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SIMPLE PHYSICAL MODELS FOR FOUNDATION VIBRATION – A GUIDED TOUR

MODELES PHYSIQUES SIMPLES POUR LES VIBRATIONS DE FONDATIONS

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SYNOPSIS: As an alternative to rigorous boundary-element solutions, simple physical models can be used to determine e.g. the interaction force-displacement relationship (dynamic stiffness) and the effective seismic input motion of foundations. Translational and rotational cones and their corresponding lumped-parameter models together with simple one-dimensional wave patterns in the horizontal plane allow surface, embedded and pile foundations even for a layered site to be analysed and thus form a major step towards developing a strength-of-materials approach to foundation-vibration analysis. The analysis can mostly be performed directly in the time domain. The physical models provide physical insight which is often obscured by the mathematical complexity of rigorous solutions, offer simplicity in application as well as in the physics and in the rigorous mathematical solution of the physical model, are sufficiently general to enable reasonably complicated practical cases to be solved, exhibit adequate accuracy and offer the potential for generalizations.

INTRODUCTION

Important tasks of foundation-vibration analysis such as determining the interaction force-displacement relationship (dynamic-stiffness) and the effective seismic input motion of a foundation can be solved exactly using some form of the *boundary-element method*. In these *rigorous* methods a formidable theoretical background is required, and a considerable amount of expertise in idealising the actual dynamic system is necessary. The user tends to be intimidated. A sophisticated computer program must be available. The computational expense involving the numerical evaluation of integrals and the solution of a large system of equations for just one run is large, making it difficult from an economical point of view to perform the necessary parametric studies. A false sense of security is thus provided. The mathematical complexity of rigorous methods often obscures the physical insight. The rigorous methods belong more to the discipline of applied mechanics than to civil engineering. These methods should only be used for large projects of critical facilities. For all other projects *simple physical models* to represent the unbounded soil should be applied. For instance, the soil below the disk is modelled as a truncated rod (bar) with its area varying as in a *cone* (Fig. 1a). Through the choice of the cone as a physical model, the complicated three-dimensional wave pattern with body and surface waves and three different velocities is replaced by the simple one-dimensional wave propagation governed by the one constant dilatational-wave velocity of the conical rod, whereby *plane sections remain plane* (theory of strength of materials).

As in the physical models the physics of the problem is simplified, *conceptual clarity* with *physical insight* results. The application performed directly in the familiar time domain is *simple*, permitting even hand calculations in some cases. The physical models permit engineering solutions to reasonably complicated practical cases. The *shape* of the foundation-soil interface, the *soil profile* and the amount of *embedment* can vary considerably. The *accuracy* is sufficient, especially when considering the many uncertainties. The *concepts* and certain *features* of the physical models can be *generalized* and the results applied in much more sophisticated calculations. Summarizing, when applying the simple physical models some loss of precision results which is more than compensated by their many advantages. As the simple models cannot cover all cases, they do not supplant the much more generally applicable rigorous boundary-element method, but rather supplement it. The physical models such as the cone models with the deformation pattern of rod theory and others to be discussed present a major step towards developing a *strength-of-materials approach to foundation dynamics*. The aim is the same as that used routinely in stress analysis of structural engineering, where e.g. for very complicated skew curved prestressed concrete bridges beam theory is applied successively and the general three-dimensional theory of elasticity is not needed. Concluding, the dynamic analyst should always «Make things as simple as possible but no simpler» (H. Einstein). Or to state it differently: *«Simplicity that is based on rationality is the ultimate sophistication»* (A.S. Veletsos).

In this paper simple physical models to determine the interaction force-displacement relationship (dynamic stiffness) and the effective seismic input motion of foundations are described. The practical results of a three-year research effort performed together with Dr. J.W. Meek are described who developed 20 years ago a rotational cone with the corresponding lumped-parameter model (Meek and Veletsos 1974) which provided the starting point for much of this and other developments. The work on physical models for foundation-vibration analysis is also summarized in a forthcoming book (Wolf 1995).

OVERVIEW

The truncated semi-infinite *translational* and *rotational cones* shown in Figs. 1a and 1b for the vertical and rocking motions can also model the horizontal and torsional motions involving shear distortions of a disk on the surface of a *halfspace* (Meek and Wolf 1992a). For practical applications the translational and rotational cones can be represented rigorously by the *discrete-element models* consisting of spring-dashpot systems with frequency-independent coefficients of Figs. 1e and 1f, respectively, whereby for the rotational cone an additional internal degree of freedom is introduced. The properties of the cones and the discrete-element models for all four motions are described in Table 1. The nomenclature is defined in Fig. 2a where a second discrete-element model (monkey-tail) for the rotational motion is also shown. For nearly-incompressible soil ($1/3 < \upsilon \leq 1/2$) a trapped mass and mass moment of inertia which can be assigned to the basemat are introduced for the vertical and rocking motions (Meek and Wolf 1993c). Based on the rotational cone, the rocking dynamic stiffness is calculated. After

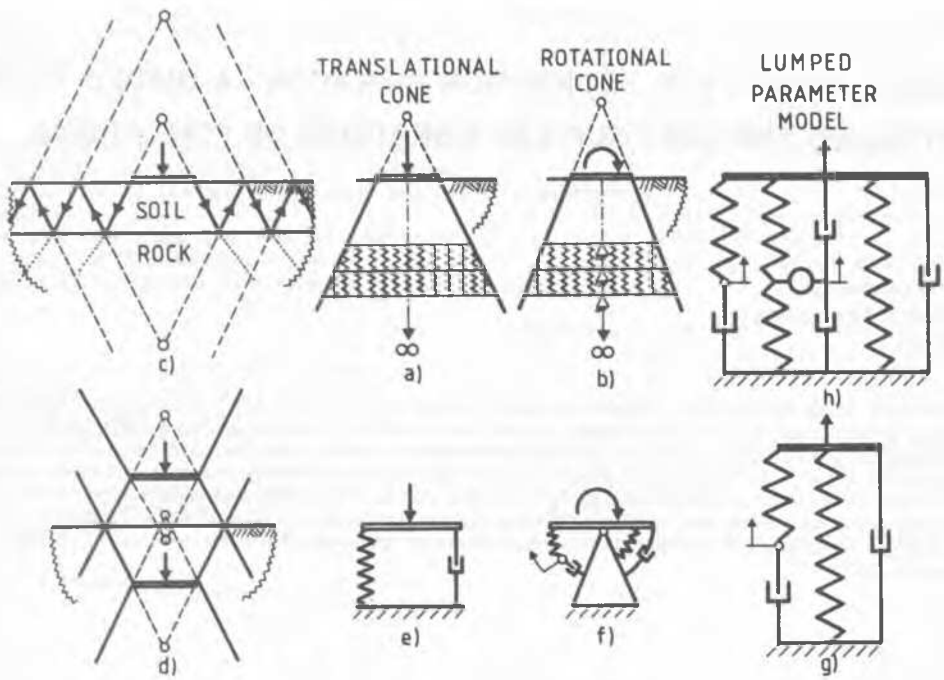

Fig. 1 Cones and lumped-parameter models

a) Disk on surface of halfspace with truncated semi-infinite translational cone.
b) Disk on surface of halfspace with truncated semi-infinite rotational cone.
c) Disk on surface of soil layer resting on flexible rock halfspace with corresponding cones.
d) Symmetry condition with respect to free surface for disk and mirror-image disk with corresponding double cones to calculate Green's functions.
e) Discrete-element model for translational cone.
f) Discrete-element model for rotational cone.
g) Lumped-parameter model consisting of springs and dashpots with one internal degree of freedom corresponding to f).
h) Lumped-parameter model consisting of springs, dashpots and a mass with two internal degrees of freedom.

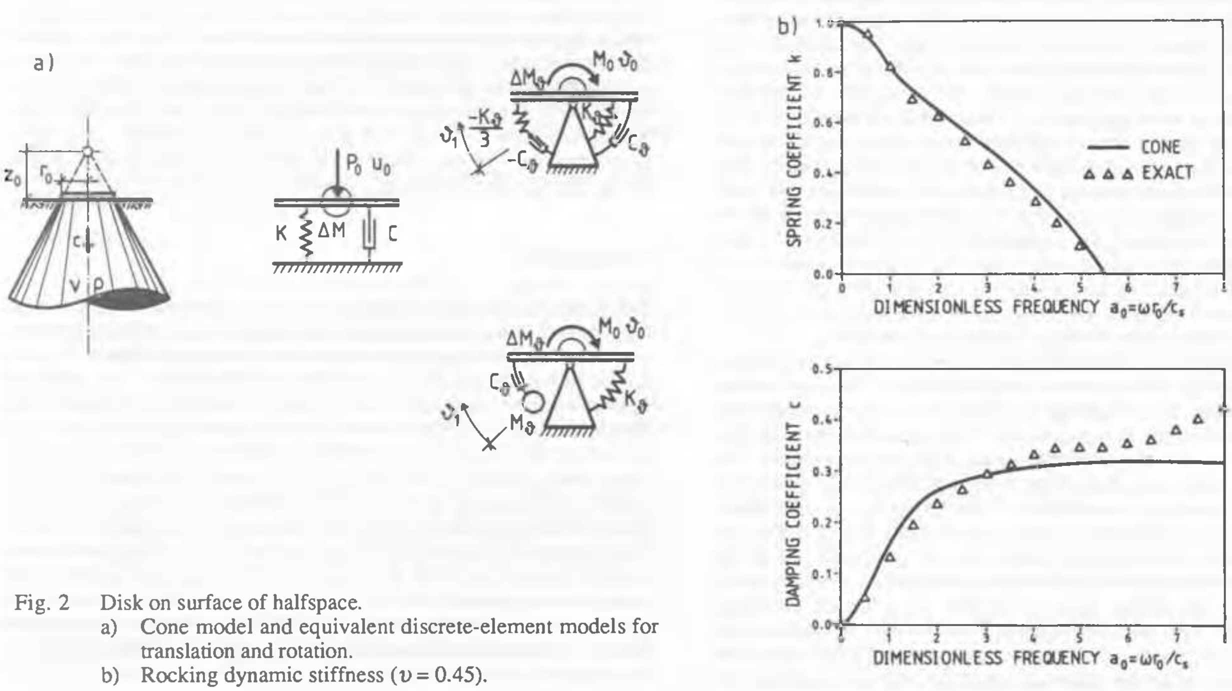

Fig. 2 Disk on surface of halfspace.
a) Cone model and equivalent discrete-element models for translation and rotation.
b) Rocking dynamic stiffness ($\upsilon = 0.45$).

Table 1: Bare essentials to model a foundation on surface of homogeneous soil halfspace

Motion	Horizontal	Vertical		Rocking		Torsional
Equivalent Radius r_0	$\sqrt{\frac{A_0}{\pi}}$	$\sqrt{\frac{A_0}{\pi}}$		$\sqrt[4]{\frac{4I_0}{\pi}}$		$\sqrt[4]{\frac{2I_0}{\pi}}$
Aspect Ratio $\frac{z_0}{r_0}$	$\frac{\pi}{8}(2-\nu)$	$\frac{\pi}{4}(1-\nu)\left(\frac{c}{c_s}\right)^2$		$\frac{9\pi}{32}(1-\nu)\left(\frac{c}{c_s}\right)^2$		$\frac{9\pi}{32}$
Poisson's Ratio ν	all ν	$\leq\frac{1}{3}$	$>\frac{1}{3}$	$\leq\frac{1}{3}$	$>\frac{1}{3}$	all ν
Wave Velocity c	c_s	c_p	$2c_s$	c_p	$2c_s$	c_s
Trapped Mass ΔM ΔM_ϑ	0	0	$2.4\left(\nu-\frac{1}{3}\right)\rho A_0 r_0$	0	$1.2\left(\nu-\frac{1}{3}\right)\rho I_0 r_0$	0
Discrete-Element Model	$K=\rho c^2 A_0/z_0$ $C=\rho c A_0$			$K_\vartheta=3\rho c^2 I_0/z_0$ $C_\vartheta=\rho c I_0$ $M_\vartheta=\rho I_0 z_0$		

(A_0 = area, I_0 = moment of inertia, c_s = shear-wave velocity, c_p = dilatational-wave velocity, ρ = mass density)

non-dimensionalizing the latter by its static value, the real part k and the imaginary part divided by the dimensionless frequency c agree well with the exact solution (Fig. 2b).

Fig. 1c shows the extension of the cone model to calculate a disk on the surface of a *soil layer resting on flexible rock halfspace* (Wolf and Meek 1993). In this layered cone model the vertical force applied to the disk produces dilatational waves propagating downwards along a cone from the disk. At the interface of the layer and the rock, a refracted wave propagating in the rock in the same direction as the incident wave along its own cone (dotted line) is created in addition to a reflected wave propagating back through the layer along the indicated cone (dashed line) in the opposite upward direction. The latter will reflect back at the free surface and then propagate downwards along its cone; upon reaching the interface of the layer and the rock, a refraction and a reflection again take place. The waves in the layer thus decrease in amplitude and spread resulting in radiation of energy towards infinity in the layer in the horizontal direction. The dynamic stiffness equals

$$S(\omega) = K \frac{1 + i\omega\frac{T}{\kappa}}{1 + 2\sum_{j=1}^{\infty}\frac{(-\alpha)^j}{1+j\kappa}e^{-ij\omega T}}$$

with the static stiffness of the halfspace K with the properties of the layer, $T=2d/c_p^L$ (d = depth of layer, L = soil layer) and $\kappa = 2d/z_0^L$. The reflection coefficient $-\alpha(\omega)$ determined with rod theory of cones is, in general, a function of ω; its low-frequency limit (static case) equals for $\nu^L = \nu^R$ (R = rock halfspace)

$$-\alpha(0) = \frac{\rho^L(c_p^L)^2 - \rho^R(c_p^R)^2}{\rho^L(c_p^L)^2 + \rho^R(c_p^R)^2}$$

For this frequency-independent reflection coefficient the dynamic analysis can be performed in the time domain. For the horizontal motion c_s replaces c_p. The horizontal dynamic stiffness of a disk on the surface of a layer resting on a halfspace non-dimensionalized by K calculated with a layered cone is quite accurate (Fig. 3).

For a *soil layer resting on rigid rock*, $-\alpha$ equals -1 (Meek and Wolf 1992b). The dynamic flexibility in the time domain, non-dimensionalized with K, i.e. the unit-moment impulse rocking response function, calculated with the layered cones (Fig. 4) exhibits jump discontinuities and is very accurate. The "exact" solution determined for a slightly damped soil by numerical Fourier transformation smears over the discontinuities.

The concepts of cone models can be expanded to the analysis of *embedded cylindrical foundations*. Again, the vertical degree of freedom is addressed in Fig. 1d, but the following argumentation is just as valid for the horizontal, rocking or torsional ones. To represent a disk within an elastic fullspace, a double-cone model is introduced (Meek and Wolf 1993a). Its displacement field defines an approximate Green's function for use in an uncomplicated (one-dimensional) version of the boundary-element method. To enforce the stress-free condition at the free surface of the halfspace (Fig. 1d), two identical double cones in the full space placed symmetrically (with respect to the free surface) and excited simultaneously by the same forces are considered. By applying principles of symmetry and superposition, the soil's flexibility matrix defined at the disks located within the embedded part of the foundation can be set up. The rest of the analysis follows via conventional matrix methods. As an example a cylinder embedded in a halfspace is examined (Fig. 5a). In the embedded part of the foundation 8 disks with double cones are introduced (one is shown). In Fig. 5b the static stiffness factors defined as the ratio of the static stiffness of the embedded foundation to that of a disk on the halfspace are plotted as a function of the embedment ratio for all motions. In Fig. 5c the (non-

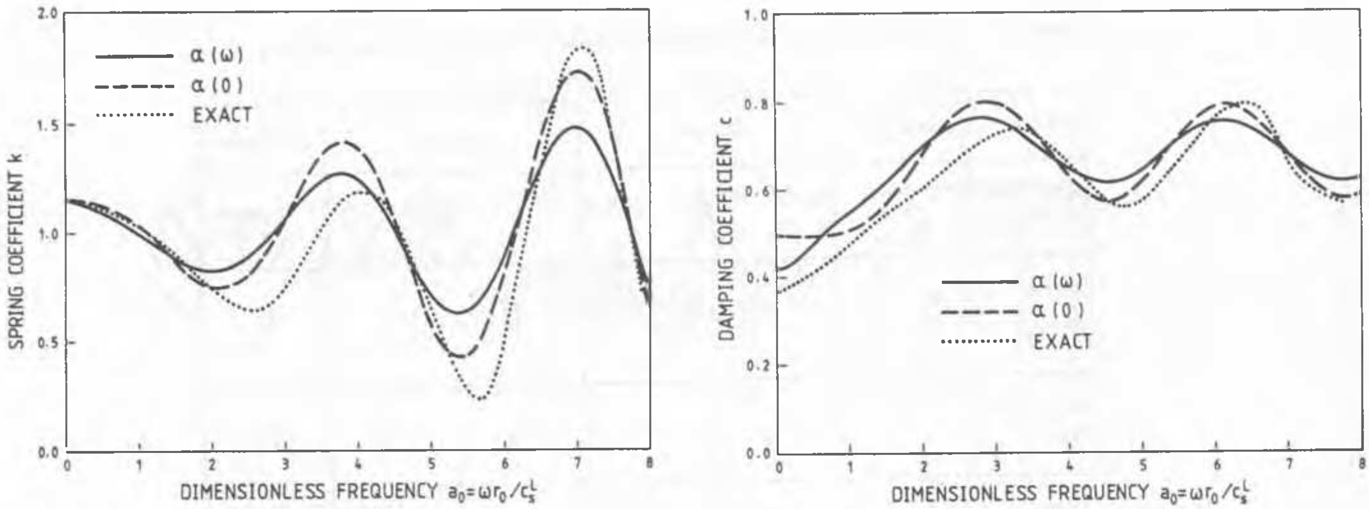

Fig. 3 Horizontal dynamic stiffness of disk on layer resting on halfspace ($r_0/d = 1$, $\nu^L = \nu^R = 0.25$, $c_s^L/c_s^R = 0.8$, $\rho^L/\rho^R = 0.85$).

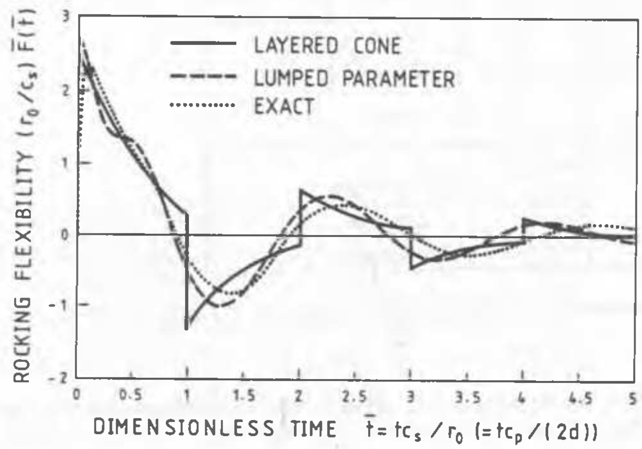

Fig. 4 Rocking dynamic flexibility in time domain of disk on layer resting on rigid rock ($r_0/d = 1$, $\nu = 1/3$).

dimensionalized) rocking dynamic stiffness is presented calculated with cones and the exact solution for $e/r_0=1$ is also shown. The horizontal and rocking components of the effective seismic input motion for vertically propagating seismic S-waves (Fig. 5a) are compared in Figs. 5d and e to the exact solution. In all cases the cone models lead to surprisingly accurate results.

The model of Fig. 1f shown in Fig. 1g for the translational motion forms the starting point to develop systematically a family of consistent *lumped-parameter models*. The direct spring is chosen to represent the static stiffness. The coefficients of the other spring and the two dashpots are selected so as to achieve an *optimum fit* between the dynamic stiffness of the lumped-parameter model and the corresponding exact value (taken from the literature, originally determined by a rigorous procedure such as the boundary-element method). To increase the number of coefficients and thus the accuracy, several systems of Fig. 1g can be placed in parallel. Fig. 1h shows the lumped-parameter model for three such systems, whereby two of them are combined to form a new system consisting of two springs, one independent dashpot (the two dashpots in series have the same coefficient) and a mass. It can be shown that the frequency-independent coefficients, which can be determined using curve fitting applied to the dynamic stiffness (involving the solution of a linear system of equations only) will be real (but not necessarily positive). The *eight springs, dashpots and mass* will represent a *stable lumped-parameter model with only two additional internal degrees of freedom*. Use is thus implicitly made of the result obtained with the state-of-the-art formulation which leads to the rigorous dynamic stiffness used in the optimum fit. To take soil-structure interaction into account, the lumped-parameter model (Fig. 1h) or the discrete-element model (Figs. 1e and 1f) representing the soil is attached to the underside of the structural model. The total dynamic system of the structure and the soil can be analysed using a standard structural dynamics computer program. For illustration, the lumped-parameter model of Fig. 1h is used to calculate the rocking dynamic flexibility of a disk on a layer on rigid rock shown in Fig. 4 (Wolf and Paronesso 1991).

The models shown in Fig. 1 prescribe a displacement pattern varying with depth along the axis of the cone. To extend the application, *displacement patterns in the horizontal plane* other than those corresponding to the strength-of-materials assumption of plane cross-sections remain plane can be introduced. One-dimensional wave propagation is again prescribed. Using nonmathematical physical reasoning and calibration with rigorous solutions, the vertical displacement on the free surface of a halfspace for a loaded source subdisk (Fig. 6a) can be derived (Meek and Wolf 1993b). This approximate *Green's function* exhibits a different dynamic behaviour in the *near and far fields*. *Arbitrary shaped surface foundations* can be treated as an assemblage of subdisks. As another displacement pattern, *cylindrical waves* can be assumed to propagate with the propagation velocities indicated in Fig. 6b to determine *dynamic interaction factors*, describing the effect of a source pile on a receiver pile (Dobry and Gazetas 1988). This allows *pile groups* taking *pile-soil-pile interaction* into consideration to be analysed. As an example the 3x3 floating pile group shown in Fig. 7a is analysed. E denotes the Young's modulus of elasticity. 25 disks with the corresponding double cones (Fig. 1d) are used to model a single pile (Wolf, Meek and Song 1992). The dynamic stiffness of a single pile agrees quite well with the exact solution (Fig. 7b). Dynamic-interaction factors are determined based on the sound physical approximation of Fig. 6b, modified appropriately for the vertical motion. As an example of the results the amplitude of the axial force at the head of the centre pile is plotted for the vertical motion of the pile group for the two pile distance to pile diameter ratios in Fig. 7c. The force amplitudes are normalized with the average static force of the pile group. Even details of the strong dependency on frequency are well represented.

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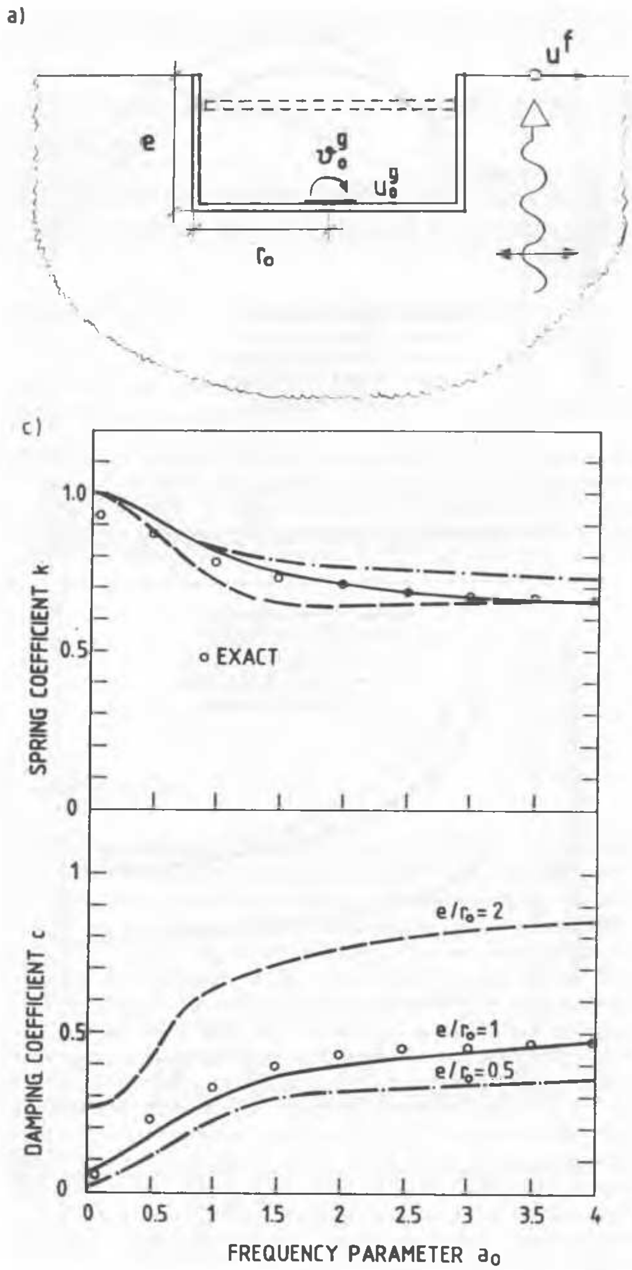

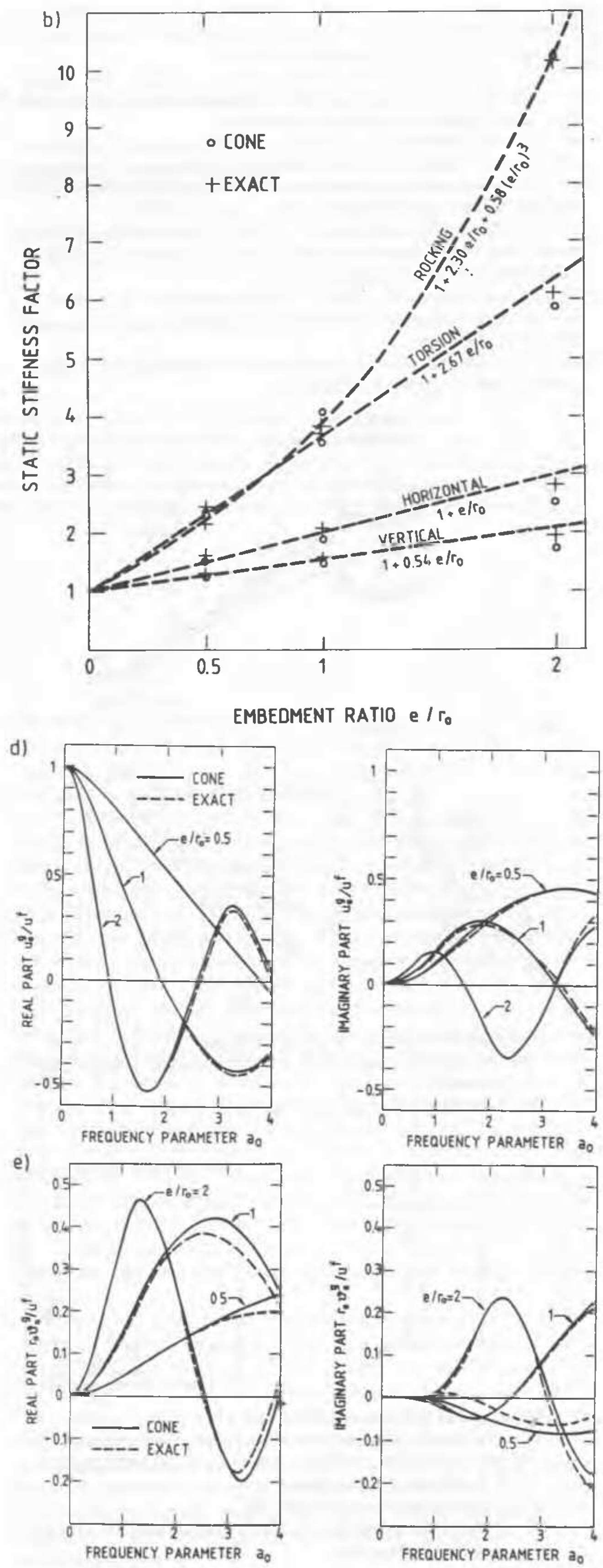

Fig. 5 Cylinder embedded in halfspace ($\upsilon = 0.25$).
a) Nomenclature.
b) Static stiffness factor.
c) Rocking dynamic stiffness.
d) Horizontal component of effective seismic foundation input motion.
e) Rocking component of effective seismic foundation input motion.

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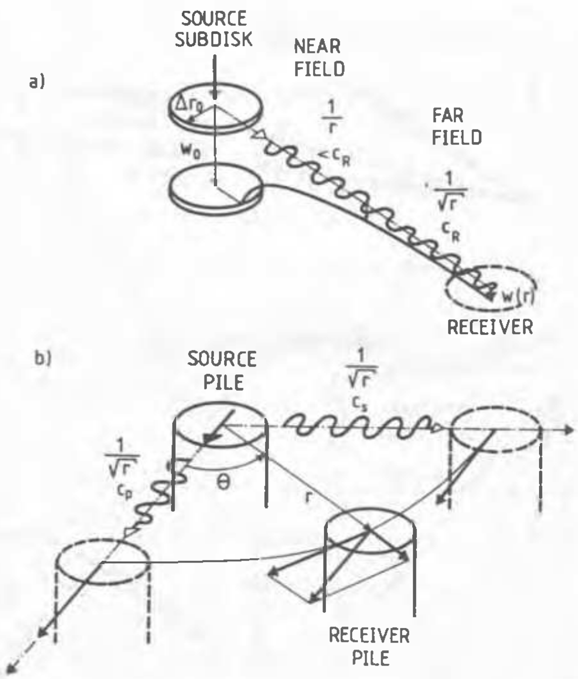
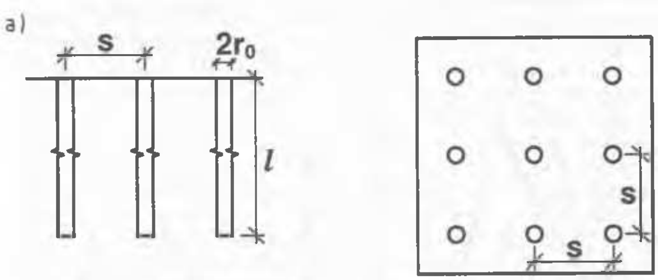
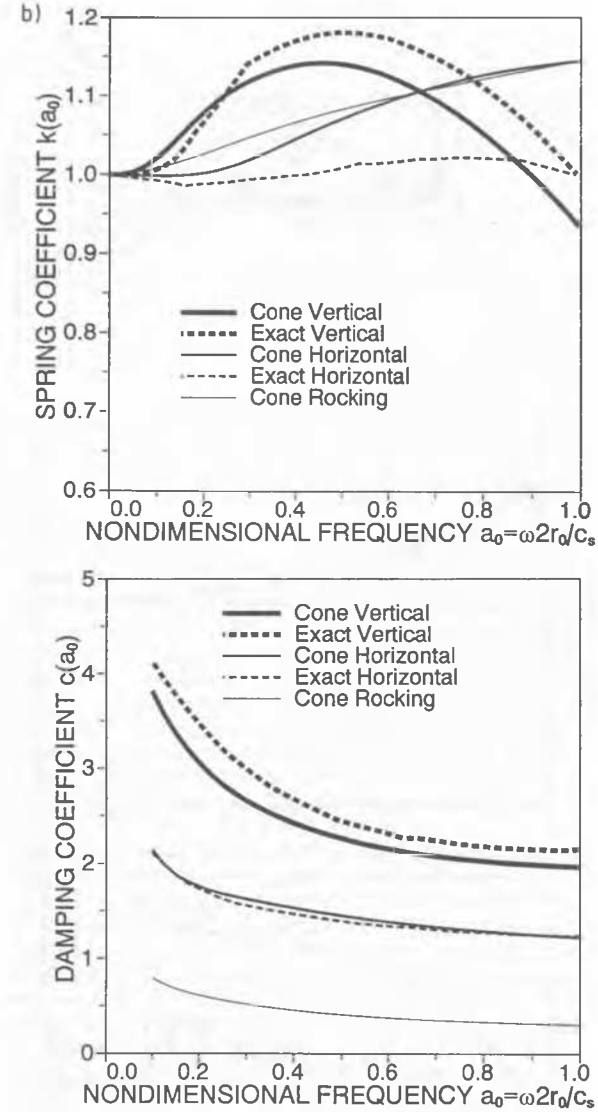


Fig 6 Displacement patterns in horizontal plane.
a) Vertical displacement on free surface from loaded subdisk.
b) Horizontal displacement from loaded pile.





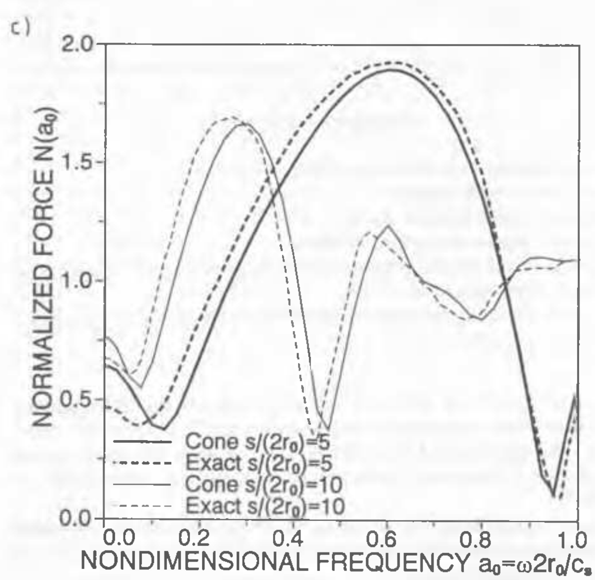


Fig. 7 Floating pile group in soil halfspace.
a) Elevation and plan view of 3x3 pile group [$s/(2r_0) = 5$ and $= 10$, $l/(2r_0) = 15$, $\upsilon = 0.4$, $E_p/E_s = 1000$, $\rho_p/\rho_s = 1.4$, 5% material damping].
b) Dynamic stiffness of single pile.
c) Axial force amplitude at head of centre pile for vertical motion of pile group.