ACTIVE PRESSURE DISTRIBUTION IN COHESIVE SOILS

DISTRIBUTION DES PRESSIONS ACTIVES DANS LES SOLS COHESIFS

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SYNOPSIS: For cohesive soils Rankine's earth pressure theory often predicts deep tensile cracks. In practice designers rarely rely on deep tensile cracks, instead they replace the theoretical pressure distribution with pressure diagrams obtained from experience and/or from empirical modifications of Rankine's theory. In this paper the effects of long term loading, saturation and shear strength at low confining stresses are presented. It was found that the classical Rankine soil pressure distribution can significantly underestimate the active pressures on retaining walls and that more conservative methods should be used for cohesive soils. This paper suggests that the curvature of the failure envelope has a significant effect on earth pressures. This effect can be approximated using a new bilinear pressure distribution or some commonly used empirical diagrams.

INTRODUCTION

In designing retaining walls, the analysis of soil pressures has been of paramount interest. In recent years the refinement of the Finite Element Method together with sophisticated constitutive models has resulted in realistic determination of earth pressures. These numerical methods can take into account construction history, soil-structure displacements, effects of variable structural rigidity, effects of variable backfill soil properties, etc... Nonetheless, design solutions for predicting earth pressures have been based, mostly on conventional theories.

There are two classical theories to compute earth pressures on retaining walls. These are the Coulomb and Rankine theories. Coulomb's earth pressure theory is commonly used for cohesionless soils and generally gives quite satisfactory results. For cohesive soils the Rankine method is typically used. Rankine's theory requires a tensile crack developing to a depth $z = z_c$ and a linear pressure distribution at $z > z_c$. According to Rankine's theory values of z_c in the order of 150 to 300 cm are not uncommon for cohesive soils. Tensile cracks to such depths are rarely stable. Furthermore, it is questionable that the absence of long term soil pressures may extend to depths below one meter. Therefore, for calculating purposes geotechnical engineers typically substitute the theoretical pressure distributions with pressure diagrams obtained from experience and/or from empirical modifications to Rankine's theory. These diagrams often lack the theoretical or experimental background, and usually incorporate a certain degree of conservatism, in order to neglect or mask some aspects of the problem in the interest of simplicity.

EARTH PRESSURE THEORIES

Numerous methods for computing earth pressures on retaining walls have been proposed. These methods can be grouped into three general classes:

1) General relationships can be used to characterize the stress-strain behavior of a soil (Lade and Duncan, 1973; Pradel and Ishihara, 1988;...). Using numerical techniques such as the Finite Element Method the values of stresses and deformations can be found throughout the soil mass (Duncan and Clough, 1990; Morgenstern and Eisenstein, 1970). This method requires extensive testing to obtain the constants of the constitutive model, access to a sophisticated computer program and is seldom used in practice.
2) Methods based on the Theory of Plasticity determine stresses assuming that plastic failure is fulfilled either in the whole mass or along specific lines or surfaces. They can be divided into Slip Line (Brinch Hansen, 1953; Sokolovski, 1960), Lower Bound (Rankine, 1857; Arai and Jinki, 1990) and Upper Bound Methods (James and Brandsby, 1971; Chen, 1975).
3) Methods based on Extremum Conditions select a slip surface and then determine the forces acting on the boundaries of the earth mass. The definite slip surface is the one that gives the maximum or minimum earth pressure (Coulomb, 1773; Janbu, 1957; Bang 1985).

In recent years numerous theoretical and experimental investigations have confirmed the validity of Coulomb's earth pressure theory for cohesionless soils. The coefficients of lateral pressures from various theories were compared by Morgenstern and Eisenstein (1970) using different angles of wall friction. The variations in the coefficient of active pressure using these theories was generally less than 10%. A recent study by Arai and Jinki (1990) using Finite Elements concluded that Coulomb's method produces active thrusts close to the lower-bound approach of the theory of plasticity regardless of wall friction, wall inclination, and backfill slope. It has also been verified experimentally that for cohesionless soils, Coulomb's earth pressures provides an adequate assessment of the distribution, as well as the magnitude, of the stresses mobilized behind a wall (Sherif and Fang, 1984). Hence, it is not surprising that this theory is commonly used by engineers for cohesionless soils. One of the shortcomings of Coulomb's theory, is that it does not consider cohesion of the backfill material. Hence, Coulomb's earth pressure theory is commonly used for cohesionless soils such as sands. The Rankine's method takes into account the cohesion of the backfill material. Therefore, when cohesive soils are encountered, engineers prefer to use Rankine's method, even though Rankine's method can be applied only to a vertical smooth wall.

RANKINE'S ACTIVE EARTH PRESSURE THEORY

Rankine (1857) studied the stress conditions corresponding to the plastic equilibrium of a semi-infinite mass with a plane surface. For simplicity, Rankine assumed a frictionless wall, and found that the limiting state of stress is reached when the Mohr circle touches the failure envelope. The active pressure condition occurs when the retaining wall is allowed to move by an amount sufficient to develop a state of limit equilibrium in the soil. Then, the lateral stress σ_h can be found geometrically from the vertical stress $\sigma_v = \gamma z$, where γ is the soil's unit weight. If the failure envelope is a simple straight line defined by the cohesion c and the friction angle ϕ, then:

$$\sigma_h = K_a \gamma (z - z_c) \qquad (1)$$

$$K_a = \tan^2\left(45^o - \frac{\phi}{2}\right) \qquad (2)$$

$$z_c = \frac{2c}{\gamma\sqrt{K_a}} \qquad (3)$$

(Effects of surcharge and groundwater pressures are not included)
The pressure analysis above defines a state of plastic equilibrium that neglects the effect of friction between the wall and the soil mass. For active earth pressures, particularly in structures less than 760 cm, the error is small and can be neglected (Sowers, 1964).

APPLICATION OF RANKINE'S THEORY

In homogeneous backfills Rankine's theory will have a pressure distribution as shown in Fig.1a, with a tensile crack developing to a depth z_c. For cohesive soils, strength parameters often result in deep tensile cracks. Deep tensile cracks are rarely stable. Furthermore, it is questionable that the absence of long term soil pressures may extend to depths below a meter. Therefore, in practice the tension zone is not relied on to reduce lateral earth pressures. For design purposes, Rankine's pressures are often replaced by empirical pressure distributions. Commonly encountered practices include:

1) Method A: For calculation purposes the cohesive soil backfill is replaced by an assumed granular soil with the active pressure diagram shown in Fig.1b (Bowles, 1982; Das, 1990).
2) Method B: It is assumed that a tension crack will form to the depth z_c and that it will fill with water which then exerts a positive pressure on the wall as shown in Fig.1c (Bowles, 1982; Clough and Duncan, 1990).
3) Method C: For walls with clayey soils in the backfill, design pressures are based on experience rather than theory (Clough and Duncan, 1990). In design, Terzaghi and Peck (1967) earth pressures charts are often used.
4) Method D: Calculations are performed with a different set of strength parameters. Often, the cohesion is neglected or largely reduced (Bjerrum and Kirkedam, 1958).
5) Method E : Some codes allow designs based on arbitrary values, regardless of the soil type or shear strength. For instance, in Los Angeles unrestrained retaining walls that are not over 460 cm and have a leveled drained backfill, may be designed for an assumed Equivalent Fluid Unit Weight of 4.71 kN/m^3 regardless of soil type and shear strength parameters.

Note that these methods generally give design loads much larger than the Rankine theory, and that they are often used in design regardless of the type of drainage installed. These methods show that the use of earth pressure theories for cohesive backfills require special precautions, because Rankine's earth pressure theory tends to underestimate the actual thrust on the retaining wall.

SHEAR STRENGTH OF COHESIVE SOILS

The use of earth pressure theories requires knowledge of the shear strength of the soil. Factors that can affect the strength of cohesive backfills include: saturation and creep. It has long been established that while the strength of cohesionless soils is not seriously affected by saturation, cohesive soils are drastically affected due to increases in the water content (Terzaghi, 1934; Sowers, 1964), and thus the earth pressures may increase significantly. Furthermore, it is well known that the strength of a soil depends on the rate of loading, and that with a slowing-down of the rate of displacement, the peak strength of soils significantly deteriorates (Vyalov, 1986). Therefore, the stress-strain behavior for very slowly applied stresses is different than that for a conventional test which requires a few hours for completion. The effects of creep include a lower peak shear strength, τ_f, and a larger strain at failure. Hence, creep can cause a considerable increase in earth pressures.

Since the stability of the soil mass behind a retaining wall is assessed in terms of the peak strength of the material, it seems necessary to introduce formulas that incorporate the evolution of the shear strength with time. These formulas must consider the relation between the laboratory rate of displacement and the field rate. Such formulas have been proposed, e.g. Skempton (1964), but are complex and require extensive testing at different rates of displacement. Since designers are usually concerned with long term behavior, only long term

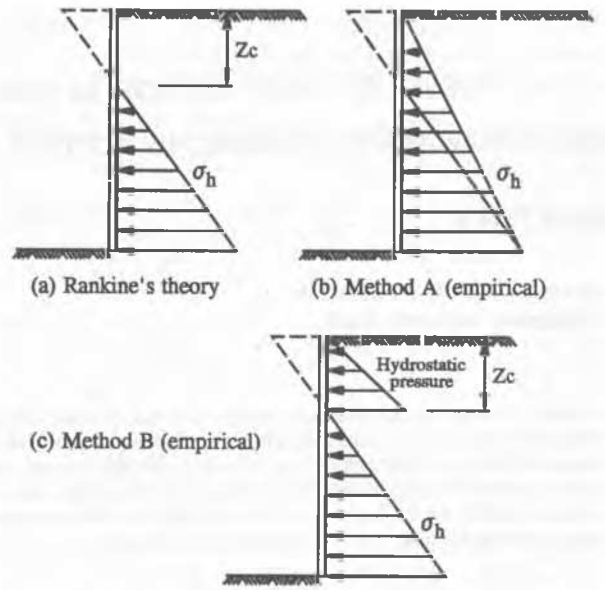

Fig 1. Active pressure distributions in cohesive soils.

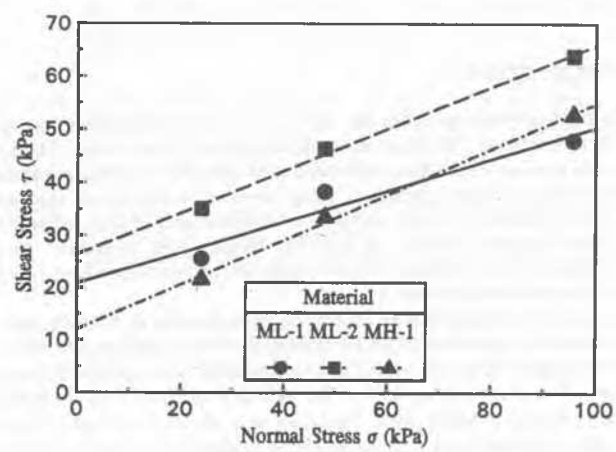

Fig 2. Shear strength data for materials ML-1, ML-2 and MH-1

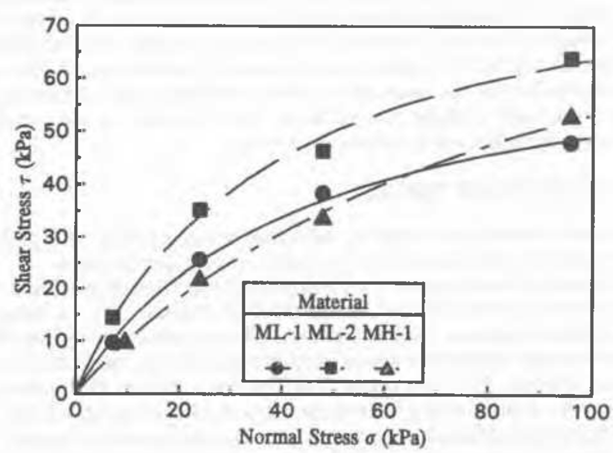

Fig 3. A curved failure envelope for materials ML-1, ML-2 and MH-1

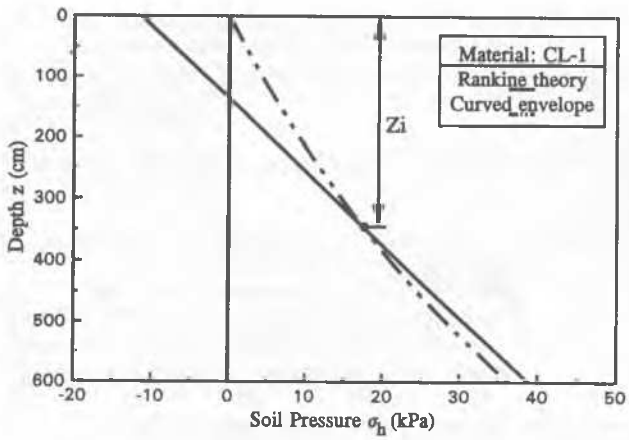

Fig 4. Active soil pressures for material CL-1.

strength needs to be assessed. For this purpose the residual strength, τ_r, of the material may be used as a criterion for mechanical stability of the backfill material. This procedure is supported by the experimental findings of Skempton (1964) and Babitskaya and Turovskaya (Vyalov, 1986). Skempton found that the residual soil strength is hardly influenced by the rate of displacement and that the use of residual strength would not produce an error in excess of 2-5% over a wide range of field displacement rates. Similarly, Babitskaya and Turovskaya found that the residual strength of plastic soils had values close to the ultimate long term strength, i.e. $\tau_r = (\tau_f)_\infty$ (Vyalov, 1986). Thus, in assessing the long term earth pressure on a retaining wall, the residual strength may be used for calculations.

EXPERIMENTAL PROCEDURES

Undisturbed specimens of materials from five sites around Los Angeles were tested. Direct shear tests, in accordance with ASTM 3080, were conducted to determine the strength of the materials. The intact specimens were flooded and sheared slowly (0.015%/min.) under a constant normal stress σ. Values of σ equal to 23.9, 47.9 and 95.8 kPa were used for the determination of the cohesion c and friction angle ϕ (Fig.2). In addition, several tests at very low normal stresses were performed with values of σ equal to 7.18 and 19.58 kPa.

A CURVED FAILURE ENVELOPE

In practice, the failure envelope of soils is typically assumed linear and the Mohr-Coulomb failure criteria is used. However, experimental evidence has shown that the failure envelope is generally curved. The curvature is particularly significant at low confining stresses, where the Mohr-Coulomb criteria tends to severely overestimate the strength of the soil. Numerous expressions have been proposed to describe the nonlinear failure envelope of soils using power, cycloid, and hyperbolic equations (Atkinson and Farrar, 1985; Maksimovic, 1989; Vyalov, 1986). In uncemented soils the interparticle attraction forces tend to be minimal and soil does not have any true cohesion. The absence of a true cohesion can be exemplified by a test in which a clay sample is submerged in water. In this test collapse of the sample typically occurs within a few hours. In order to capture this behavior the curved failure envelope must contain the origin: $\sigma = \tau = 0$. The results of the direct shear tests described above were analyzed using the following hyperbolic relation:

$$\tau_f = \frac{\sigma_f}{a \cdot \sigma_f + b} \tag{4}$$

Parameters a and b where obtained from the experimental data by linear regression of (σ_f/τ_f) vs. σ_f. Fig.3 shows that this hyperbolic relation gives very good predictions for $\sigma \leq 100$ kPa. Note that for a given normal stress σ_f on the slope of the failure surface is:

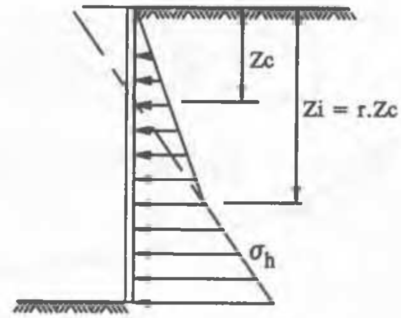

Fig 5. New method for the evaluation of active pressures

$$\frac{d\tau_f}{d\sigma_f} = \frac{b}{(a \cdot \sigma_f + b)^2} \tag{5}$$

LATERAL EARTH PRESSURES

For a curved failure surface the horizontal and vertical stresses from Rankine's active state can be obtained as follows:

$$\sigma_h = \sigma_f + \tau_f\left(\frac{d\tau_f}{d\sigma_f} - \sqrt{1 + \left(\frac{d\tau_f}{d\sigma_f}\right)^2}\right) \tag{6}$$

$$\sigma_v = \sigma_f + \tau_f\left(\frac{d\tau_f}{d\sigma_f} + \sqrt{1 + \left(\frac{d\tau_f}{d\sigma_f}\right)^2}\right) \tag{7}$$

In Fig.4 the lateral soil pressures predicted by Eq.6 are compared with those from Rankine theory. These figures show that the use of a curved failure envelope implies higher soil pressures at shallow depths. They also indicate that below the depth z_i (Fig.4) both theories give nearly the same pressures. It is also interesting to note that for depths shallower than z_i the curvature of the soil pressure distribution is small. Hence, for $z \leq z_i$ lateral stresses can be computed with a linear distribution. Based on the observations above, a new method for computing active pressures is proposed. This method is illustrated in Fig.5 and consists of the following bilinear pressure distribution:

$$if \quad z \leq z_i \qquad \sigma_h = K_a \gamma z\left(\frac{z_i - z_c}{z_i}\right) \tag{8}$$

$$if \quad z \geq z_i \qquad \sigma_h = K_a \gamma (z - z_c) \tag{9}$$

These expressions are a function of z_i and a simple expression of this depth is necessary for practical applications. As exemplified in Fig.6, the depth z_i appears to be a function of the cohesion c and of the depth z_c defined in Eq.3. For practical applications, when $c \geq 5$ kPa, z_i may be obtained as follows:

$$z_i = r \cdot z_c \tag{10}$$

$$r = \frac{10.24}{c^{0.6544}} \tag{11}$$

DISCUSSION

In the City of Los Angeles many freestanding retaining walls that are not over 460 cm have been designed for an assumed Equivalent Fluid Unit Weight of 4.71 kN/m³. This procedure is accepted by local codes and is used regardless of soil type or shear strength parameters. Although, 4.71 kN/m³ may appear low for clayey and silty soils, there does not seem to be a generalized problem with retaining walls as a result of this practice in this region. The total active force P_a computed from different methods have been normalized and are compared in Fig.7 for a 460 cm wall. Fig.7 shows that the classical Rankine

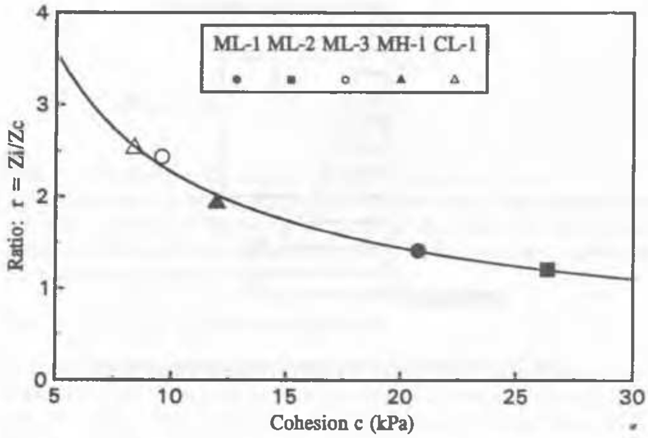

Fig 6. Effect of cohesion on the ratio r

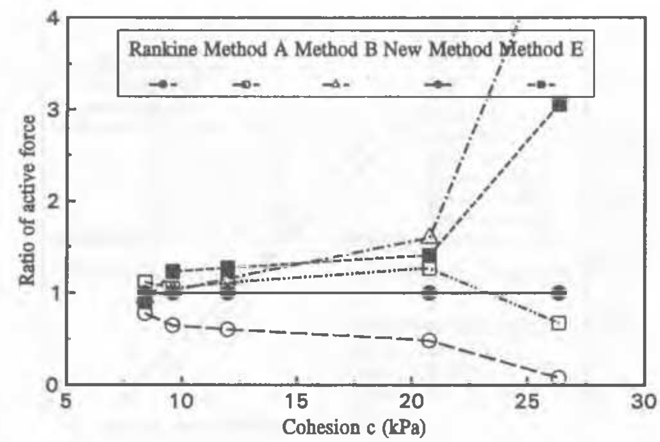

Fig 7. Comparison between active forces Pa (new method =1).

soil pressure distribution can seriously underestimates the active thrust on retaining walls. Hence, there is a need to replace Rankine's earth pressures with more conservative methods. That is exactly what methods A through D try to accomplish. Fig.7 also shows that for soils with a cohesion less than 25 kPa typical design methods result in forces that are either within the same range or slightly above those predicted by the new method. Hence, this study suggests that the discrepancies between Rankine's theory and commonly used design practices are largely due to the curvature of the failure envelope.

Although earth pressures for design purposes can be estimated based on theory, several factors must be taken into account, including tolerable wall movement, drainage conditions, earth pressures due to compaction, surcharge loads, etc... Cohesive backfills tend to creep and walls designed for active soil pressures may move continuously throughout their lives. Fig.7 shows that commonly used design methods tend to slightly overestimate the actual load and result in a certain degree of conservatism. Although these practices lead to designs that are conservative they may not reduce the effects of creep sufficiently. Hence, the amount of movement that can be tolerated often remains the major question that must be answered. Although existing methods may provide acceptable distributions for many practical problems they can seriously underestimate pressures in walls where very little movement can be tolerated. In circumstances where little or no movement is acceptable, at-rest earth pressures should generally be considered, and for practical purposes the formula proposed by Jaky (Morgenstern and Eisenstein, 1970) may be used:

$$\sigma_h = K_o \sigma_v \approx (1 - \sin\phi)\sigma_v \qquad (12)$$

CONCLUSIONS

This study suggests that the curvature of the failure envelope has a significant effect on earth pressures. This effect can be approximated using a bilinear pressure distribution and new equations are proposed. It was found that Rankine's soil pressure distribution seriously underestimates the active pressures and that more conservative methods should be used in cohesive soils. This study also suggests that commonly used diagrams which often lack a theoretical or experimental background, result in active forces that are generally within the same range of those predicted by the new method.

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