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DYNAMIC RESPONSE OF EMBEDDED FOUNDATIONS

REACTION DYNAMIQUE DE FONDATIONS EN PLEINE TERRE

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SYNOPSIS : Several theories have been developed to predict the dynamic response of embedded foundations. With embedment, the stiffness increases and also the side friction increases the stiffness. In this paper theoretical solutions for the response of embedded foundations subjected to vertical vibration have been obtained considering the effect of increase in stiffness with depth and also the friction around the foundation. Experimental study has also been carried out to compare the test results with the theoretical predictions. Experiments have been carried out with concrete footings of area 45cm x 45cm with various depths of embedment of 0cm, 60cm, 120cm and 180cm.

INTRODUCTION

Several theories have been developed to predict the dynamic response of embedded foundations. These theories assume that the dynamic response is affected by the changes in (a) the stiffness coefficient of soil with depth and (b) the interfacial frictional force between the vertical face of the foundation and the surrounding soil.

Using elastic half-space theory and making use of lumped analog model, solutions have been obtained by Barnov (1967) and Novak and Beredugo (1972). Assuming the infinite soil media as a finite model and using numerical methods, solutions have been obtained by Lysmer and Kuhlemeyer (1969) and Waas and Lysmer (1972). By representing the friction force developed at the vertical interface between the foundation and the surrounding soil by a Coulomb friction damper and using mass-spring-dashpot model, solutions have also been obtained by Den Hartog, (1931), Anandakrishnan & Krishnaswamy (1971) and Sridharan etal (1981). All these methods consider that the stiffness coefficient is constant with depth.

The increase in stiffness coefficient with depth has been obtained by Kaldjian (1969, 1971), using finite element method and by Ramaiah etal (1977) and Sridharan etal (1983), using Mindlin's (1936) expression for the displacement of a point inside a homogeneous isotropic solid.

In this paper, theoretical solution for the response of embedded foundation subjected to vertical vibration has been obtained, considering the effect of both the above parameters, which can be represented in the form of nondimensional charts. Experimental study has also been carried out to compare the test results with the theoretical predictions.

THEORETICAL APPROACH

By equating the work done by the viscous damping force and the interfacial frictional force between the foundation and surrounding soil to that of an equivalent damping force, the expression for the vertical displacement, Z_o, of an embedded foundation, based on mass spring dashpot model, has been obtained as (Sridharan et al 1981)

$$Z_o = \frac{\frac{Q_o}{K}\left[\frac{-4CwF}{\pi K Q_o} \pm \left[\left(\frac{-4CwF}{\pi K Q_o}\right)^2 - Y\left\{\left(\frac{4F}{\pi Q_o}\right)^2 - 1\right\}\right]^{\frac{1}{2}}\right]}{Y} \quad [1]$$

where $Y = \left(1 - \frac{mw^2}{K}\right)^2 + \left(\frac{Cw}{K}\right)^2$

$$F = (\pi K_o \mu_f \Gamma_s r_o^3) H^2 \quad [2]$$

for cohesionless soil

Q_o = externally applied dynamic force
K = spring constant
w = operating frequency
C = damping co-efficient
m = total mass, z = embedment depth
K_o = earth pressure co-efficient at rest
μ_f = friction co-efficient between soil & the foundation surface
Γ_s = unit weight of soil
r_o = equivalent radius of foundation area
H = embedment ratio = z/r_o

Using Mindlin's equation (1936) for the displacement of a point at a depth due to a point load acting at a depth inside an elastic medium, the ratio of stiffness coefficient below the load to stiffness coeffcient at the surface for rigid base, uniform and parabolic type of loading conditions have been obtained by Sridharan etal (1983) as

$$K_{zR} = \frac{4\pi(1-\mu)^2}{K_1} \quad [3]$$

$$K_{zU} = \frac{8(1-\mu)^2}{K_2} \qquad [4]$$

and

$$K_{zP} = \frac{16(1-\mu)^2}{3K_3} \qquad [5]$$

respectively, where

$$K_1 = [T_3 \frac{\pi}{2} + T_4 \sin^{-1} (\frac{1}{H_1}) + \frac{T_5 . H}{2H_1^2} + \frac{H(1+12H^2)}{H_1^4}] \qquad [6]$$

$$K_2 = [T_3 + T_4 (H_1 - 2H) - T_5 (\frac{H^2}{H_1} - \frac{H}{2}) - 8 (\frac{H^4}{H^3_1} - \frac{H}{8})] \qquad [7]$$

$$K_3 = [\frac{2}{3} T_3 + \frac{2}{3} T_4 (H^3_1 - H(3+8H^2)) + \frac{T_5}{2} (H + 8H^3 - 4H^2 H_1) + \frac{16H^4}{H_1} + H(1 - 8H^2)] \qquad [8]$$

where $T_3 = 3-4\mu$, $T_4 = 5-12\mu+8\mu^2$, $T_5 = 10-16\mu$, $H_1 = (1+4H^2)^{\frac{1}{2}}$ and μ = Poisson's ratio

The equation of motion for an embedded foundation considering both (a) increase in the stiffness with depth and (b) effect of frictional force at the interface of the foundation and the surrounding soil is expressed as

$$m\ddot{Z} + C_{eq} \dot{Z} + K_{eq} Z = Q_o \sin wt \qquad [9]$$

where

K_{eq} = stiffness coefficient at the foundation depth

$Z = Z_o \sin wt$, w = frequency of Dynamic force

C_{eq} = equivalent damping coefficient.

The solution for displacement amplitude, Z_o is same as given in Eq. 1, with K = Keq

The magnification factor for frequency dependent excitation, M_r [$=Z_o m/m_e e$) is

$$M_r = \frac{-V_1 + (V^2_1 - V_2 V_3)^{\frac{1}{2}}}{V_2} \qquad [10]$$

in which $V_1 = 2D\eta H^2 a_o$, $V_2 = (K_{zR} - a_o^2)^2 + (2Da_o)^2$, $V_3 = (\eta H^2)^2 - a_o^4$, $n = F/(H^2 m_e e w_n^2)$, $a_o = w/w_n$, w_n = Natural frequency, D = Damping Factor.

For steady state vibration, the following condition has to be satisfied

$$\frac{4F}{\pi m_e e\ w_n^2} < a_o^2 \qquad [11]$$

The variation of magnification factor at resonance with embedment ratio for various values of damping factors are presented for rigid base type of loading for μ=0.25, and for friction force coefficient, η =0.1 in Fig. 1. Similar variations for uniform and parabolic loading conditions have been presented by Nagendra (1982).

It is observed that the magnification factor at resonance increases with embedment ratio in the beginning and then decreases. The increase in magnification factor at resonance is only up to an embedment ratio equal to 1.0. It can be seen in Eqn. 2 that the increase in friction force is proportional to the square of the embedment ratio, which means that the friction force increases slowly till embedment ratio is 1.0 and rapidly for embedment ratio greater than 1.0. Thus, the effect on stiffness increase is more for lower embedment ratios and the effect on friction force is predominant at higher embedment ratios.

The effect of friction force combined with viscous damping is to decrease the displacement amplitude and increase the resonant frequency. The increase in stiffness coefficient increases the critical damping, which results in increase of resonant amplitude and decrease of resonant frequency. This reduction is small when compared with the increase in resonant frequency due to increase in stiffness coefficient. Thus the net effect of increase in stiffness coefficient is to increase both resonant amplitude and resonant frequency.

Hence, the net effect of embedment is to increase the resonant frequency and the net effect on resonant amplitude depends on the relative effect of increase of frictional force and stiffness coefficient. Since at lower embedments when the effect is more on stiffness coefficient, the magnification factor increases, where as at higher embedments when the effect is more on friction force, the magnification factor decreases.

It is also seen in Fig.1 that the magnification factor at resonance decreases with increase in damping factor. Fig. 2 indicates that the frequency factor at resonance increases with embedment ratio and damping factor. The increase is more for higher damping factors compared to lower damping factors.

It is seen from Fig.3 that for low embedment ratio when the friction force coefficient is small, magnification factor increases and then decreases. For higher values of friction force coefficient, such an increase is not noticed at lower embedments.

It is observed from Fig. 4 that the frequency factor increases with increase of embedment ratio and friction force coefficient. Similar results have been obtained for other Poisson's ratios and damping factors (Nagendra 1982).It has been found that at resonance the magnification factor decreases with increase in Poisson's ratio and the frequency factor decreases with increase in Poisson's ratio, when the friction force is small but increases when the friction force increases.

EXPERIMENTAL PROGRAMME

TEST SITE

The site chosen near Soil Mechanics laboratory, Indian Institute of Science, Bangalore was about 5m x 8m. From the subsurface exploration of the test site by four bore holes (5m depth) the soil condition was found to be red earth of lateritic origin (Raman 1975). Water table was not encountered up to 5m depth.

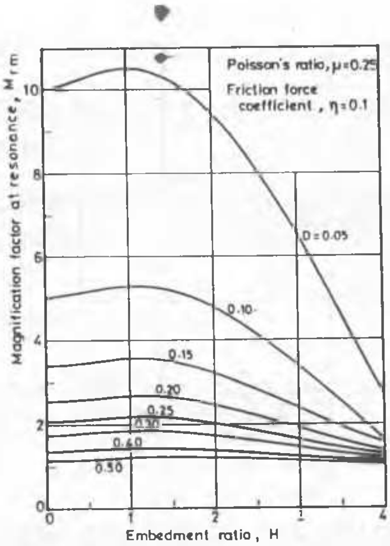

Fig.1 - Variation of Magnification Factor at Resonance, M_{rm} with Embedment Ratio, H.

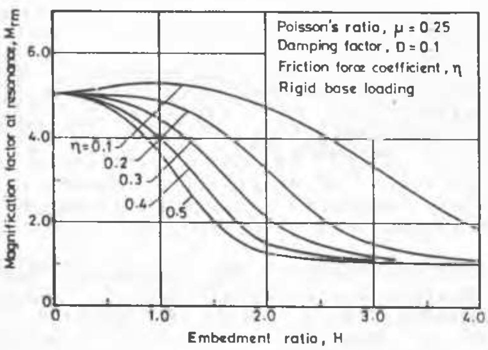

Fig. 3 - Variation of Magnification Factor at Resonance, M_{rm} with Embedment Ratio, H.

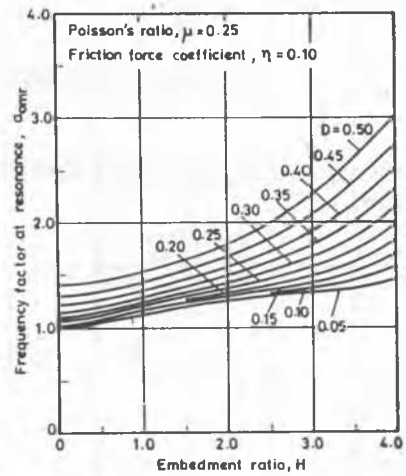

Fig.2 - Variation of Frequency Ratio at Resonance, a_{omr} with Embedment Ratio, H.

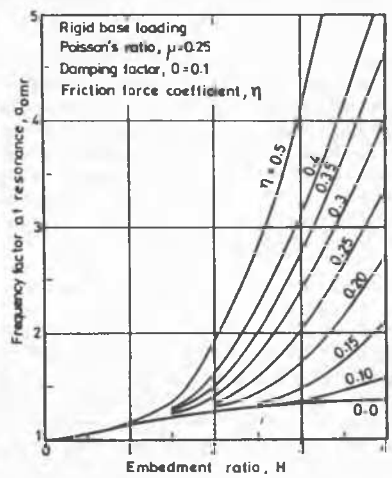

Fig. 4 - Variation of Frequency Ratio at Resonance, a_{omr} with Embedment Ratio, H.

EXPERIMENTS

Tests have been carried out on surface footing and embedded footings at different depths, with and without soil around the footing for different static loads and excitation levels.

The effect of increase in stiffness due to depth has been evaluated by comparing the results of the surface footing and foundation at certain depth without soil around the footing. Such a foundation will not be acted upon by frictional force along the vertical face. The effect of side frictional force has been evaluated by refilling the soil into the trench around the footing to its natural density.

The footings were made up of reinforced concrete with nominal reinforcement. Square footings of same base area 45cm x 45cm but with different embedment depths (60 cm, 120 cm and 180 cm) were cast. Lazan oscillator model LA-1 having eccentric masses driven by a motor was used for the investigation. The displacement amplitude of vibration was measured using PHILIPS electrodynamic pickup to an accuracy of 1 micron.

The foundation was subjected to vibration in vertical direction and the frequency and corresponding displacement amplitude were recorded. In all 156 tests were conducted on footings of different embedments under various static loads and excitation levels and the resonant amplitude and corresponding frequency were obtained. Only limited results are presented.

RESULTS AND DISCUSSION

(a) Effect of increase in stiffness coefficient with depth (without soil around the footing): This series of experiments were conducted on four foundations with soil having been removed upto their bottom level, for four static loads and four excitation levels. The test results showed that resonant frequency decreases with increase in both static load and excitation level, while resonant amplitude decreases with increase in static load and increases with excitation level, as it ought to be.

A comparison of frequency ratio (frequency-embedded/frequency-surface) and amplitude ratio (amplitude-embedded/amplitude-surface) at the same static load and excitation level has been made in Fig. 5. It is seen that frequency ratio

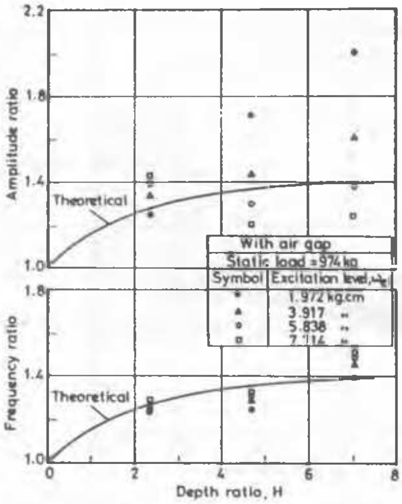

Fig. 5 - Comprison of Frequency and Amplitude Ratios at Resonance from Theory and Experiment

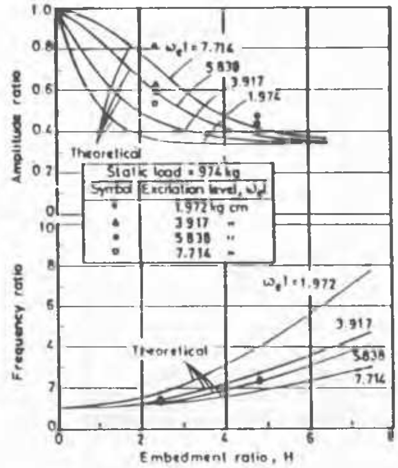

Fig. 6 - Comparison of Frequency and Amplitude Ratios at Resonance from Theory and Experiment

and amplitude ratio increase with depth. Further the effect of depth ratio is significant in the initial stages and marginal with increase in depth ratios. The increase in the frequency ratio with depth also compares well with the theoretical prediction for the increase in stiffness coefficient with depth considering rigid base pressure distribution.

(b) Foundation with soil friction around: This series of experiments was conducted on seven foundations for four static loads and four excitation levels. Fig. 6 presents the variation of resonant frequency ratio (fre.emb/fre-sur) and resonant amplitude ratio (amp. emb/amp-sur) with embedment ratio for four static loads. Resonant frequency ratio increases and resonant amplitude ratio decreases with embedment ratio, as it ought to be. It is clear that embedment effect on resonant frequency is significant, whereas its effect on resonant amplitude is less at higher embedment ratios. Hence, it is not worthwhile to increase the embedment indefinitely in order to reduce the amplitude, while the resonant frequency could be significantly increased with embedment. It could also be seen in Fig.6 that the experimental results also compare well with the theoretical prediction (Eqn. 10). Similar results have been obtained for other static loads (Nagendra 1982).

CONCLUSIONS

Theoretical solution for the response of embedded foundations subjected to vertical vibration has been obtained considering the effect of increase in stiffness with depth and also the friction around the foundation. Experiments carried out with and without soil around the embedded foundations and foundation on the surface agree well with the theoretical predictions. The resonant frequency increases and resonant amplitude decreases for embedded foundations. It has been found that the effect of embedment is significant at lower values of depth of embedment.

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