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# LATERAL VIBRATION OF TAPERED PILES

# VIBRATION LATERALE DE PIEUX A BOUT CONIQUE

***Sudhendu Saha***

Professor of Civil Engineering
Bengal Engineering College
Howrah, India

**SYNOPSIS :** An approximate computer-aided numerical analysis for tapered piles under lateral vibration is presented. The method of analysis accounts for soil-pile interaction in a relatively simple manner. Since cross-section and dynamic soil reaction per unit length of tapered piles vary with depth, the solution for dynamic response is obtained by finite difference method. The dynamic stiffness and damping parameters are frequency dependent and vary with several other factors including slenderness ratio. In all cases, however, stiffness increases and damping decreases with increase of angle of taper. The stiffness and damping coefficients of single tapered piles can be easily utilised to determine the dimensionless amplitudes of footings supported by tapered piles and exposed to frequency dependent horizontal excitation. It has been concluded that the use of tapered piles would be generally advantageous to meet the design criteria for footings subjected to unbalanced lateral dynamic forces.

## INTRODUCTION

The lateral dynamic characteristic of piles has been receiving considerable increase in interest particularly in machinery foundations and structures exposed to unbalanced lateral dynamic forces. Designing pile foundations to resist dynamic lateral loads primarily requires limiting the pile deflection to permissible values and also to avoid resonance. Much of the research into this subject has been prompted by the need to design energy related large and expensive structures. The response of structures to dynamic lateral loads depends to a great extent on the stiffness and damping that the pile-soil system can provide.

A number of analyses have been developed to study the lateral dynamic behaviour of piles. Winkler type formulations were developed by Prakash and Chandrasekharan (1973), Kagawa and Kraft (1980) and others. Penzien (1970) and Matlock et al (1978) used lumped mass idealisation as a mathematical model to represent the pile-soil system. Recent approaches consider pile-soil interaction in terms of continuum mechanics and account for propagation of elastic waves, (Novak and Nogami 1977, and others). Kuhlemeyer (1979), Angelides and Roesset (1981), and others have attempted dynamic finite element formulations with energy-absorbing boundaries to simulate the effect of outward spreading waves. The above studies on piles have indicated that dynamic pile-soil interaction modifies pile stiffness which is frequency dependent and generates geometric damping through energy radiation.

Although considerable advancement has been made in developing methods of analysis and understanding the complex dynamic soil-pile interaction, the phenomena are yet to be fully understood. Moreover, in the above analyses, attentions have been given to study dynamic characteristics of cylindrical piles only. This paper utilises a simple computer-aided approximate numerical approach (Saha and Ghosh 1986 a, b) to explain the coupled horizontal and rocking vibration of tapered piles which have relative advantage over the cylindrical piles subjected to lateral dynamic forces.

## THEORETICAL ANALYSIS

The analysis is based on the assumption that the pile is vertical, elastic and of circular cross section, but uniformly tapered down with depth and has perfect contact with elastic homogeneous isotropic semi-infinite soil medium. The viscous damping per unit area of projected vertical surface and that on the projected annular horizontal surfaces of the pile elements are equal to the equivalent viscous damping of a rigid disk vibrating vertically and horizontally respectively on semi-infinite elastic medium. For convenience of computer aided numerical analysis, the tapered pile has been discretised into cylindrical elements of reducing diameter with depth to form idealised step tapered pile (Fig. 1).

A pile element undergoing a complex horizontal displacement u(z, t) at a depth z meets horizontal soil reactions on projected vertical and horizontal surfaces of pile elements. The side soil reaction on vertical surfaces per unit length of pile is assumed as complex function of soil parameters, frequency and depth in the form as

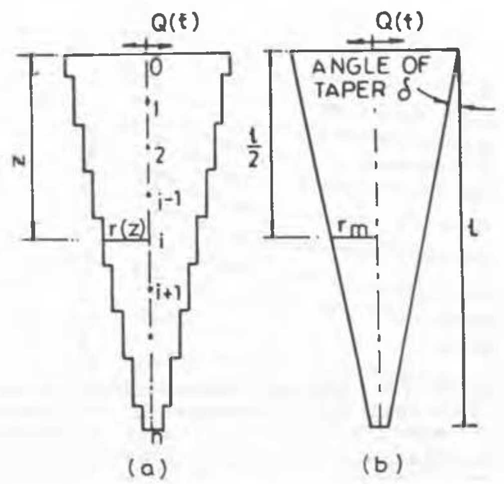


FIG.1 : TAPERED PILE (a) DISCRETISED (b) REAL

$$S_w(z,t) = [F_w(z) + iw\, C_w(z)]\, u(z,t) \qquad ..\ (1)$$

where $F_u(z)$ = horizontal elastic stiffness function varying with depth per unit length of pile,

$$u(z,t) = u(z)\, e^{iwt} \qquad ..\ (2)$$

w = circular frequency

t = time, $i = \sqrt{-1}$

$C_w(z)$ = equivalent viscous damping function varying with depth per unit length of pile

$$= \frac{2G\, C_{w2}(z)}{\pi\ w} \qquad ..\ (3)$$

$C_{w2}(z)$ = half space damping parameter varying with depth z, Poisson's ratio $\nu$ and dimensionless frequency $a_o$ (Fig. 2).

$$a_o = wr_o(z)/v_s \qquad ..\ (4)$$

$r_o(z)$ = radius of pile at depth z,

$V_s$ = shear wave velocity of soil medium.

The soil reaction on projected annular horizontal surface per unit length of pile has been derived after Arnold et al (1955) and Hsieh (1962), and is written as

$$S_u(z,t) = [F_u(z) + iw\, C_u(z)]\, u(z,t) \qquad ..\ (5)$$

where $F_u(z)$ = elastic stiffness function varying with depth per unit length of pile

$$F_u(z) = Gr_o(z)\, C_{u1}(z) \qquad ..\ (6)$$

$C_{u1}(z)$ = half space stiffness parameter varying with depth, Poisson's ratio and dimensionless frequency (Fig. 2)

$C_u(z)$ = equivalent viscous damping function varying with depth, per unit length of pile

$$= \frac{4GC_{u2}(z)}{w\, r_o(z)}\, [r_o(z) \tan\delta - \tan^2\delta\,] \qquad ..\ (7)$$

$C_{u2}(z)$ = half space damping parameter varying with z, $\nu$ and $a_o$(Fig.2)

$\delta$ = angle of pile taper

The differential equation of the damped horizontal vibration of discretised tapered pile has been written as

$$m(z)\,\frac{\partial^2 u(z,t)}{\partial t^2} + C\,\frac{\partial u(z,t)}{\partial t} + S_w(z,t) + S_u(z,t) + EI(z)\,\frac{\partial^4 u(z,t)}{\partial z^4} = 0 \qquad ..\ (8)$$

where, m(z) = mass per unit length of pile varying with depth

C = Co-efficient of pile material damping

I(z) = moment of inertia of pile cross-section varying with depth

E = Young's modulus of pile material

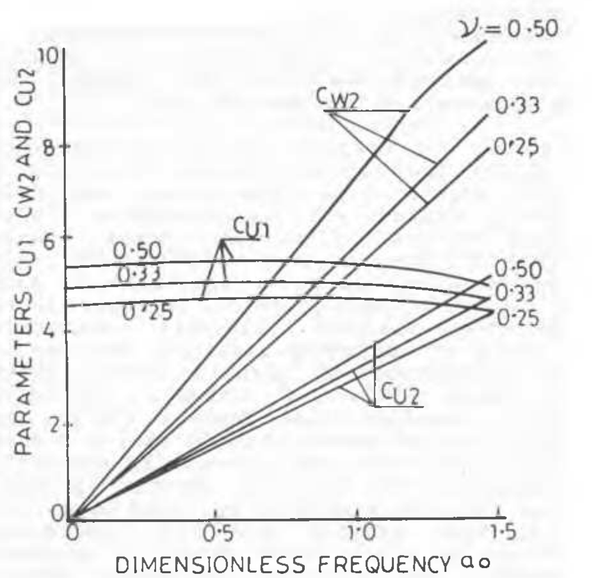


FIG. 2 : HALF SPACE STIFFNESS AND DAMPING PARAMETERS

Due to harmonic horizontal and/or rotational excitation, the pile undergoes complex horizontal vibration expressed by Eqn. (2) in which

$$u(z) = u_1(z) + u_2(z) \quad .. \quad (9)$$

where $u_1$ and $u_2$ are the real and imaginary parts respectively of complex displacement u(z). Substituting Eqn. (2) in Eqn. (8) and neglecting pile internal damping, the following, ordinary differential equation has been obtained :

$$EI(z) \frac{d^4u(z)}{dz^4} + [F_w(z) + F_u(z) - m(z)\, w^2 + iw \left\{ C_w(w) + C_u(z) \right\} ]\, u(z) = 0 \quad .. \quad (10)$$

**FINITE DIFFERENCE SOLUTION**

The boundary conditions for the piles at top may be free or fixed. The constraint at the pile head may arise from the presence of a pile cap or through direct connection of the piles with the superstructure. The dynamic stiffness of pile is determined as the end force or moment producing unit displacement or rotation of the pile head. Discretising the pile (Fig. 1) into a number of elements, and satisfying the appropriate boundary conditions, the Eqn. (10) has been solved for nodal lateral displacements using computer aided finite difference technique. From the complex lateral nodal displacements for respective modes of vibration, the horizontal, cross and rotational stiffness and damping co-efficients of pile at its head are calculated (Saha and Ghosh 1986b).

Horizontal stiffness co-efficient

$$K_x = \frac{EI_o}{r_m} \cdot f_{u1} \quad .. \quad (11)$$

Horizontal damping co-efficient

$$C_x = \frac{EI_o}{r_m^2 \cdot V_s} \cdot f_{u2} \quad .. \quad (12)$$

Rotational Stiffness co-efficient

$$K_\theta = \frac{EI_o}{r_m} \cdot f_{\theta 1} \quad .. \quad (13)$$

Rotational damping coefficient

$$C_\theta = \frac{EI_o}{V_s} \cdot f_{\theta 2} \quad .. \quad (14)$$

Cross stiffness coefficient

$$K_{x\theta} = K_{\theta x} = \frac{EI_o}{r_m^2} \cdot f_{c1} \quad .. \quad (15)$$

Cross damping coefficient

$$C_{\theta x} = C_{x\theta} = \frac{EI_o}{r_m \; V_s} \cdot f_{c2} \quad .. \quad (16)$$

where $f_{u1}$, $f_{\theta 1}$ and $f_{c1}$ are the horizontal, rotational and cross stiffness parameters respectively; $f_{u2}$, $f_{\theta 2}$ and $f_{c2}$ are the horizontal, rotational and cross damping parameters respectively. $r_m$ is the mean radius or the radius at mid depth of the tapered pile. $I_o$ is the moment of inertia of the pile cross section at pile head. The stiffness and damping coefficients of single piles are utilised to determine the equivalent stiffness and damping coefficients of footing supported on group of piles. These are determined as the forces at the centroid of the footing to produce corresponding unit displacement of the centroid (Saha and Ghosh 1985).

**DISCUSSION**

The accuracy of the theory for piles of uniform diameter has been satisfactorily examined by comparing theoretical results with the experimental results reported by Novak and Grigg (1976). These comparisons for lateral dynamic responses of single pile and pile group has been described else where (Saha and Ghosh 1985, 1986b). Some typical variations of horizontal, rotational and cross stiffness and damping parameters of tapered concrete piles with dimensionless frequency $a_o = wr_m/V_s$ and slenderness ratio $l/r_m$ are shown in Figs. 3 and 4 respectively. It has been observed that these parameters are frequency dependent and vary with several other factors like slenderness ratio, wave velocity ratio $V_s/V_c$, $V_c$ being compressional wave

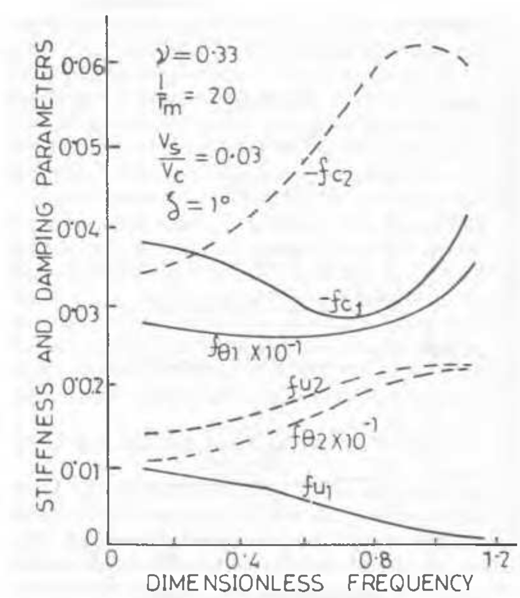


FIG. 3 : VARIATIONS OF STIFFNESS AND DAMPING PARAMETERS OF PILE WITH DIMENSIONLESS FREQUENCY

velocity in pile material, Poisson's ratio and angle of taper. With the increase of angle of taper, stiffness increases and damping decreases at any particular frequency. The variations of stiffness and damping parameters are insignificant beyond a certain slenderness ratio.

## CONCLUSION

From the analysis and discussion of the results, it may be concluded that the proposed simple approximate method predicts reasonably the lateral dynamic response characteristics of tapered pile and of footings supported on tapered piles. Use of tapered pile is generally advantageous for increasing the resonant frequency and decreasing the resonant amplitude of a pile supported footing. Since with decrease of slenderness ratio, stiffness increases and damping decreases, use of tapered piles would be helpful to meet the design criteria. The stiffness and damping parameters can be easily computed and used to predict the dynamic behaviour of laterally excited tapered piles.

## REFERENCES


Penzien, J. (1970) - Soil-Pile Foundation Interaction, Chapter-14, Earthquake Engg. Ed. Wiegel, F.L., Prentice-Hall, Englewood cliffs, N. J.

Hsieh, T. K. (1962) - Foundation Vibrations, Proc. Inst. of Civil Engineers, London, Vol. 22.

Matlock, H., Stephen, H.C. and Bryant, L.M. (1978) - Simulation of lateral pile behaviour under earthquake motion, Proc. of speciality Conf. on Earthquake Engg. and Soil Dynamics, ASCE, Pasadena.


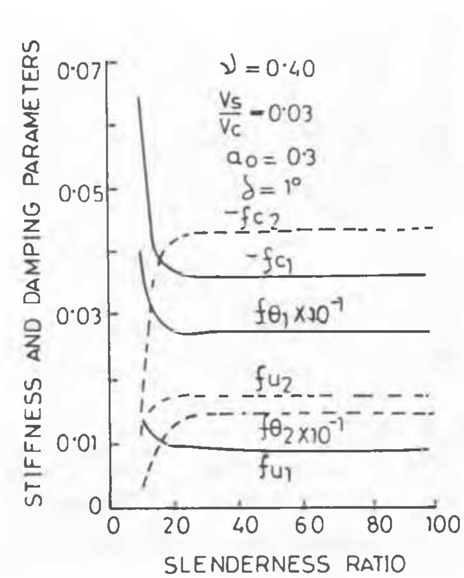


FIG. 4 : VARIATIONS OF STIFFNESS AND DAMPING PARAMETERS OF PILE WITH SLENDERNESS RATIO


Prakash, S. and Chandrasekharan, V. (1973) - Pile Foundation under lateral dynamic loads, Proc. 8th ICSMFE, Vol. 2.1.

Saha, S. and Ghosh, D. P. (1985) - Analysis of Pile groups under lateral dynamic loads, Proc. IGC-85, Roorkee, Vol.1.

Angelides, D.C. and Roesset, J.M. (1981) - Non-linear lateral dynamic stiffness of Piles, Jr. of Geotech. Engg. Divn., ASCE, Vol. 107, GT-11.

Arnold, R.N., et al (1955) - Forced vibration of a body on an infinite elastic solids, Jr. of Applied Mechanics, ASME, Vol.22, No. 3.

Kagawa, T. and Kraft, L. M. (1980) - Lateral Load-deflection relationship of piles subjected to dynamic loadings, Soils and Foundations, Vol. 20, No. 4.

Kuhlemeyer, R. L. (1979) - Static and dynamic laterally loaded Floating Piles, Jr. of Geotech. Divn., ASCE, Vol. 105, GT - 2.

Novak, M. and Grigg, R. F. (1976) - Dynamic Experiments with small pile foundations, Canadian Geotechnical Journal, Vol. 13.

Novak, M. and Nogami, T. (1977) - Soil-Pile interaction in horizontal vibration, Int. Jr. of Earthquake Engg. and Structural Dynamics, Vol. 5, No. 3.

Saha, S. and Ghosh, D. P. (1986a) - Vertical vibration of Tapered Piles, Jr. of Geotech Engg., ASCE, Vol. 112, No. 3.

Saha, S. and Ghosh, D. P. (1986b) - Dynamic Lateral response of piles in coupled mode of vibration, Soils and Foundations Japan, Vol. 26, No. 1.