CALCULATION OF STRUCTURAL FORCES FOR PROPPED RETAINING WALLS

CALCUL DES FORCES STRUCTURALS POUR DES MURS DE SOUTENEMENT ETAYES

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SYNOPSIS : This paper presents a summary of the results of a numerical parametric study on the effects of wall stiffness and K_o on the behaviour of single propped embedded retaining walls. By modifying Rowe's wall flexibility number, ρ, to include both the wall and soil stiffness it is shown that results from analysis based on different soil stiffness profiles and of different wall geometries can be normalised. Finite element predictions of maximum wall bending moment and prop force are compared with values from simple limit equilibrium calculations. Coefficients relating the finite element and limit equilibrium results are established and the manner in which they vary with both wall stiffness and K_o value are presented. A design procedure combining these coefficients with standard limit equilibrium analysis is proposed. This results in the enhancement or reduction of bending moments and prop forces calculated by limit equilibrium depending on wall stiffness and in situ stress conditions.

INTRODUCTION

Limit equilibrium methods of analysis are widely used in the design of propped embedded retaining walls. However, such methods do not implicitly account for the effects of wall stiffness or the initial stress conditions in the ground. While neither of these affect the depth of wall embedment for stability they can have a large influence on bending moments in the wall and prop forces. Limited experimental (Rowe, 1952) and numerical (Potts & Fourie, 1985) research has shown that wall bending moments and prop force increase with increasing wall stiffness and/or in situ coefficient of earth pressure, K_o. For stiff walls installed in high K_o soils, bending moments and prop forces can substantially exceed values predicted by the limit equilibrium approaches currently used for design. For flexible walls and/or low K_o soils the opposite is true and the limit equilibrium methods produce conservative estimates.

In an attempt to clarify the situation further, a numerical parametric study has been performed in which non linear finite element analyses have been carried out varying the stiffness of the wall, the stiffness of the soil, the length of the wall and the K_o value. The results of these analyses are presented and compared below.

GEOMETRY AND MATERIAL PROPERTIES

All analyses consider an excavated wall and, apart from wall stiffness, soil stiffness, wall length and K_o value, all other material properties and assumptions were identical and were the same as those employed by Potts & Fourie (1984). Drained conditions with no water have been assumed in the soil which has been modelled using a linearly elastic-perfectly plastic constitutive model. A Mohr-Coulomb yield surface with strength parameters c' and ϕ' has been adopted. Associated flow conditions in which the angle of dilation ν is equal to ϕ' have been assumed, which allows the results to be compared with classical earth pressure solutions based on plasticity theory. The common parameters are summarised in Table 1.

Table 1: Summary of common parameters

Width of Excavation	20m
Wall thickness	1m
Poisson's ratio of the wall, μ_w	0.15
Angle of shearing resistance of soil, ϕ'	25°
Soil cohesion, c'	0
Poisson's ratio of the soil, μ_s	0.2
Bulk unit weight of soil, γ	20 kN/m^3
Coefficient of active earth pressure ($\delta=\phi'$), K_a	0.32
Coefficient of passive earth pressure ($\delta=\phi'$), K_p	3.9

The following equation has been adopted to represent the Young's modulus of the soil:

$$E_s = \alpha + \beta z \quad (\text{MN/m}^2) \tag{1}$$

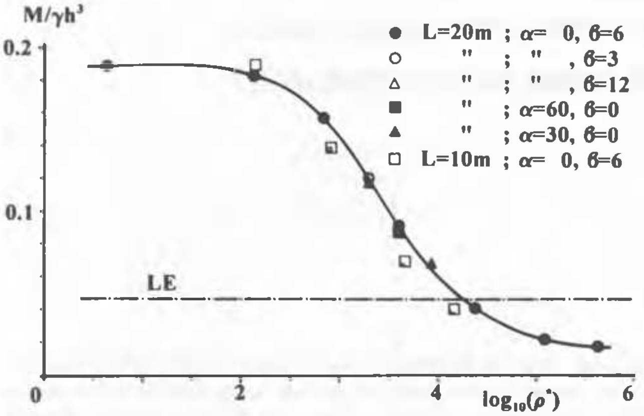
where α and β are constants and z is equal to the depth below the original ground surface. By selecting $\beta=0$ the soil stiffness is constant with depth. Alternatively, if $\alpha=0$ the stiffness increases linearly with depth below the soil surface.

The wall has been assumed to be linear elastic and was assigned different values of Young's modulus, E_w, to represent walls of different bending stiffness. A rigid horizontal prop acts at the top of the wall.

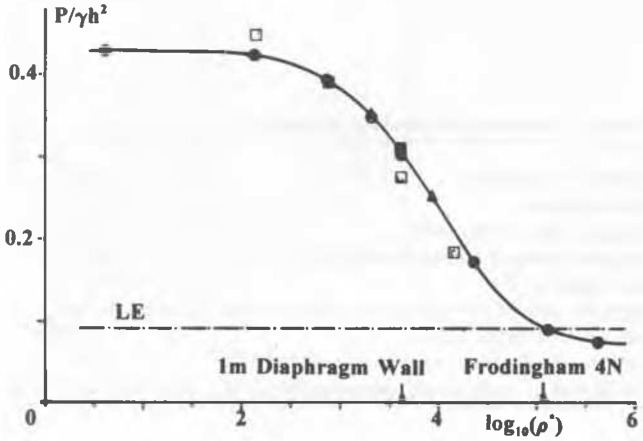
The parametric study involved analyses in which the soil stiffness, wall stiffness, wall length and K_o values were varied. Due to restrictions on space only selected results from this study are presented below.

FLEXIBILITY NUMBER

Rowe (1952) defined a wall flexibility number ρ ($=L^4/(E_w I)$), where L is the length of the wall and I its moment of inertia. Unfortunately this

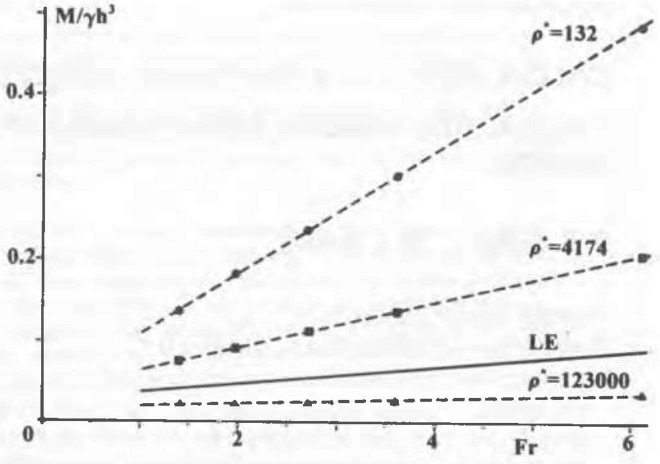


a) Maximum Wall Bending Moment

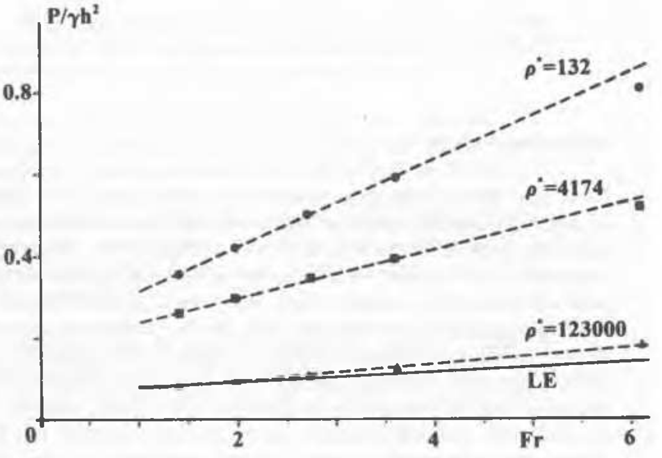


b) Prop Force

Fig. 1: Variation of $M/\gamma h^3$ and $P/\gamma h^2$ with $\log_{10}(\rho^*)$.
($K_o = 2$ and $Fr = 2$)



a) Maximum Wall Bending Moment



b) Prop Force

Fig. 2: Variation of $M/\gamma h^3$ and $P/\gamma h^2$ with Fr.
($K_o = 2$ and $L = 20m$)

number is not dimensionless having units of $m^4/(kNm^2$ per meter) and does not include the stiffness of the ground. Intuitively it is reasonable to expect that the stiffness of the soil as well as that of the wall and the value of K_o must influence both the bending moments and prop forces. The following revised definition of the flexibility number is therefore proposed:

$$\rho^* = (L^4.E_s^{av})/(E_w.I) = \rho.E_s^{av} \qquad (2)$$

where E_s^{av} is the average stiffness of the soil over the full length, L, of the wall. This definition has the advantage that ρ^* is dimensionless and includes both the soil and wall stiffness. As will be demonstrated below, this definition allows the normalisation of results from analyses with different soil stiffness and wall lengths.

EFFECTS OF WALL STIFFNESS

The solid circles in Figure 1 show results of analyses with K_o=2, wall length L=20m, soil stiffness increasing with depth (α=0, β=6), at a retained height h=13.3m and with a range of wall stiffness values. Values of maximum wall bending moment, M, normalised by γh^3 and prop force, P, normalised by γh^2 are plotted against $\log_{10}(\rho^*)$ in Figures 1a and 1b respectively. The wall stiffness values cover the full range likely to be encountered in practice. Values of ρ^* associated with a typical 1m thick concrete diaphragm wall and a Frodingham 4N sheet pile are indicated on Figure 1b.

For a 20m long wall with a retained height of 13.3m and the soil properties given in Table 1, the factor of safety Fr is equal to 2, where Fr is defined by Burland, Potts & Walsh (1981). Values of maximum bending moment and prop force calculated from a limit equilibrium (LE) calculation using the actual wall length and this value of Fr are also shown on Figures 1a and 1b. Clearly these values are independent of wall stiffness. Such a calculation is typical of that currently proposed in many design manuals.

The numerical results indicate that for stiff walls (small values of ρ^*) the maximum wall bending moment and the prop force are approximately four times larger than the equivalent limit equilibrium values. As wall stiffness reduces (ρ^* increasing) both the maximum wall bending moment and prop force reduce following the reverse 'S' shaped curves indicated on Figures 1a and 1b. For very flexible walls the values of maximum wall bending moment and prop force fall below the limit equilibrium values.

EFFECT OF SOIL STIFFNESS

Also shown on Figures 1a and 1b are results from analyses with different soil stiffness. The actual stiffness distributions are indicated by the values of α and β. Both the predicted maximum wall bending moment and prop force for these analyses follow the same 'S' shaped curves discussed above. These results are typical of those performed in the parametric study and they support the use of ρ^* as a normalising parameter. It may be noted that such normalisation is not possible with Rowe's flexibility number, ρ, which is independent of soil stiffness.

EFFECT OF WALL LENGTH

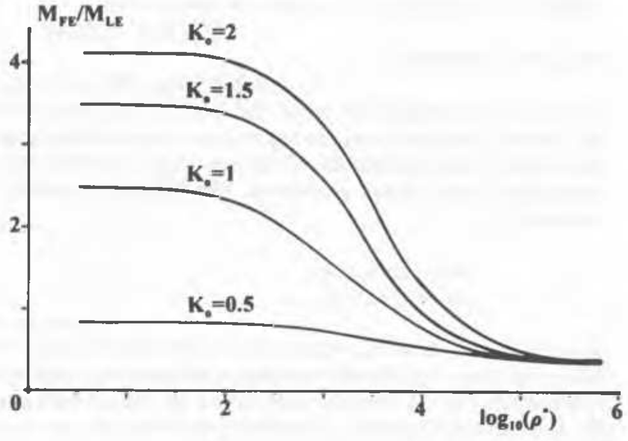
Analyses with a wall length, L of 10m are also shown on Figures 1a and 1b. At low values of ρ^* these results plot above the 'S' shaped curves defined by the trend of the results from 20m long walls. The opposite is true at higher values of ρ^* where the results plot below the 'S' shaped curves. It may be noted that where analyses with different wall heights are being compared Rowe's flexibility number, ρ, provides a better normalising parameter for walls with low wall stiffness. However for walls with a higher stiffness the use of ρ^* is superior. Consequently while it is appreciated that there is room for improvement in the definition of the modified flexibility number ρ^*, its use is advocated for the present. Clearly further research is required to clarify the situation.

EFFECT OF RETAINED HEIGHT

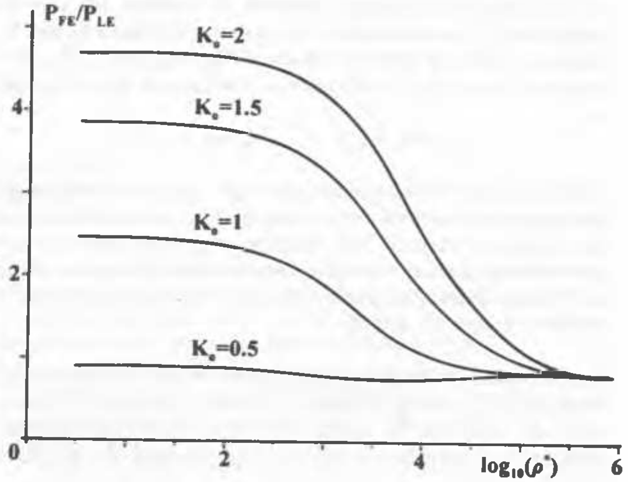
The results presented above are for retained heights which give a factor of safety Fr=2. As the retained height changes for a fixed wall length then so does Fr and the maximum wall bending moment and prop force. Figures 2a and 2b indicate how the normalised maximum wall bending moment and prop force vary with Fr, and therefore retained height, for a selection of the above analyses in which K_o=2. These results indicate that for typical design values of Fr the maximum wall bending moment and prop force vary approximately linearly with Fr. Such linear relationships are typical of all analyses performed in the parametric study. Also shown on Figures 2a and 2b are the variations of maximum wall bending moment and prop force obtained from a limit equilibrium calculation. These too indicate a linear variation with Fr.

EFFECT OF K_0

The majority of the above analyses were repeated with K_o values of 0.5, 1, 1.5 and 2. The results follow the same trend as those outlined above but the magnitudes of both maximum wall bending moment and prop force decrease with reducing K_o value. The results plotted as M_{FE}/M_{LE} versus $\log_{10}(\rho^*)$ are presented in Figure 3a. M_{FE} and M_{LE} are the maximum wall bending moments from the finite element analyses and a limit equilibrium calculation respectively. A similar plot showing the variation of P_{FE}/P_{LE} with $\log_{10}(\rho^*)$ is given in Figure 3b. Again these results are for Fr=2. The limit equilibrium values, M_{LE} and P_{LE} are independent of K_o and are used to normalise the finite element results. Their values are as indicated on Figures 1a and 1b.



a) Maximum Wall Bending Moment



b) Prop Force

Fig. 3: Variation of M_{FE}/M_{LE} and P_{FE}/P_{LE} with $\log_{10}(\rho^*)$

Inspection of Figures 3a and 3b indicates that as the stiffness of the wall reduces both the maximum wall bending moment and prop force approach a common value. For the maximum wall bending moment this value is approximately equal to $0.4M_{LE}$ whereas for the prop force it is $0.83P_{LE}$. As the wall stiffness increases (ρ^* reduces) both M_{FE} and P_{FE} increase. The magnitude of this increase depends on K_o, the higher K_o the larger the increase. For all cases, except K_o= 0.5, values of

M_{FE} and P_{FE} eventually exceed the limit equilibrium values M_{LE} and P_{LE}. At high wall stiffness the curves level off and there is little further increase in M_{FE} and P_{FE} with increase in wall stiffness.

DESIGN APPROACH

The results of the numerical parametric study can be used to modify the values of maximum wall bending moment and prop force obtained from simple limit equilibrium calculations which currently form the backbone of most design procedures. The following equations are proposed:

$$M_D = (1+a_{bm})M_{LE} \tag{3}$$
$$P_D = (1+a_{pf})P_{LE} \tag{4}$$

where M_D and P_D are design values of maximum wall bending moment and prop force respectively and the coefficients a_{bm} and a_{pf} are obtained from the results of the numerical study and are dependent on the wall flexibility number, ρ^*, and the value of K_o.

The design procedure then involves the following steps. Firstly the values of ρ^*, Fr, M_{LE} and P_{LE} are calculated for the design geometry and soil and wall conditions. The values of K_o and Fr are then used to interrogate the numerical database to establish the appropriate normalised 'S' curves similar to those shown in Figure 3a and 3b. It is then possible to establish values of M_{FE}/M_{LE} and P_{FE}/P_{LE} for the calculated value of ρ^*. Coefficients a_{bm} and a_{pf} can then be calculated as:

$$a_{bm}=(M_{FE}/M_{LE}) - 1; \qquad a_{pf}=(P_{FE}/P_{LE}) - 1$$

The coefficients will have either positive or negative values depending on whether the structural forces calculated by limit equilibrium are to be enhanced or reduced. The interrogation process can be simplified by combining the curves given in Figure 3a and 3b which are for Fr=2 with simple linear relationships between M_{FE}, M_{LE}, P_{FE}, P_{LE} and Fr as shown in Figures 2a and 2b.

The procedure can also be generalised so that only a proportion of the coefficients a_{bm} and a_{pf} is applied. However in such a situation care must be exercised to ensure that the reduced coefficients are compatible. A procedure for achieving this compatibility is as follows:

- First, the base value of a_{bm} is determined assuming 100% enhancement/reduction.
- Second, a value of ρ^* is found that gives the desired proportion of a_{bm}, and this is noted as ρ^*_{eq}.
- Finally, the value of a_{pf} corresponding to ρ^*_{eq} is evaluated.

The above procedure is equivalent to finding the prop coefficient, a_{pf}, for a wall with a flexibility number, ρ^*_{eq} which gives the required proportion of a_{bm}. Clearly alternative procedures can be established.

The numerical database can also be used to establish a coefficient, a_{sf}, for the maximum shear force in the wall. Further modification of the values of M_D and P_D may be necessary to account for other phenomena such as over dredging or progressive failure of anchors as recommended in the appropriate design codes. A modified form of the above procedure has been implemented in the program GCG ReWaRD, Bond & Potts (1992).

CONCLUSIONS

The results of an extensive numerical parametric study of the behaviour of propped retaining walls have shown that both wall stiffness and the value of K_o have considerable influence on the maximum wall bending moment and prop force.

A modified wall flexibility number, ρ^*, has been defined which is dimensionless and includes both the soil and wall stiffness. It has been shown that use of this flexibility number allows results from analyses with different soil and wall stiffness and wall length to be directly compared.

For stiff walls in soils with K_o greater than 0.5, both the maximum wall bending moment and prop force exceed values based on conventional limit equilibrium design calculations. For example, for K_o values of 1 and 2 the numerical results exceed the limit equilibrium values by over 200% and 400% respectively. As the stiffness of the wall reduces both the maximum wall bending moment and prop force also reduce and when plotted against $\log_{10}(\rho^*)$ follow reverse 'S' shaped curves. For walls with low stiffness, the results are independent of the value of K_o, with the maximum wall bending moment and prop force approaching approximately 40% and 80% of the values calculated by limit equilibrium.

The results of the numerical study can be used to modify simple design procedures and a method for achieving this is outlined. This involves enhancing/reducing values of the maximum wall bending moment and prop force obtained from limit equilibrium calculations using coefficients derived from the parametric study. At present this procedure is tentative and requires verification by comparison with both field and model measurements. It does however provide a simple method for accounting for both the effects of wall stiffness and K_o value.

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