THE EXTENDED CRISS METHOD FOR CALCULATING SLOPE STABILITY

METHODE CRISS MODIFIEE DE CALCUL DE LA STABILITE DES PENTES

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SYNOPSIS: The CRItical Slip Surface or CRISS method has been developed to calculate the safety factor for the overall stability of slopes from finite element stress analyses. In the simple version a multi-linear failure surface is propagated in both directions from an internal point of high mobilised stress level. For each segment of the surface the slope giving the lowest value of local safety factor is sought. In the extended version a complete circular or multi-linear failure surface is required as input. The safety factor for this surface is calculated and the geometrical parameters are then adjusted by a pattern search optimization routine until the surface giving the minimum F is found. Examples are given to demonstrate that the new method gives values of safety factor comparable with those obtained from the best limit equilibrium methods or other finite element approaches.

INTRODUCTION

The finite element method has been applied to stress-strain analyses of slopes and earth/rock dams with considerable success, though the calculation from these stresses of safety factors against shear failure has often been performed in a relatively simplistic manner. Attempts to convert F.E. stresses into stability calculations have been made by many researchers, e.g. Kulhawy et al (1969), Wright et al (1973), Zienkiewicz et al (1975), Donald et al (1985), Donald and Giam (1988), Yamagami and Ueta (1988), using a variety of approaches and definitions of safety factor. In the present paper a method is presented by which the safety factor against shear failure along any arbitrary surface may be calculated readily from F.E. stresses, for three definitions of F. A simple algorithm for selecting the critical shear surface is then described, followed by an improved version in which the critical failure mechanism is determined automatically, using a multivariable unconstrained optimization routine. Several example problems are presented with comparative analyses by the proposed method, conventional limit equilibrium methods and the F.E. based Nodal Displacement Method. With the F.E. methods there is no requirement to assume F = constant around the failure surface, no assumptions are needed regarding side forces between soil blocks or slices and any piece-wise linear failure surface may be used.

DEFINITIONS OF F

The results presented have been calculated for three reasonable definitions of safety factor, the first two of which have some currency in the literature. In limit equilibrium analyses the safety factor is normally based on soil shear strength and is given by

$$F = \frac{\text{available shear resistance}}{\text{mobilised shear stress}} = \frac{s}{\tau} \qquad (1)$$

There are several ways in which average stresses around a slip surface may be estimated, leading to the following definitions:

(a) F based on stress level

$$F_{FE1} = \frac{\Sigma \, \Delta l}{\Sigma \dfrac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f} \Delta l} \qquad (2)$$

where the stress level or mobilised strength is given by $\dfrac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')f}$

and Δl is the length of a segment of the failure surface. F_{FE1} is essentially the inverse of a weighted average stress level along the length of the slip surface. The value of σ_3' is assumed the same for the mobilised and failure states.

(b) F based on shear stress

$$F_{FE2} = \frac{\Sigma \, (c' + \sigma' \tan \phi') \, \Delta l}{\Sigma \, (\tau \, \Delta l)} \qquad (3)$$

This definition is closest to that of conventional limit equilibrium methods. In equation (3) σ' and τ are the normal and shear stresses acting on segment Δl. The value of σ' is assumed to be the same in the mobilised and failure states.

(c) F based on strength weighted stress level

$$F_{FE3} = \frac{\Sigma \, (c' + \sigma' \cdot \tan \phi') \, \Delta l}{\Sigma \dfrac{(\sigma_1' - \sigma_3')}{(\sigma_1' - \sigma_3')_f} (c' + \sigma' \tan \phi') \, \Delta l} \qquad (4)$$

This definition acknowledges that for stress level calculations a strong soil layer makes a proportionately larger contribution to stability than a weak soil layer (Donald et al, 1985).

THE CRISS APPROACH

Most early attempts to calculate safety factors for slopes from F.E. stresses used an assumed failure surface, generally circular, and either F_{FE1} or F_{FE2}. Extensive trial and error calculations were necessary to find the critical surface yielding the minimum safety factor. Obvious extensions to this approach include non-circular slip surfaces and automatic search routines to attempt to find the global minimum safety factor. Giam and Donald (1988) presented the program CRISS (CRItical Slip Surface) as an effective and systematic procedure for obtaining the critical slip surface from a known stress distribution, with simultaneous determination of the minimum safety factor.

The principle of CRISS is illustrated in Figure 1. Starting from a point of high stress level, P, a failure surface is propagated to the top and bottom of the slope. The slope is divided into a number of stages, for each of which the inclination of the possible failure surface is incremented in 1° steps. The factor of safety against shear failure is calculated for each incremented linear segment until a minimum value is found for that stage. The process is then repeated until the failure surface daylights in the crest and toe regions. Several positions of the starting point P should be used to find the global critical surface.

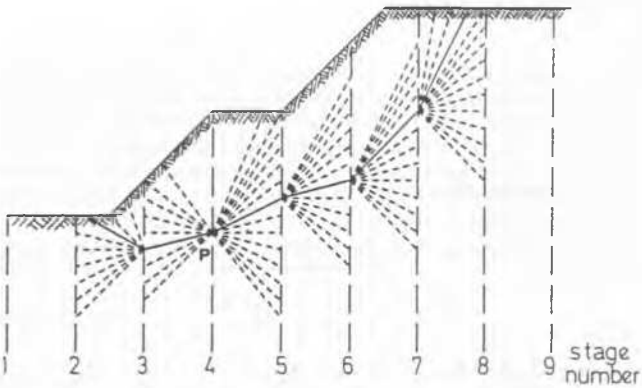


Fig. 1. Simple search scheme

A typical stage and segment are shown in Figure 2. The segments are normally divided into five intervals and stresses calculated at the control points in the centre of each interval. The stresses σ'_{xx}, σ'_{yy} and τ'_{xy} at the control points are determined by interpolation from the calculated stresses at the four closest integration points in the finite element mesh. Normal and shear stresses are calculated from:-

$$\sigma'_{ni} = \frac{1}{2}(\sigma'_{yy} + \sigma'_{xx}) + \frac{1}{2}(\sigma'_{yy} - \sigma'_{xx})\cos 2\alpha - \tau_{xy}\sin 2\alpha \qquad (5)$$

$$\tau_{ni} = \frac{1}{2}(\sigma'_{yy} - \sigma'_{xx})\sin 2\alpha + \tau_{xy}\cos 2\alpha \qquad (6)$$

leading to a safety factor for the segment of

$$F = \frac{\Sigma\,(c' + \sigma'_{ni}\tan\phi')\,\Delta l}{\Sigma\,(\tau_{ni})\,\Delta l} \qquad (7)$$

The overall safety factor for the complete multi-linear failure surface is then given by

$$F_{FE2} = \frac{\int_L (c' + \sigma'_n \tan\phi')\,dL}{\int_L \tau_n\,dL} \qquad (8)$$

where L = total length of failure surface.

Note that equations (7) and (8) may be written in various ways to reflect the three chosen definitions of safety factor.

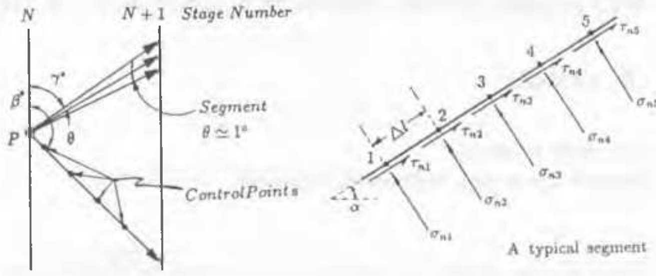


Fig. 2. A stage segment - details

OPTIMIZATION ROUTINES

Wedge Stability Analyses

The use of multivariable unconstrained optimization algorithms in selecting the critical failure mechanism for wedge-type stability analyses has been discussed in some detail by Giam (1989). Examples of a range of stability calculations have been presented by Donald and Giam (1990), (1992), demonstrating the power of a number of optimization approaches in seeking out efficiently the mechanism leading to the lowest safety factor.

In the last decade many non-linear programming approaches have been applied in soil mechanics, all assuming a unimodal objective function, G (Fig. 3), and therefore frequently requiring several initial trial failure surfaces to ensure a high probability of capturing the global minimum safety factor. Dynamic programming methods have been used by Baker (1980) and Yamagami and Ueta (1986), (1988), but although they avoid the unimodality restriction and follow a rapid multi-stage strategy for optimization, they are not derivative free (direct search) algorithms and are more difficult to program than the non-linear programming methods. Giam (1989) compared five direct search approaches - Simplex, Hooke and Jeeves, Rosenbrock, Powell, Pattern Search -applied to multi-wedge analysis programs GWEDGEM and EMU, before deciding that the Pattern Search method (Hooke and Jeeves (1961)) was the best approach for problems with up to 50 geometric variables requiring optimization.

Extended CRISS Method

The success of the Pattern Search optimization algorithm with multi-wedge analyses encouraged its application to an extended version of the CRISS approach. Two types of failure surface were considered - circular, with only three variables x,y and r to be optimized and non-circular, approximated by an n-segment multi-linear surface, with 2n geometric variables. These surfaces are shown in Figure 3 and the only difference from wedge analyses is that internal shearing interfaces are not included in the calculations.

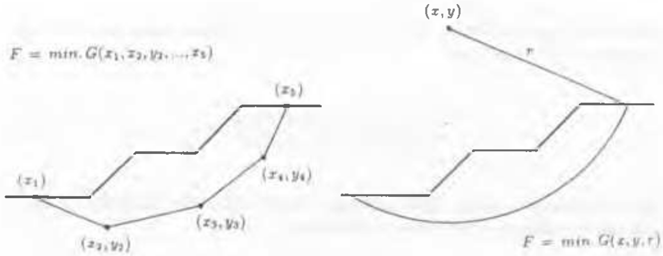


Fig. 3. Formulation of problem for optimization

Formulation for Optimization

Each segment of the failure surface is divided into a number of intervals, much as described earlier for each stage line in the original CRISS program. The normal and shear stresses at the central control points for each interval are again interpolated from the four nearest integration points, using equations of the form

$$\sigma_{xx}' = a_1 + a_2 x + a_3 y + a_4 xy \qquad (9)$$

where x,y = coordinates of integration points
and a_i = interpolation coefficients

Equations (5) to (8) still apply and a value of F for the initial trial surface may be evaluated readily. The Pattern Search routine is then applied to the trial surface and the geometric variables adjusted by small steps until the minimum F is found, using any chosen definition of safety factor (equations (2) to (4)). Several different initial trial surfaces should be used to expose local minima.

EXAMPLES

The finite element stress calculations for all examples presented were made using the Cambridge University program CRISP, with some minor modifications to calculate the appropriate stress components needed for the various definitions of F. Gravity turn-on, built-up and excavated slopes were examined, with the stability investigated by CRISS, limit equilibrium and nodal displacement methods.

Example 1.

Figure 4 shows a simple slope for which the safety factor is 1.00 (Donald and Giam (1989)). Table 1 summarises the results from the extended CRISS analyses, with both built-up and gravity turn-on values exhibiting excellent accuracy and agreement. The values for the excavated slope are slightly high, a trend also found by Brown and King (1966).

Table 1. Comparative Factors of Safety; Example 1

	Factor of Safety		
	Multi-linear		
Type	F_{FE1}	F_{FE2}	F_{FE3}
Built-up	1.001	1.001	1.001
Gravity turn-on	1.000	1.003	1.000
Excavation	1.082	1.044	1.044

The most accurate value of F in Figure 4 arose from a seven segment extended CRISS failure surface, at 1.001 slightly better than a value of 1.004 from a Bishop circular analysis, equal to that from a CRISS simple (original version) analysis. A circular surface CRISS analysis gave F = 1.015. The critical failure surfaces are seen to be in good agreement.

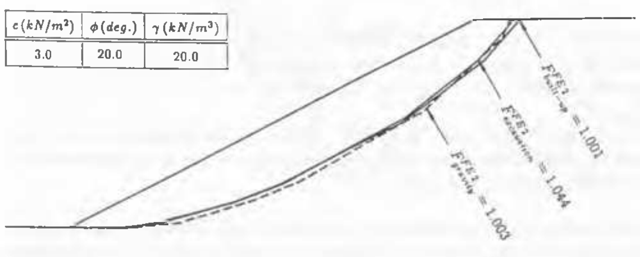


Fig. 4. Example 1 - Simple slope

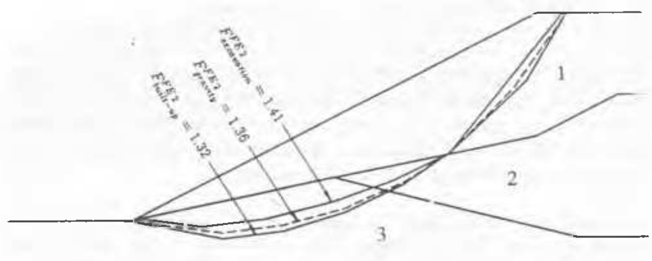
Example 2

Figure 5 shows a three layer profile for which the accepted referee value of safety factor is F = 1.39 (Donald and Giam (1989)). The results are summarised in Table 2. Analysis by the Nodal Displacement Method (N.D.M. - F.E. based) gave F = 1.39 (Giam (1989)).

Table 2. Comparative Factors of Safety; Example 2

	Factor of Safety			
	Multi-linear			Circle
Type	F_{FE1}	F_{FE2}	F_{FE3}	F_{FE2}
Built-up	1.29	1.32	1.26	1.35
Gravity turn-on	1.32	1.36	1.26	1.39
Excavation	1.53	1.41	1.51	1.43

All methods tabulated show an increase in F from built-up to excavated slopes, with circular failure surfaces generally giving higher values than multi-linear ones. The average F_{FE2} for the six values in Table 2 is F = 1.38, very close to the limit equilibrium referee value of 1.39. The F_{FE2} definition is closest to limit equilibrium definitions and reasonable agreement in F values would be expected. The differences between F_{FE2} values for circular and non-circular surfaces are not great and it is likely that the non-circular values are a little low because no shearing on internal interfaces has been included in the analysis. The values based on mobilised stress ratio definitions of F vary somewhat erratically, casting some doubt on their reliability. Physical reasons for some of the differences have been discussed by Kulhawy et al (1969). The critical failure surfaces in Figure 5 for multi-linear CRISS analyses are seen to be in close agreement.



Soil 1 c = 0 kPa, φ = 38°, γ = 19.5 kN/m³
Soil 2 c = 5.3 kPa, φ = 23°, γ = 19.5 kN/m³
Soil 3 c = 7.2 kPa, φ = 20°, γ = 19.5 kN/m³

Fig. 5. 3-layer slope

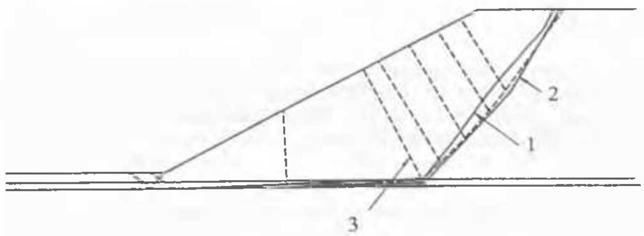
Example 3

The final example to be presented, Figure 6, has been detailed to produce a highly non-circular failure surface, through the inclusion of a thin, weak layer at depth (Donald and Giam (1989)). The referee value, from the ACADS report, is F = 1.26 and an Nodal Displacement Method analysis gives F = 1.25. The soil properties are

Main soil c' = 28.5 kPa, φ' = 20°
Weak layer c' = 0, φ' = 10°

CRISS analyses gave:-

extended CRISS F_{FE2} = 1.25 (for built-up slope)
simple CRISS F_{FE2} = 1.30 (for built-up slope)



1. Simple search, $F_{FE2} = 1.31$
2. Extended CRISS, $F_{FE2} = 1.25$
3. Limit Equilibrium, $F_{FE2} = 1.27$ (GWEDGEM)

Fig. 6. Example 3 - slope with weak layer

The multi-wedge limit equilibrium program GWEDGEM yielded F = 1.27 and the extended CRISS and GWEDGEM analyses predicted almost identical failure surfaces, though the simple-search CRISS surface diverged somewhat in the slope above the weak layer. The values of F for extended CRISS, N.D.M., GWEDGEM and the ACADS referee value cover a narrow range from 1.25 to 1.27, according high credibility to all methods.

DISCUSSION

The above examples demonstrate the adequacy of the search schemes described for selecting critical failure surfaces in slope stability analyses, both for circular and highly non-circular surfaces. The extended CRISS analysis also predicts failure surface geometries which compare well with those derived from limit equilibrium analyses. The simple search scheme works adequately for straight forward problems, it requires no trial failure surface and it is computationally efficient. However, it cannot guarantee a good optimal solution for complex problems, for which a multi-variable unconstrained method, such as the modified Pattern Search, is recommended. Using the extended CRISS approach a trial failure surface is required as input, but the power of the optimization routine is such that the trial surface need not be chosen with great precision. However, for speedy convergence to an accurate result it has been found advisable to base the initial trial surface on that predicted by some form of limit equilibrium analysis.

The stress history of the slope obviously has an influence on the safety factor. For slopes on the point of failure, such as Example 1, the effect is less noticeable, with built-up and gravity turn-on analyses giving F = 1.00, though the excavated slope value is a little higher at 1.04 to 1.08. When F is significantly greater than 1.00, as for Example 2, the differences between built-up and gravity turn-on remain insignificant, but the excavation analysis gives values of F up to 0.25 higher. As limit equilibrium analyses make no allowance for stress history, the question arises as to what situation they are approximating. From the admittedly limited evidence presented here it is reasonable to claim that limit equilibrium analyses agree most closely with gravity turn-on or built-up analyses, using the F_{FE2} definition. If the indicated trends concerning excavated slope analysis are correct, then limit equilibrium analyses should generally be conservative for excavated slopes, though more work is obviously required to prove the point.

The role of F_{FE1} and F_{FE3} is less clear. F_{FE1} could be regarded as a safety factor against the initiation of local failure and could therefore be of significance in soils subject to progressive failure, with F_{FE3} serving the same purpose for soil profiles with widely varying strength properties. With present data it is impossible to make more definite recommendations.

CONCLUSIONS

Two versions of the CRISS method have been presented for deriving values of safety factor from finite element analyses of slopes. The simple method is adequate for straightforward problems, but for more complex slopes the extended CRISS program, with automatic search for the critical failure surface, is recommended for improved accuracy. Values of F derived from CRISS analyses agree well with limit equilibrium and Nodal Displacement Method values, justifying the applicability and accuracy of the method.

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