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# PILE-SOIL INTERACTION DUE TO STATIC AND DYNAMIC LOAD
# PIEU-SOL INTERACTION SOUS CHARGE STATIQUE ET DYNAMIQUE

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SYNOPSIS : The reliability of the pile bearing capacity predictions from dynamic load test results was studied. A piecewise linear elasto-plastic solution was presented to the nonlinear pile-soil interaction (PSI) model. The PSI parameters were identified by solving a nonlinear least-square data fitting problem on the basis of optimization theory. The Euclidean norm of the nonlinear residual function was minimized by a variant of the generalized secant method. The identifiability of the model parameters was tested on synthetic noise-free data set. The PSI parameters were identified from static load test results, and they were compared with PSI parameters identified from dynamic load test results. The validity of the static PSI model proved to be very good for the shaft resistance, and seemed improvable for the pile-tip resistance.

## INTRODUCTION

The validity of the results of dynamic load tests (DLT) on foundation piles has always been discussed since this quality assurance method was introduced into practice in the early 70's, as an economical testing procedure for pile foundations. The main source of doubts was the well-known fact that the pile-soil interaction (PSI) system behaves totally different under dynamic and static loading conditions. It is a great matter in dispute whether the static load-settlement behaviour can be predicted from DLT results. An evident way of testing it is to perform both dynamic and static load tests (SLT) on the same pile, and to compare the results. This comparison can be made on the 'observation level' i.e. to compare the load-settlement relations predicted from DLT and observed during SLT. However, this comparison can be made on a deeper level ('model level'), i.e. to compare the dynamic and the static PSI models identified from DLT and SLT results respectively. The PSI parameters were identified from field test data in this case, and conclusions are drawn both from the predicted and the observed load-settlement behaviour of the PSI system and from the estimated PSI parameters.

A static bearing capacity prediction contest was organized on the 4th International Conference on the Application of Stress Wave Theory to Piles (The Hague, The Netherlands, 1992) to make it possible to all interested participants to predict the SLT results before SLT have been performed on the test piles. The author of this paper also participated on this contest (Berzi, 1992), and the results are discussed herein.

A solution is presented to a static PSI model. This model was based on the dynamic PSI model proposed by Smith (1960). The PSI structure was analyzed according to the rules of solid mechanics. A closed form solution was derived for the linear elastic model, and the elasto-plastic model was solved by piecewise linearization (generalized Newton-Rhapson iteration). The static PSI parameters were estimated on the basis of optimization theory. A nonlinear least square data fitting problem was solved by a variant of the generalized secant method (Wolfe, 1965; Berzi et al. 1992). The results were compared both on the 'model' and on the 'observation' level, and conclusions were drawn on the validity of the static PSI model from the results of these comparisons.

## STATIC PILE-SOIL INTERACTION MODEL

The load-settlement relationship of the pile head was simulated by an elasto-plastic PSI model that was based on the dynamic PSI model proposed by Smith (1960). The viscous property of the PSI system was ignored in case of static loading, and it was assumed that the overall non-linearity of the load settlement relationship was purely due to the development of plastic sliding at the pile shaft and at the pile end. The pile was represented by a series of lumped masses connected by linear elastic springs to each other, while the PSI model consisted of the composites of linear elastic springs and friction elements attached to all pile elements (Fig.1.). The PSI model parameters

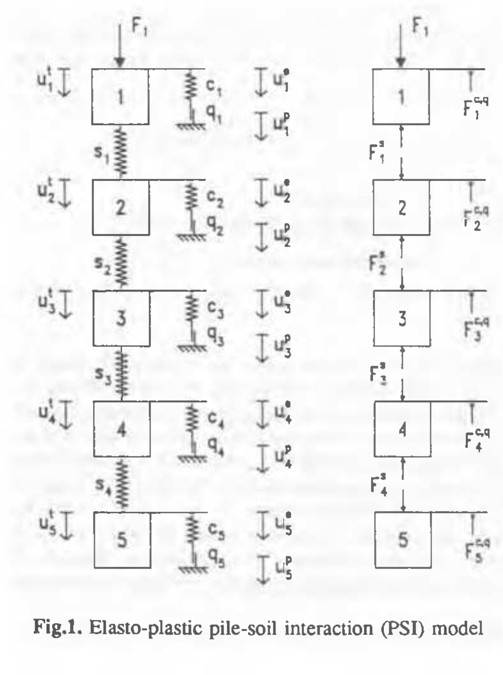
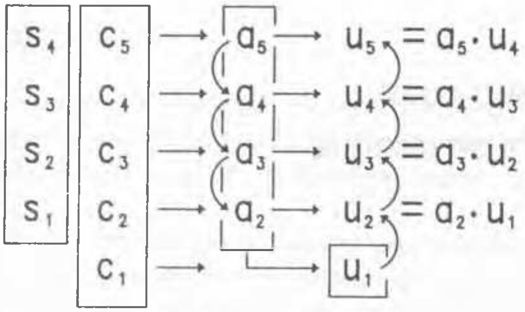


**Fig.1.** Elasto-plastic pile-soil interaction (PSI) model



$$a_j = \frac{S_{j-1}}{S_{j-1} + C_j + \sum_{k=j+1}^{n} C_k \cdot \prod_{l=j+1}^{k} a_l}$$

$$u_1 = \frac{F_1}{C_1 + \sum_{k=2}^{n} C_k \cdot \prod_{j=2}^{k} a_j}$$

Fig.2. Solution of the linear elastic PSI model

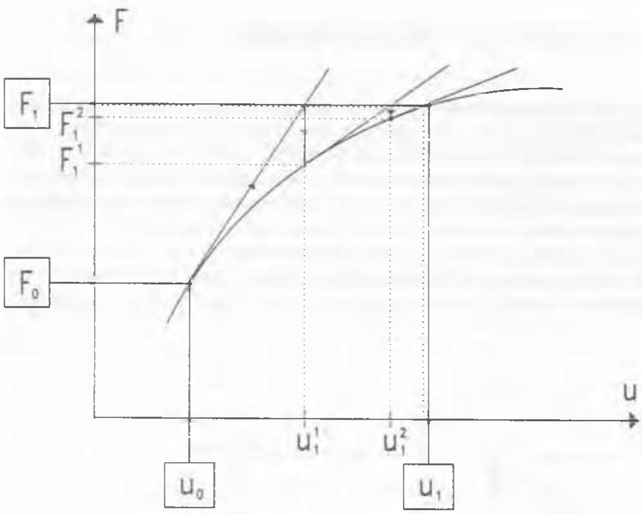



Fig.3. Solution of the elasto-plastic PSI model

were the spring stiffnesses ($C_i$), and the quake values ($Q_i$) of the pile elements.

The elasto-plastic analysis was performed by a piecewise linear elastic calculation process. A closed form solution and calculation scheme (Fig.2.) was derived from the equilibrium, compatibility and constitutive equations to the linear elastic system. The displacement of the pile top was expressed by the force acting at the pile top divided by an expression that characterized the **K(u)** stiffness of the pile head against this force. The **K(u)** pile head stiffness was depending only on the stiffnesses of the linear elastic springs characterizing the pile material ($S_i$) and the elastic PSI ($C_i$) (i=1,..,k, where k is the number of pile elements). The calculation scheme (Fig.2.) corresponds to the recursive inversion of the **K(u)** tridiagonal stiffness matrix.

The non-linear elasto-plastic PSI system was analyzed through an iterative procedure where linear elastic analysis was performed in each iteration step. The iteration was based on a generalized version of the Newton-Rhapson method, where the $\mathbf{F}^s$ and $\mathbf{F}^{c,q}$ reaction forces and the $\mathbf{u}^t$, $\mathbf{u}^e$ and $\mathbf{u}^p$ displacements of the pile elements (force and displacement vectors) were involved in the iteration (Fig.1.).The s, c and q superscripts of the reaction forces refer to the PSI parameters, while the t, e and p superscripts of the displacements refer to the total, elastic and plastic displacements, respectively. The iterative procedure is shown on Fig.3. in the scalar case. The initial $\mathbf{K}(\mathbf{u}_0)$ linear elastic stiffness was determined for the elato-plastic PSI structure that characterized the structure at the $\mathbf{u}_0=\mathbf{u}^t(t_0)$ initial displacement, where $t_0$ is the initial time instant. The

$$u_1^1 - u_0 = K^{-1}(u_0) \cdot (F_1 - F_0) \qquad \textbf{(1)}$$

displacement increment was determined from the $\mathbf{F}_1$-$\mathbf{F}_0$ load increment and from the initial $\mathbf{K}(\mathbf{u}_0)$ stiffness of the PSI structure, where $\mathbf{F}_1$ is the actual load on the top of the pile and $\mathbf{F}_0$ is the preload that was acting in the previous load level. The $\mathbf{K}(\mathbf{u}_1^1)$ linear elastic stiffness was updated according to the $\mathbf{u}_1^1$ actual displacements, and the $\mathbf{F}_1$-$\mathbf{F}_0$ original load increment was reduced to the $\mathbf{F}_1$-$\mathbf{F}_1^1$ load increment belonging to the $\mathbf{u}_1^1$-$\mathbf{u}_0$ actual displacement increment. The $\mathbf{u}_1^2$-$\mathbf{u}_1^1$ linear elastic response of the new structure was recalculated using the $\mathbf{F}_1$-$\mathbf{F}_1^1$ reduced load and the $\mathbf{K}(\mathbf{u}_1^1)$ updated linear elastic stiffness of the PSI structure. Then the $\mathbf{K}(\mathbf{u}_1^2)$ linear elastic stiffness was updated and the load increment was reduced to $\mathbf{F}_1$-$\mathbf{F}_1^2$ again. This procedure had been repeated until the $\mathbf{F}_1$-$\mathbf{F}_1^i$ load increment reduced to zero.

## PARAMETER IDENTIFICATION

The $C_i$ and $Q_i$ (i =1..k) parameters of the static PSI model were of interest in the identification procedure, where k was the number of pile elements in the PSI model. The least square method (Fig.4.) was applied to minimize the

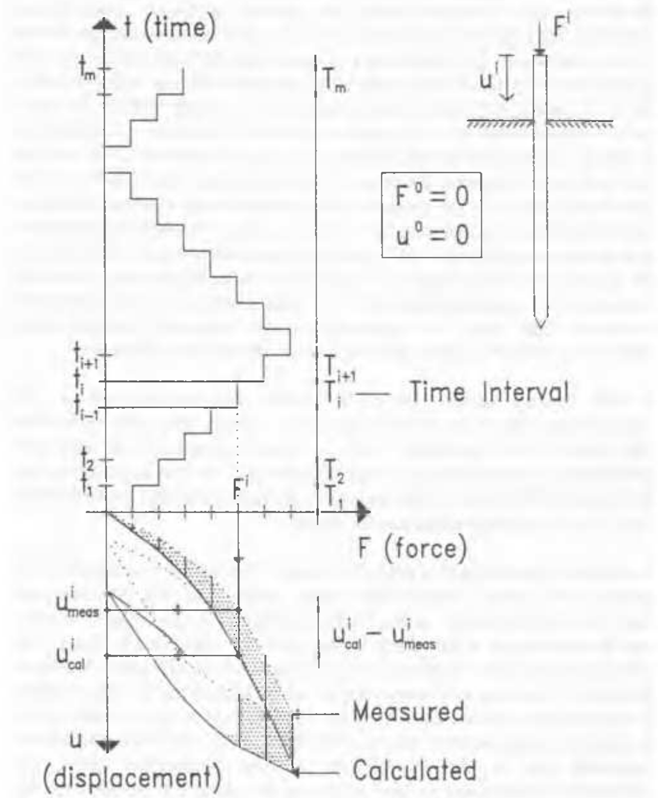



Fig.4. Objective of parameter estimation

$$R(p) = \left(\int_{t_0}^{t_1} [r(F(t),p)]^2 dt\right)^{\frac{1}{2}} = MIN. \qquad (2)$$

Euclidean norm of the

$$r(F(t),p) = u_o(F(t)) - u_s(F(t),p) \qquad (3)$$

nonlinear residual function, where $u_o(F(t))$ is the observed and $u_s(F(t),p)$ is the simulated load-settlement relationship of the pile head, F(t) is the load history applied on the pile head, t is the time, and **p** is the parameter vector with n unknown components. The residual function was sampled in m > n discrete values of its t independent variable.

The problem of parameter estimation lead to the solution of a nonlinear least square data fitting problem that was solved by a variant of the generalized secant method (Wolfe, 1965; Berzi et al. 1992). The n unknown components of the model parameter vector $\mathbf{p}^*$ was sought which provided **r(p)** sampled residual function values (residual vector with m components) as closely to zero as possible. The **r(p)** residual vector was linearly interpolated between n+1 points of the m dimensional vector space. These $\mathbf{r}(\mathbf{p}_i)$ vectors were determined from the $\mathbf{p}_i$ (i=0,..,n) trials for the $\mathbf{p}^*$ unknown parameter vector. The best fit $x=[x_1,...,x_n]^T$ linear combination of these interpolation points was determined by solving a linear least square problem that corresponded to the solution of the **Ax=b** overdetermined system of linear equations, where the $\mathbf{r}(\mathbf{p}_i)$-$\mathbf{r}(\mathbf{p}_0)$ (i=1,..,n) vectors formed the mxn type **A** coefficient matrix, and $\mathbf{b}=\mathbf{r}(\mathbf{p}_0)$ was a vector with m components. The $p_{n+1}$ new approximation to the model parameters was given by the same **x** linear combination of the $\mathbf{p}_i$-$\mathbf{p}_0$ vectors as it was determined for the residual vectors. Then again n+1 initial trials were selected for the $\mathbf{p}^*$ unknown parameter vector to calculate the next approximation. This new set of trial vectors was created by the perturbation of the $\mathbf{p}_{n+1}$ approximating vector that was determined in the present iteration step.

**Table 1.** PSI Parameters for Identifiability Tests

| Pile element | Soil Stiffness kN/cm | Quake cm |
|---|---|---|
| 1, 2, 3, 4 | 100 | 0.400 |
| 5, 6, 7, 8 | 250 | 0.300 |
| 9, 10 | 900 | 0.600 |

**Table 2.** Estimated Parameters (n = 2)

| Iteration | $C_i$ (i =5,6,7,8) kN/cm | $Q_i$ cm | Residual (R(p)/m) mm |
|---|---|---|---|
| 0 | 150.000 | 0.10000 | 42.62795 |
| 1 | 170.178 | 0.29974 | 5.53462 |
| 2 | 179.709 | 0.30857 | 2.45527 |
| 4 | 208.541 | 0.30945 | 0.64580 |
| 6 | 249.518 | 0.30040 | 0.00089 |
| 8 | 250.000 | 0.29999 | 0.00003 |

**Table 3.** Estimated parameters (n = 1)

| $C_i$ (i = 5,6,7,8) kN/cm | $Q_i^*$ cm | Residual (R(p)/m) mm | $C_i \cdot Q_i$ kN |
|---|---|---|---|
| 749.742 | 0.10000 | 0.13543 | 74.9742 |
| 374.946 | 0.20000 | 0.06874 | 74.9892 |
| 250.000 | 0.30000 | 0.00002 | 75.0000 |
| 187.510 | 0.40000 | 0.06553 | 75.0040 |
| 150.023 | 0.50000 | 0.12579 | 75.0115 |
| 125.051 | 0.60000 | 0.17947 | 75.0306 |

* fixed value

## IDENTIFIABILITY TESTS

The aim of identifiability tests was to check whether the $u_o(F(t))$ observed load-settlement relation of the pile head (SLT results) contained sufficient information about the unknown $C_i$ and $Q_i$ model parameters. Numerical tests were performed on synthetic noise-free data set. The load-settlement relation of the pile head was simulated on the basis of the static PSI model by assuming a set of PSI model parameters shown in Table 1., and this load-settlement relation was considered as 'observation' for identifiability tests.

Then the iteration was started with an initial guess on the model parameters (Table 2.). The values of the parameters was systematically modified until the (2) Euclidean norm of the (3) residual vector decreased to a sufficiently small prescribed level. The variation of the estimated parameters are shown in Table 2 through the iterative process. The true values of the $C_i$ and $Q_i$ model parameters (250, 0.3) were located in a very efficient way in 8 iteration step in spite of the bad initial guesses on the parameters (150, 0.1).

The results of these numerical tests also showed, that the estimated $C_i$ and $Q_i$ parameters were very sensitive to the noise originated either from measurement errors or from incorrect modeling. This phenomena comes from the nature of the (2) objective function of the parameter estimation problem. This function has similar characteristics as the well-known Rosenbrock's test function (Fletcher, 1987). The contours of this function look like a long, narrow, curved valley in case of a two unknown parameters problem (n=2). This fact is illustrated by an example, where the $C_i$ parameters were estimated for incorrect fixed values of the $Q_i$ parameters (Table 3.).

The multiplication of the $C_i$ and the $Q_i$ parameters (local soil resistance around the pile) defined the 'bottom' of the valley. It follows from this fact, that these parameters cannot be identified properly from static load test results, but their multiplications can be identified well. In other words, the observed and the best-fit simulated system responses are very close to each other while the values of the parameters are still very far from the true values, but their multiplications agree well with the corresponding multiplications of the true values.

## RESULTS

The results of DLT and SLT were compared for a 19 m long precast concrete driven pile with 625 $cm^2$ cross sectional area (square). The Young's modulus of the pile material was 4000 $kN/cm^2$. The PSI parameters (Table 4.) were estimated from the DLT results (Fig.5.) before the SLT had been performed.

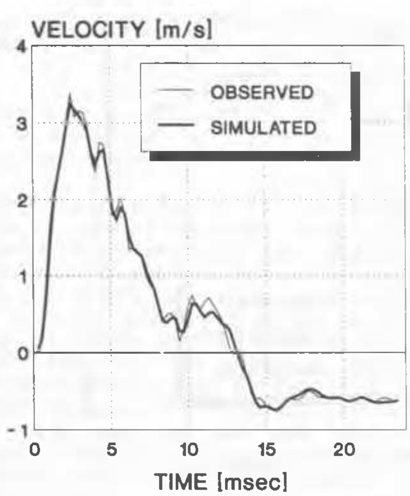



Fig.5. Dynamic load test results

**Table 4.** Estimated Dynamic PSI Parameters (Berzi, 1992)

| Pile element | Soil Stiffness kN/cm | Quake cm | Local Resistance kN | Total Resistance kN |
|---|---|---|---|---|
| 1, 2 | 20 | 0.707 | 14 | 28 |
| 3, 4 | 322 | 0.381 | 123 | 274 |
| 5, 6 | 25 | 0.021 | 1 | 276 |
| 7, 8 | 135 | 0.318 | 43 | 362 |
| 9, 10 | 1437 | 0.253 | 364 | 1090 |

**Table 5.** Estimated Static PSI Parameters

| Pile element | Soil Stiffness kN/cm | Quake cm | Local Resistance kN | Total Resistance kN |
|---|---|---|---|---|
| 1, 2 | 139 | 0.058 | 8 | 16 |
| 3, 4 | 905 | 0.074 | 67 | 150 |
| 5, 6 | 314 | 0.099 | 31 | 212 |
| 7, 8 | 836 | 0.079 | 66 | 344 |
| 9, 10 | 1301 | 0.261 | 340 | 1024 |

**Table 6.** Settlements from Dynamic and Static Tests

| Load kN | Settlement mm | | | Prediction Error* | |
|---|---|---|---|---|---|
| | Dynamic Test | Static Test | | mm | % |
| | Predicted | Observed | Simulated | | |
| 100 | 0.702 | 0.300 | 0.424 | 0.402 | 57.3 |
| 200 | 1.408 | 0.800 | 0.872 | 0.608 | 43.2 |
| 300 | 2.114 | 1.450 | 1.387 | 0.664 | 31.4 |
| 400 | 2.820 | 2.100 | 2.104 | 0.720 | 25.5 |
| 500 | 3.526 | 2.850 | 2.941 | 0.676 | 19.2 |
| 600 | 4.232 | 3.700 | 4.012 | 0.532 | 12.6 |
| 700 | 4.938 | 4.550 | 5.092 | 0.388 | 7.9 |
| 800 | 5.766 | 5.700 | 6.172 | 0.066 | 1.1 |
| 900 | 6.716 | 7.550 | 7.252 | 0.834 | 12.4 |
| 1000 | 7.698 | 9.500 | 8.375 | 1.802 | 23.4 |

* Predicted-Observed

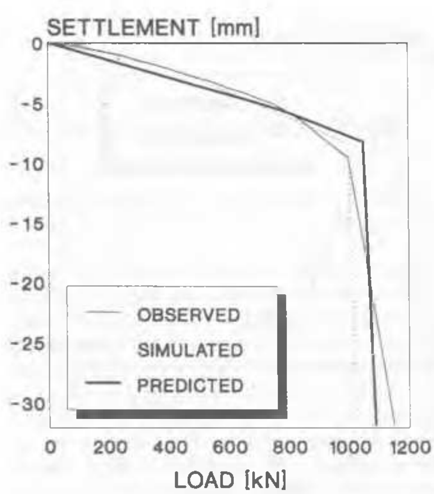



**Fig.6.** Load-settlement relations

The local soil resistances were determined for all pile elements, and they were added up along the pile from the top to the bottom (total soil resistance). The predicted total bearing capacity of the pile was 1090 kN, and the end bearing of the pile was dominant (728 kN) according to the DLT. After performing the SLT, the parameter estimation procedure was repeated on the basis of the static PSI model that was presented in this paper. The estimated parameters and soil resistances are shown in Table 5.

The values of the PSI parameters obtained from DLT and SLT were significantly different from each other, the identified soil resistances agreed well, though. The simulated total bearing capacity of the pile was 1024 kN, and the end bearing of the pile was dominant (680 kN) according to the estimated PSI parameters from the SLT. The load-settlement relations are shown in Table 6. and Fig.6. These relations were predicted from DLT results, they were observed during SLT and they were simulated according to the PSI parameters estimated from SLT results. The prediction error in Table 6. was defined as the difference between the predicted and the observed load-settlement relations. This error was less then 1 mm until the the end bearing of the pile was mobilized. The prediction error was much higher when the end bearing of the pile was fully mobilized.

## DISCUSSION

An elasto-plastic PSI model was presented based on the PSI model suggested by Smith (1960) for the evaluation of DLT results. The PSI parameters were identified from SLT results, and they were compared with the values obtained from DLT results. Though the static and dynamic PSI parameters were significantly different, the predicted static load-settlement relation agreed well with the observed one until the end bearing of the pile was mobilized. The inaccuracy of the prediction in case of fully mobilized end bearing indicates that the end bearing model is still improvable. However, the reliability of the PSI model proved to be very good in case of the practically more important shaft resistance that arises due to the working load.

Difficulties were encountered when the PSI parameters were estimated from SLT results due to the nature of the objective function applied for parameter estimation. The PSI parameters were not properly identifiable. However, this fact didn't disturb the identified soil resistance values very much.

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