\p_{3}L_{j}\\0\end{bmatrix}d\mathcal{S}\Delta t \tag{38}$$

$p_{i}$, $V$ and $\mathcal{S}$ are components of surface loadings, volume and surface of the finite element respectively.

**PRACTICAL APLICATION ON FAMILY HOUSES**

Based on above-stated theoretical solutions the computer program for the soil-structure interaction problem (named SPACESOIL) was developed. By its use we have analysed a numerical example of the prototype constructition, erected in 1988/89 on very soft soils in the Ljubljana marsh.

From 1989 to 1991 twenty six terraced family houses were build on the spread hinge-tied foundations. The foundation was carried out using reinforced concrete hinge- tied dilatation plates between individual houses.

Geometrical and stratigraphicall data are given in Fig. 1. Some results are presented graphicaly in Fig. 2.

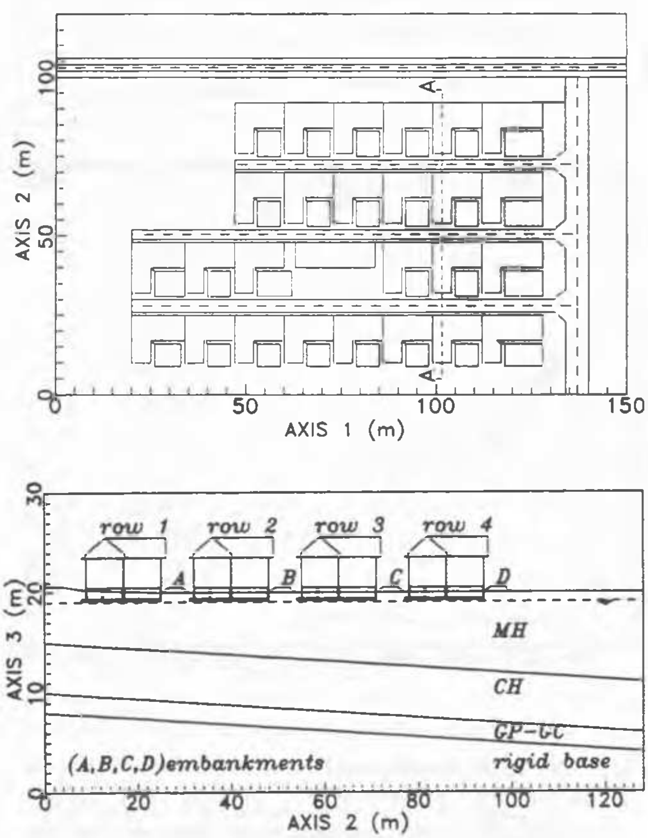
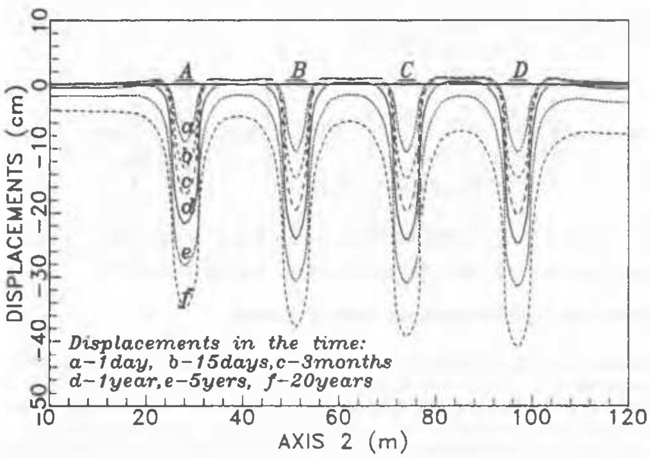
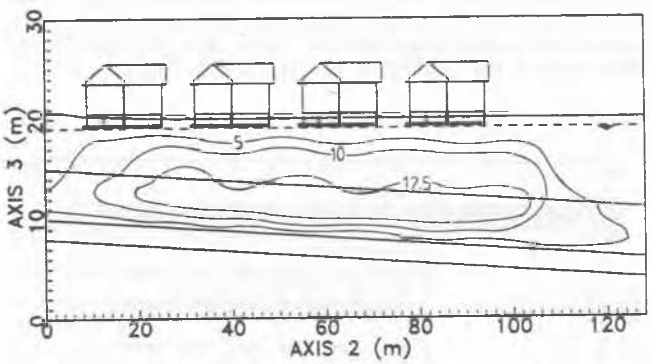



Fig. 1. Geometrical and stratigraphical data

(a)




(b)




(c)

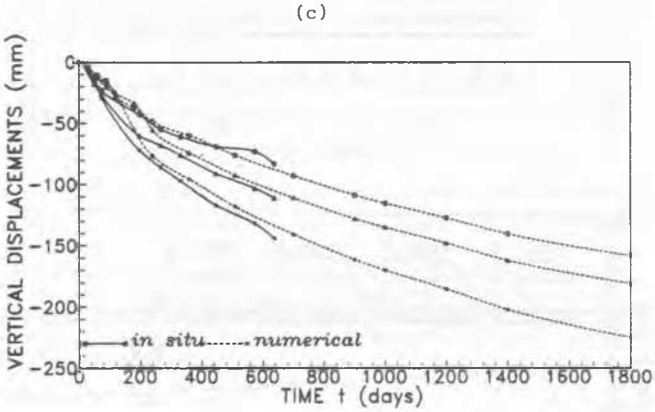



**Fig.2.** (a) Time dependent vertical displacements of the half space surface with embankments; loading only (b) isobares of pore pressures; six months after construction, (c) comparison of numerical results and in situ measurements for settlements

**Fig.3.** Details of spread hinge-tied foundation construction

**CONCLUSION**

Obtained theoretical solutions and the model of the spread hinge - tied foundation construction provide sufficiently accurate soil-structure solutions in engineering practice. In addition, nonlinear and time dependent soil characteristics, water diffusion and pore pressure dissipation, construction geometry and soil stratigraphy, various boundary conditions as well as time variation of loading can be considered simultaneously. The results presented confirm our assumptions. Such a foundation can be used on very soft soils, where conventional shallow foundation (on a separate foundation elements) is no more feasible but the deep foudation (on piles) is too expensive.

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