STRESSES UNDER AN ECCENTRICALLY LOADED FOOTING ON A SAND LAYER
PRESSIONS SOUS UN PIED BIZARRÉNET CHARGE SUR UNE COUCHE DE SABLE

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SYNOPSIS: Model tests were conducted to investigate the distribution of stresses beneath an eccentrically loaded strip footing. The footing rests on a sand layer overlying a rigid stratum. Tests were performed on sand layers of different thicknesses in a glass sided sand bed tank. Plane strain conditions were maintained using lubricated rubber membranes on the side walls of the tank. A mechanical sand spreader was used to form the sand bed layer of uniform density. The relative density and friction angle of the bed sand was 88 percent and 36.5 degrees respectively. The footing base has the dimensions of S2, Grit No. 40) were glued on to the footing base. The terminology used in this paper for physical quantities are defined in Fig. 1. The important results of the investigation concerning eccentricity ratio (e/B) upto 1/6 are reported.

EXPERIMENTAL SYSTEM

The investigation was carried out on a laboratory model. The experimental system consists mainly of a tank, a sand spreader, a loading device, an instrumented footing, displacement measuring transducers, a data recording system, a sand levelling apparatus and a photographic camera. Details of experimental system can be found in Abedin (1986) and Ameen (1990).

INTRODUCTION

The design of any footing foundation requires the knowledge of both magnitude and distribution of stresses acting in the foundation system. The internal stresses in the footing have to be known in order to provide protection to high stress zones.

Many theoretical and experimental attempts have been made on the distribution of stresses beneath the foundations, particularly, those constructed over a semi-infinite layer. However, only a limited amount of work on this particular aspect dealing with finite layer thickness is reported in the literature. Short review of the previous work on stress distribution considering various aspects can be found in Al-Omari (1984), Abedin (1986) and Ameen (1990). This study on stress distribution using eccentric loading, and finite soil layer thickness can be considered to be very rare. In this respect, the present study can claim itself to be one of the pioneering works in the field.

In this investigation, tests were carried out using a rigid strip footing resting on a sand layer overlying a rigid stratum. Load cells were placed in across the footing width (B) to study the distribution of normal and/or shear stresses. Strain controlled loading device was used and photographs of the soil layer were taken through side wall of the tank. The deformation of the layer was studied using the stereo-photogrammetric technique of Andrawes (1976). Load eccentricities, as a ratio to the width of footing (e/B), of 0, 1/12, 1/6 and 1/3 were used. The layer thickness, H, was maintained at H/B ratios of 0.5, 1.0, 1.5 and 2.5. Both surface and shallow depth footings were tested. Depth of surcharge, D was 0.5 times the footing width. The terminologies used in this paper for physical quantities are defined in Fig. 1. The important results of the investigation concerning eccentricity ratio (e/B) upto 1/6 are reported.

The Loch Aline sand used in this research is a white medium sand. The maximum and minimum proxities are 34% and 45% respectively. Sand beds of different thicknesses were formed in the tank where a constant relative density of 88% had been maintained, using a sand spreader of the type described by Walker et. al. (1967). The triaxial friction angle of the sand bed sand was 36.5 degrees at a confining pressure of 138 kPa.

The frames of sand bed tank were constructed using steel angles, bars, and plates. The internal dimensions of the tank are 1.20 m x 1.908 m x 1.00 m. The longer sides of the tank were bound by glass plates to allow for taking photographs of the sand mass during the tests. The shorter sides are by plywood. While testing the side glass walls were lubricated using silicon grease and rubber membrane. The tank base frame was covered with thick steel plates. The base plate of the tank was covered by gluing glass paper (BS Grade 32, Grit No. 40) on to it to achieve rough interface condition.

A rigid footing having base dimensions of 0.12 m x 0.9 m was used. The base had a side wall height of 0.10 m. The base of the footing was made of 9 mm thick steel plate and load cell blocks. The sides were also from steel plates. V-shaped grooves were provided at the base plate of the footing to apply loads through knife edged loading blades. They were provided at places of desired eccentricities of e/B = 0, 1/12, 1/6 and 1/3. Glass papers (BS Grade S2, Grit No. 40) were glued on to the footing base.

Three load cell blocks were housed in the footing base at different locations, Fig. 2. The two blocks (Block 1 and Block 2, Fig. 2) near the centre and the...
The load was applied to the footing using a strain controlled loading rig (50 kN capacity) through two loading blades. The vertical displacement of the footing, at the loading point, was recorded using two Linear Variable Displacement Transducers (LVDT). A cylindrical load cell was used in the loading rig to measure the total load. A more detailed discussion on the experimental system is available in Abedin (1986) and Ameen (1990).

NORMAL STRESS UNDER THE FOOTING

The distribution of normal stress under the footing was investigated by using 13 load cells (Block 2, Fig. 2). The normal stress distribution obtained by using the four cells of Block 1 served as a check on the detailed distribution. The study includes vertical central and eccentric loads, and surface and shallow depth footings. It revealed that the distribution of normal stress under the footing has an influence of sand layer thickness, load eccentricity, depth of surcharge and the stage of loading. The layer thickness and load eccentricity are described as the ratios to the width of the footing. It is observed from the study that distribution of stresses is almost independent of layer thickness if \( H/B \) ratio goes up and beyond a value of 1.0. The layer thicknesses at these cases are termed as thick. Otherwise, it is a thin layer. The test results are presented in Figs. 3, 4 and 5. They are discussed below.

Vertical Central Load

Thick layer (\( H/B \geq 1.0 \))

For a footing subjected to a vertical central load the stresses were symmetrical about the centre line of the footing.

Fig. 2. Arrangement of load cells on footing.

For a surface footing, the stress at the footing edges were zero and increased to a maximum at a distance of 0.2B to 0.25B from the edges, Fig. 3. The stress at the centre was slightly less than the maximum value, which is qualitatively similar to the "Saddle" type stress distribution predicted by Gorbunov-Possadov (1965), but distinctly less pronounced. The stress distribution between the two outer points of maximum stresses appears to have a wavy shape of small amplitude.

With the increase of applied load the stresses near the edges of the footing increased some what more slowly than those near its centre. This is due to the fact that the sand starts yielding at the footing edge at the early stage of loading (0.5 peak load). It was observed that at the early stage of loading the stress distribution was more uniform than that at peak load.

For a shallow depth (\( D/B = 0.5 \)) footing the normal stress distribution had a similar shape to that of a surface footing but with an increased value of stress. Fig. 3. However, small stresses were measured at the edge of the footing.

Thin layer (\( H/B < 1.0 \))

The distribution of normal stress was symmetrical about the centre of the footing, Fig. 3. For a surface footing the stresses at the edges were almost equal to zero, Fig. 3. At peak load the normal stress increased almost linearly from the edge to the centre. At the early stage of loading (0.5 peak load) the stress distribution was similar.

Similar pattern of normal stress distribution with the maximum stress at centre and the minimum at the edge was observed in tests on shallow footing (\( D/B = 0.5 \)), Fig. 3. The values of normal stresses were found to be higher as compared to a surface footing.

Vertical Eccentric Load

The distribution of normal stress presented in this paper were investigated using load eccentricities (\( e/B \)) of 0, 1/12 and 1/6. The results are presented in Figs. 4 and 5.

Thick layer (\( H/B \geq 1.0 \))

For a surface footing, the extreme edge cell opposite to the eccentric side recorded very small stress approximately equal to zero, Fig. 4. The zone of zero stress increased with the increase of load eccentricity and was found approximately equal to \( B-e \). The examination of stereo pictures also support this observation. The maximum stress occurred between a distance of 0.2B to 0.25B from the edge of the eccentric side.
The conventional analysis assumes maximum stress at the outer edge on the eccentric side. This means that the assumed maximum bending moment using the conventional distribution is larger than its actual value.

For a shallow depth footing (D/B = 0.5) the maximum stress occurred near the eccentric edge and reduces approximately to zero in a similar manner as it happened in case of a surface footing, Fig. 4.

Thin layer (H/B < 1.0)

For a vertical eccentric load the locus of maximum stress was found to shift with load eccentricity and occurred approximately beneath the loading point with reduced maximum value of stress as compared to the case of a centrally loaded footing, Fig. 5. The stresses on the edges were low with smaller values on the eccentric side than that of a centrally loaded footing, and almost zero on the opposite edge of eccentricity.

It was observed that at early stage of loading (0.5 peak load) the stress distribution tends to be more uniform than that at peak value. The effect of overburden surcharge (D/B = 0.5) was to increase the value of the stresses across the footing width without affecting the shape of the distribution.

**Maximum Normal Stress**

Of all the important aspects on distribution of stresses the magnitude of the maximum stress is of utmost interest from the design point of view. In the present investigation the maximum normal stress, σ_{max} at peak load, is expressed as a ratio to the average stress, σ_{av}. This is termed as maximum normal stress ratio. Table 1 presents the maximum normal ratios along with those estimated by using Meyerhof (1953) and conventional method. The results are also plotted against layer thickness in Fig. 6. The important findings are stated in the following paragraphs.

For a footing on a thin layer (H/B < 1.0) and subjected to a vertical central load the maximum normal stress ratio increases with the decrease of layer thickness.

**Table 1. Maximum Normal Stress Ratio for a Thick Layer**

<table>
<thead>
<tr>
<th>e/B</th>
<th>D/B=0</th>
<th>D/B=0.5</th>
<th>Meyerhof (1953)</th>
<th>Conventional</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.35</td>
<td>1.30</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>1/12</td>
<td>1.70</td>
<td>1.50</td>
<td>1.20</td>
<td>1.50</td>
</tr>
<tr>
<td>1/6</td>
<td>2.05</td>
<td>1.90</td>
<td>1.50</td>
<td>2.00</td>
</tr>
</tbody>
</table>

For a footing resting on a thick layer and subjected to eccentric loading, the maximum normal stress ratio increases with increasing load eccentricity. Similar results were observed for thin layers, which were accompanied by the increase with decreasing layer thickness for small eccentricities (e/B < 1/12).

There is a small influence of overburden surcharge on the maximum stress ratio. The application of surcharge causes a reduction in the value of the ratio.

The conventional theory involving linear stress distribution principles predicts the maximum normal stress ratio with reasonable accuracy for eccentricities (e/B<1/6) considered in this study. The theory of Meyerhof (1953) underestimates the value.

**SHEAR STRESS UNDER THE FOOTING**

The shear stress under the footing was measured using four of Block 1, Fig. 2, placed across the width of the footing. The single cell of Block 3 (Fig. 2) served as to check the average shear. Assuming zero stress at the extreme edges and considering the stresses recorded by the four individual cells as the values at the mid point of the corresponding cells, the results are summarized in Fig. 7 and discussed in the following paragraphs.

**Vertical Central Load**

It was observed that for a centrally loaded surface footing the distribution of shear stress across the base of the footing is skew symmetric. For a footing resting on a thick layer (H/B<1.0), the maximum stresses occurred at the outer quarter, while for a footing resting on a thin layer (H/B < 1.0) the maximum stress occurred at the inner quarter, Fig. 7. The value of maximum shear stress for a thin layer was larger than the corresponding value for a thick layer. For shallow footings (D/B = 0.5) the shape of shear stress diagram has been found to be similar to the corresponding surface footing. The surcharge was found to increase the values of shear stresses.

**Vertical Eccentric Load**

The distribution of shear stress across the footing width under an eccentric load are shown in Fig. 7. For a footing resting on a thick layer, the interior...
point where the shear stress change direction moved towards the side of the eccentricity and the distribution was unsymmetrical, Fig. 7. For a footing resting on a thin layer the distribution of stresses were similar to those for a thick layer but with increased value to stresses while compared to that of a thick layer. Similar to central load the effect of surcharge was only to increase the value of shear stresses. the pattern of distribution remaining unaffected.

**Maximum Shear Stress**

As for normal stress the maximum shear stress, \( \tau_{\text{max}} \) observed across the footing width at peak load is also expressed in terms of maximum shear stress ratio given by \( \frac{\tau_{\text{max}}}{\tau_{\text{av}}} \), where \( \tau_{\text{av}} \) is the average shear stress. The sign of shear is ignored here. The maximum shear stress ratio at different eccentricities are plotted against \( H/B \) in Fig. 8. It is observed that for both surface and shallow depth footing (D/B=0.5) the ratios are scattered for the eccentricities considered. However they are found to lie within a certain band and the boundaries may be defined by two straight lines. For surface footing the boundary equations are \( \frac{\tau_{\text{max}}}{\tau_{\text{av}}} = 1.25 \) and \( \frac{\tau_{\text{max}}}{\tau_{\text{av}}} = 2.25 \).

**Fig. 8. Maximum shear stress ratio**

For shallow depth footing (D/B = 0.5) the corresponding boundary lines may be represented by \( \frac{\tau_{\text{max}}}{\tau_{\text{av}}} = 1.15 + 0.1 H/B \) and \( \frac{\tau_{\text{max}}}{\tau_{\text{av}}} = 2.3 + 0.1 H/B \) respectively. It should be mentioned that the limits of H/B are 0.5 and 2.5. It worthwhile to mention that the average shear stress under the footing at peak load was observed to be approximately 20% to the average normal stress. The range being 14% to 25%. The only exception is the case of thin surcharged (H/B = 0.5, D/B = 0.5) layer with zero eccentricity where the average shear stress is approximately double.

**CONCLUSIONS**

From the results of the model tests the following conclusions can be drawn:

i) For a centrally loaded footing there is a significant influence of the layer thickness (H/B) on the distribution of normal stresses under the footing. When the layer thickness is greater than or equal to the width of the footing (thick layer) the pattern of normal stress distribution was found to be similar.

ii) For a centrally loaded surface footing on a thick layer the normal stresses at the edges are zero and maximum at a distance of approximately 0.2B from the edge. The maximum stress is approximately 1.3 times the average stress. For design purposes the distribution can be assumed trapezoidal, having a maximum value in the middle 3/5 th and zero at the edges. The effect of surcharge is to increase the values of stresses. Otherwise, the pattern of the distribution remains similar to that of the surface footings.

iii) For a centrally loaded surface footing resting on a thin layer the normal stresses increased almost linearly from the edges to the centre. Similar pattern of normal stress distribution with maximum stress at centre and the minimum at the edges were observed for a shallow footing.

iv) For an eccentrically loaded surface footing on thick layer the maximum stress occurred at a distance of 0.2B from the edge and the stress diminishes to zero at a distance of (B-e) from the edge opposite to eccentricity. The surcharge causes a shift of the point of maximum stress to the eccentric edge.

v) For a thin layer (H/B < 1.0) the stress is maximum approximately beneath the loading point. The stress at the edge of footing are low and approximately zero at the opposite edge of eccentricity.

vi) For a centrally loaded surface footing the distribution of shear stress across the footing is skew symmetric. For a thick layer, the maximum shear occurs at the outer quarter of the footing. Whereas, for a thin layer it occurs at the inner quarter. Shear stress increases with decreasing layer thickness. The effect of surcharge is to increase the value of stress only.

vii) For a eccentrically loaded footing the point of contraflexure (the point where the shear stress changes sign) shifts towards the side of eccentricity resulting in an unsymmetrical distribution. Otherwise, the distribution is similar to that of a central load.

viii) The average shear stress under the footing is approximately 20% of the average normal stress. For a very thin layer the maximum shear stress is in the range of 1.15 to 2.25 for surface footing, and 1.2 to 2.4 for shallow depth footing (D/B=0.5).

**REFERENCES**


