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SETTLEMENT OF RIGID CIRCULAR CYLINDERS LYING ON A BED OF SAND

TASSEMENT DE CYLINDRES CIRCULAIRES ET RIGIDES, POSES DANS UN LIT DE SABLE

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SYNOPSIS: The area of contact between a rigid circular cylinder with horizontal axis and a bed of sand on which the cylinder is lying, depends on the load of the cylinder. It is related to the settlement of the cylinder. The settlement of such cylinders under proportionally increasing load is studied theoretically and experimentally. The sand is considered to consist of convex irregular rigid grains of limited strength and is represented as rigid-plastic strain-hardening material (extended psammic material). Considerations of topology and of functional analysis, as well as algebraic and geometrical reasoning leads to power functions relating the monotonically increasing load and the settlement of very long and of very short cylinders. The exponents of the power functions are rational functions of the minimal exponent of hardening of psammic material. The theory is checked upon by small scale model tests. Possible applications are discussed.

INTRODUCTION

Small distortions in granular soils may be decomposed uniquely into a sum of contributions due to several micromechanisms. P.W. Rowe (1962) pointed out three of them: elastic deformation of grains, interparticle slip and crushing. Elsewhere the authors (Dietrich and Arslan, 1985) have shown that the latter mechanism may be subdivided into contributions due to abrasion and fragmentation. Consider a sequence of similar structures founded in or on dry sand, subject to similar loading processes and ordered according to scale. The average foundation stresses and subsoil stresses pertaining to these structures vary as the scale. As the scale decreases the ratios among the contribution of the various micromechanisms to the displacement of the structure change. First, the contribution due to elastic deformation of grains becomes negligible. Next, this happens to the contribution due to fragmentation and then to the contribution due to abrasion. So finally the displacement of a structure sufficiently small is brought about by interparticle slip only. The sand then behaves as if composed of rigid unbreakable grains (Dietrich/Arslan, 1985). In terms of continuum mechanics this behavior is modeled by a rigid plastic strainhardening material called "psammic material" (Dietrich, 1977). It possesses an internal constraint (due to the rigidity of grains) and all its stresses are extra stresses undetermined by deformation. The latter determines only the ratios of stresses (more precisely the direction of the stress tensor in tensor space) or - in traditional soil mechanics terminology - the degree of mobilization of internal friction. The rate of hardening of psammic material with respect to length of strain path exhibits an increase beyond limit whenever the strain path bends sharply i.e. whenever its radius of curvature vanishes, so e.g. when a sample in the triaxial apparatus changes from compression to extension or vice versa. This behavior of the hardening function is approximated immediately after the bend by a power law of hardening (Dietrich, 1982) exhibiting increase beyond limit at the bend. The exponent of the power law is the smaller the larger is the angle by which the direction of the strain path changes. The power law degenerates to a linear law when the strain path is continued smoothly, i.e. the hardening function is differentiable at the point of the strain path considered. The minimal exponent of hardening ν proves to be an important soil property depending on granulometry but not on the configuration of grains and especially not on relative density. If psammic material is extended to cover limited strength of grains, the aforementioned behavior is modified by a factor only, depending on the average intensity of stress (i.e. on the scale of the respective structure). The extended material still deserves the name "rigid-plastic strainhardening" because there is no storage of elastic energy (Dietrich/Arslan, 1985). (If the influence of elastic deformation of grains were to be covered also, an additional summand had to be added to the displacement due to interparticle slip, fragmentation and abrasion.)

The behavior of certain classes of structures in or on homogeneously sedimented psammic halfspace under proportional loading turns out to be governed by power laws the exponents of which are rational functions of ν. Two such classes are represented by long and short rigid cylinders on top of psammic halfspace. They are analyzed below.

As explained before psammic behavior ist the more prevalent the lower is the intensity of stresses. The latter is low in small scale models. Therefore small scale models on sand may model cylinders on top of psammic halfspace. A series of such tests is reported in the sequel. The purpose of the tests is not to obtain design data but to gain insight in the mechanics of the systems considered, and to test the soundness of the underlying considerations. The insight may be partial only but it is based on mechanical principles.

THEORY

Consider a weightless rigid cylinder resting on a horizontal bed of sand and loaded by a vertical load F (see Fig. 1). The sand's granulometry, represented by a set of numbers, is indicated by "G", its initial configuration, also a set of numbers, by "K" and its initial bulk density by "γ". The sand's constitution is considered to be that of psammic material corresponding to an assembly of rotund rigid unbreakable grains.

F is the actual value of a load process as it proceeds in time or - because the system is considered rate-independant - as the length of the load path develops.

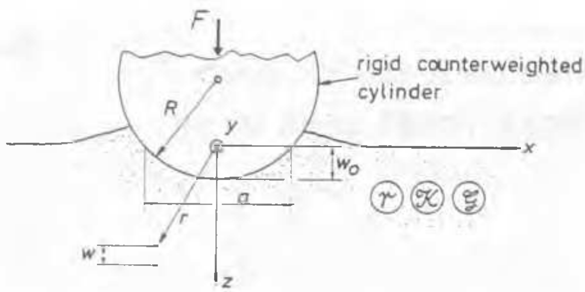

Fig. 1. Rigid cylinder indenting a psammic half-space under a total vertical load F

For moderate loads, the indenting portion of the circular cross section of the cylinder of radius R may be replaced by a parabola such that (see Fig. 1) the settlement w_0 of the lowermost point of a cross section of the cylinder

$$w_0 = \frac{1}{8R} a^2 \tag{1}$$

where a is the width of the indenting portion. Obviously

$$\frac{w_0}{R} \to 0 \Rightarrow w << a << R \tag{2}$$

Supposing displacements to be small compared to a and distortions of the soil small compared to unity, we introduce two different qualities of length (quality is equivalent but not equal to physical dimension. We consider equations between physical quantities (as one does intuitively), not between measuring numbers. Quality is a property of physical quantities whereas physical dimension is a property of a transformation of measuring numbers. See e.g. Görtler, 1975): U for displacements such as w_0 and L for the dimensions of the system considered such as a. We thus obtain from (1)

$$\mathrm{qul}(w_0) = \mathrm{U} \wedge \mathrm{qul}\,(a) = \mathrm{L} \quad \Rightarrow \mathrm{qul}\,(R) = \mathrm{qul}\,(a^2/w_0) = \mathrm{L}^2\,\mathrm{U}^{-1} \tag{3}$$

Due to the local law of parabolic hardening of psammic material the bulk density of the homogenously sedimented psammic half-space has the quality

$$\mathrm{qul}\,(\gamma) = \mathrm{K}\,\mathrm{L}^{\nu-3}\,\mathrm{U}^{-\nu} \tag{4}$$

as long as distortions are small. Here K is the quality of force and ν is the minimal exponent of hardening of psammic material. Not all of the distortions produced by loading the cylindrical structure are small. There are even always some at every stage of the loading process, which are large beyond limit. But considerations of topology and of functional analysis indicate, that the distortions in those regions of the halfspace, that govern the displacement of the cylinder, are small as long as the displacements of the cylinder are small compared to its governing dimensions (Dietrich, 1982).

Replacing the load F by the load P per unit of length l of the cylinder

$$P := F/l \tag{5}$$

we write down the following symbolic expression for the settlement w of a point r of the sandbed under the loaded cylinder

$$w = f\,(r, \boldsymbol{P}, P, R, l, \gamma, G, K) \tag{6}$$

where $\boldsymbol{P}$ is the loading process up to the present and $P, := P(1)$, is the present load such that

$$\boldsymbol{P} := \{(\lambda, P(\lambda)/P(1)) \mid 0 \le \lambda \le 1\} \tag{7}$$

For monotonic loading $\boldsymbol{P}$ is the identity function on the interval [1; 0]. For cyclic loading $\boldsymbol{P}$ may be replaced - in case we are content with the states of the system at equal phases of cycles - by the number of cycles.

To bring out the symmetries of the system the load P is replaced by the lengthlike quantity l_P, where

$$l_P := \left(\frac{P}{\gamma R^2}\right)^{\frac{1}{2+\nu}} R \tag{8}$$

and

$$\mathrm{qul}\,(l_P) = \mathrm{L} \tag{9}$$

Thus we proceed from Eq (6) to

$$w = g\,(r, l_P, \boldsymbol{P}, R, l, \gamma, G, K) \tag{10}$$

l_P is uniquely determined by (3), (4), (9), the list of arguments of (10) and the requirement to contain P. The qualities of the 8 arguments of (10) are made up of the three basic qualities K, L, U. Therefore (10) may be replaced, making use of the π - Theorem by a function of 8 - 3, = 5, numberlike arguments as follows

$$\frac{w}{R} = \left(\frac{P}{\gamma R^2}\right)^{\frac{2}{2+\nu}} \Psi\left(\frac{r}{l_P}, \frac{l}{l_P}, \boldsymbol{P}, G, K\right) \tag{11}$$

The general case, symbolized by (11), allows of 2 selfsimilar asymptotic degenerations; the long cylinder ($l/l_P \to \infty$) and the short cylinder ($l/l_P \to 0$). In the first case the dependance on (l/l_P) ceases, whence

$$l/l_P \to \infty \Rightarrow \Psi \to \lim \Psi = \Psi_\infty\left(\frac{r}{l_P}, \boldsymbol{P}, G, K\right) \tag{12}$$

and

$$\frac{w}{R} = \left(\frac{P}{\gamma R^2}\right)^{\frac{2}{2+\nu}} \Psi_\infty\left(\frac{r}{l_P}, \boldsymbol{P}, G, K\right) \tag{13}$$

Two points r_1 and r_2 are called "homologous" if at two states of loading P_1 and P_2

$$r_1 / l_{P1} = r_2 / l_{P2} = \rho \tag{14}$$

For proportional loading $\boldsymbol{P}$ is constant (see Eq.(7)) therefore

$$w\,(r_1, P_1) = \left(\frac{P_1}{\gamma R^2}\right)^{\frac{2}{2+\nu}} R\ \Psi_\infty(\rho) \tag{15}$$

$$w\,(r_2, P_2) = \left(\frac{P_2}{\gamma R^2}\right)^{\frac{2}{2+\nu}} R\,\Psi_\infty(\rho) \tag{16}$$

or

$$w\,(r_2, P_2) = \left(\frac{P_2}{P_1}\right)^{\frac{2}{2+\nu}} w\,(r_1,\ P_1) \tag{17}$$

respectively. The displacements due to P_1 given, the displacements due to P_2 in homologous points may be obtained consequently, multiplying the former by the factor $(P_2/P_1)^{\frac{2}{2+\nu}}$. Thus the displacement-fields at different stages of proportional loading are similar and this is why the system is called "self-similar" or more precisely "self-similar under monotonic loading".

In case of the short cylinder (i.e. a disc) the indenting portion of the moderately loaded cylinder behaves as a rodlike structure in psammic halfspace. A rodlike structure, when loaded, imposes a quasiplane motion on the halfspace (Dietrich, 1981). Due to this and to the circumstances considered after Eq (4) γ and l must combine to $\sqrt[\nu]{\gamma\, l^{1-\nu}}$. Consequently,

$$l/l_P \to 0 \Rightarrow \Psi \to \lim \Psi = (l_P / l)^{\frac{1-\nu}{2\nu+1}} \; \Psi_0\left(\frac{r}{l_{P0}}, P, G, K\right) \tag{18}$$

Replacing Ψ in Eq (11) by $\lim \Psi$ from Eq (18) and making use of Eq (8) one obtains

$$\frac{w}{l} = \left(\frac{P}{\gamma l \sqrt{lR}}\right)^{\frac{2}{2\nu+1}} \Psi_0\left(\frac{r}{l_{P0}}, P, G, K\right) \tag{19}$$

where

$$l_{P0} := (l_P / l)^{\frac{1-\nu}{2\nu+1}} \, l_P = \left(\frac{P}{\gamma l \sqrt{lR}}\right)^{\frac{1}{2\nu+1}} \sqrt{lR} \tag{20}$$

Self similarity of the short cylinder on psammic half-space is governed by the parameter l_{P_0} instead of l_P as above.

The minimal exponent of hardening ν and the exponents in Eqs (8), (13), (19) and (20), depending on it, are unaffected by the transition from psammic material to extended psammic material (covering fragmentation and abrasion), whereas the factors Ψ_∞ in Eq (13) and Ψ_0 in Eq (19) will be modified (Dietrich/Arslan, 1985), as pointed out already in the introduction.

EXPERIMENTS

Experiments have been done on four cylinders. The dimensions of these are given in table 1.

Table 1 Dimensions of the circular cylinders, used in the experiments

cylinder Nr	length l [mm]	radius R [mm]	ratio l/R
1	600	30	20
2	130	60	2.18
3	57	157	0.36
4	17.5	200	0.088

The cylinders rested on a bed of dense medium sand (Darmstadt-sand). The minimal exponent of hardening ν of this sand has been determined with great accuracy in former tests (Franke / Muth, 1985; Franke / Klüber, 1989) to be

$$\nu = 1 / 1.97 = 0.5076$$

Proportional vertical loading was chosen such that

$$45 \geq \frac{l}{l_P} = \frac{l}{R} \Big/ \left(\frac{P}{\gamma R^2}\right)^{\frac{1}{2+\nu}} \geq 0.07$$

Settlement of cylinders versus intensity of load is plotted on log-log-grid in Figs. 2 to 5. The theoretical power functions Eqs (13) and (19) based on the assumption of psammic behavior (rigid unbreakable grains) or of extended psammic behavior (rigid breakable grains) would be represented by straight lines on a log-log-grid, the slopes of which are given by the exponent $2 / (2 + \nu) = 0.798$ for the long and $2 / (2\nu + 1) = 0.992$ for the short cylinder. These slopes are drawn for comparison in Figs. 2 to 5 as well.

DISCUSSION

Due to the particulate nature of sand the relative accuracy of the continuous description of the indentation process is low when the indentation is small. The limitations of the measuring devices tend to produce the same effect. Correspondingly there is considerable scatter in the lower left parts of the diagrams of Figs. 2 to 5. A further effect of finite grain size may be seen in Fig. 5 where the stiffness of the sand proves very small at the beginning of indentation. This is due to the fact, that at the beginning the largest linear dimension of the circumference of the indenting portion of the cylinders is only 17.5 mm (see Table 1), whence the cylinder is carried by some dozen of grains only. As the indentation proceeds, 17.5 mm becomes the smallest linear dimension of the circumference and the indenting portion of the cylinder acts as a rodlike structure on psammic halfspace (or on extended psammic halfspace respectively). It is of the essence of a rodlike structure, that the distribution of contact stresses perpendicular to the rod's axis does not matter. Therefore the influence of absolute grain size vanishes as the indentation goes on, although the smallest linear dimension of 17.5 mm is smaller than permitted by the experimental criterion of Ovesen (1980).

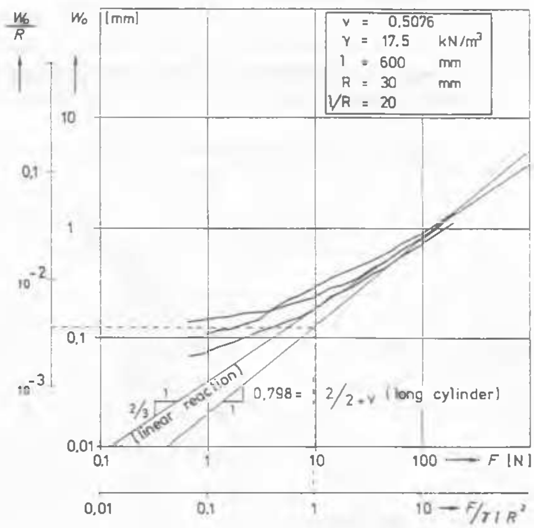

Fig. 2 Settlement w_0 of cylinder No. 1 due to total vertical load F, increasing monotonically

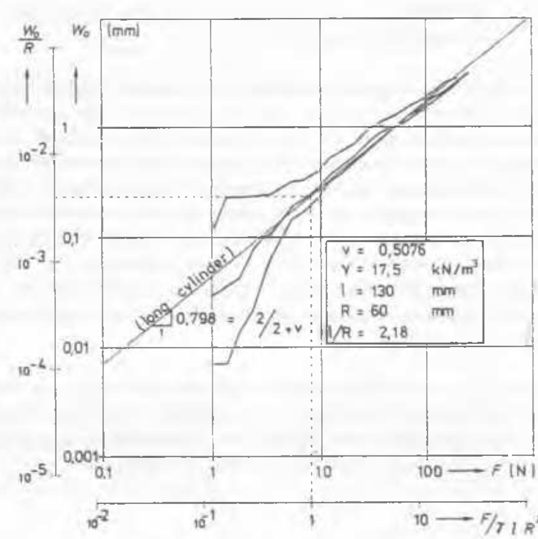

Fig. 3 Settlement w_0 of cylinder No. 2 due to total vertical load F, increasing monotonically

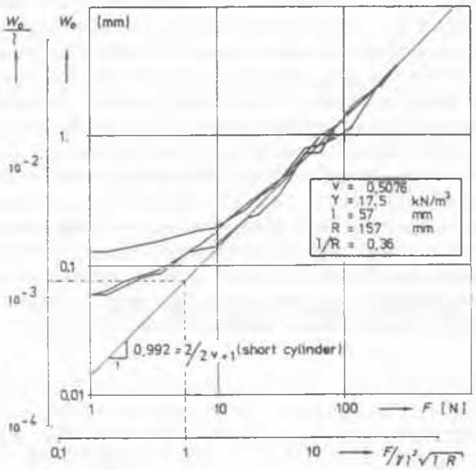

Fig. 4 Settlement w_0 of cylinder No. 3 due to total vertical load F, increasing monotonically

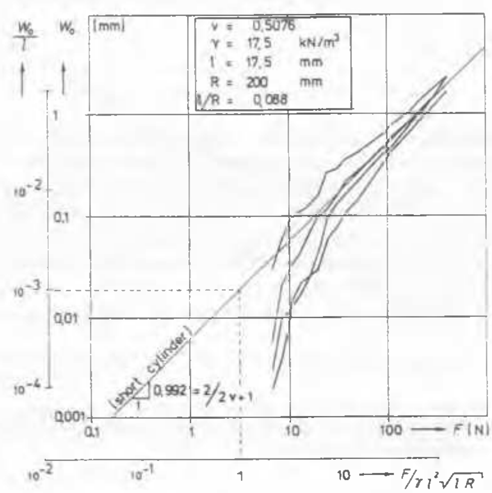

Fig. 5 Settlement w_0 of cylinder No. 4 due to total vertical load F, increasing monotonically

Figs. 3, 4 and 5, after the initial deviations just discussed, confirm the exponents of the power laws Eqs (13) and (19) quite well. The slope of the settlement-versus-load-curves in Fig. 2 however falls outside of the predicted range. It is even smaller than the smallest slope possible according to the theory in the second chapter. This might be attributed to the preponderance of elastic settlement over the settlement due to interparticle slip, fragmentation and abrasion. For elastic behavior ν tends to unity. In this case both the exponents of Eqs (13) to (19) are reduced to 2/3. The corresponding slope is indicated in Fig. 2. Influence of grain elasticity would be reduced by reducing the relative density or by uplift when the sediment is submerged.

The experiments yield the numberlike coefficients of Eqs (13) and (19), the values of which are not attainable yet by analysis. They may be read off from the load-settlement-curves in Figs. 2 to 5 observing the reduced scales. One obtains for the long cylinder on dry dense Darmstadt-sand

$$\frac{w}{R} = 0.0037 \left(\frac{P}{\gamma R^2}\right)^{0.798}$$

and for the short cylinder on dry dense Darmstadt-sand

$$\frac{w}{l} = 0.0012 \left(\frac{P}{\gamma l \sqrt{lR}}\right)^{0.992}$$

Other experiments with Darmstadt-sand (Franke/ Muth, 1985; Franke/Klüber, 1989) indicate, that the influence of fragmentation on the above coefficients is negligible in contrast to the influence of abrasion, which is considerable. To determine its size, a family of similar models at different scales must be investigated.

Nonlinear response of piles in sand has been reported by many investigators. Likewise nonlinear response of block foundations is well recorded. Therefore it might easily be overlooked how unexpected is a regular nonlinear response of a material body under three dimensional loading.

As has been pointed out in the first section of the introduction, the nonlinear response of psammic material (or extended ps. m. resp.) depends on the angle by which the strain path changes its direction abruptly. If this angle is zero the instantaneous response is linear. Since at first loading the response of sand is strictly nonlinear, the question arises for the direction of strain path which is to be attributed to the sedimentation process, so to say. It is the strain path direction of the oedometer strain. This strain is impossible for psammic material (or extended ps. m. resp.). Therefore every strain path induced by first loading begins with an abrupt change of direction the angle of which varies greatly over the psammic halfspace (or ext. ps. halfspace resp.). Consequently the considerations of topology and functional analysis, mentioned after Eq (4), are indispensable to obtain results such as Eqs (13) and (19). So these results are not at all trivial.

For practical purposes the above investigations should be completed by the investigation of the increase of settlement due to repeated loading. Repeated loading may multiply the settlement under first loading many times (see e.g. Holzlöhner, 1978). The increase is effected by the micromechanism of abrasion. The thus completed investigation will have interesting applications with pipelines but also with flexible foundation slabs and strips under concentrated loads, e.g. with railroad. The area of support of a railroadtrack depends on the moving load. If the track is stiff enough the halfspace cannot tell straining by train traffic from straining by a stationary load repeatedly applied. Then the kinship between the short cylinder under repeated loading and the railroad under traffic gets obvious.

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