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# A PROCEDURE FOR PREDICTING SEISMIC DISPLACEMENTS OF EARTH DAMS

## PROCEDURE DE PREVISION DE DEPLACEMENT SEISMIQUE DES BARRAGES EN TERRE

Peter M. Byrne<sup>1</sup> Hendra Jitno<sup>1</sup> Donald L. Anderson<sup>1</sup> Jeremy Haile<sup>2</sup>

<sup>1</sup>Department of Civil Engineering, The University of British Columbia, Vancouver, B.C., Canada

<sup>2</sup>President, Knight & Piesold, Vancouver, B.C., Canada

### SYNOPSIS

An analysis procedure is presented for predicting the earthquake induced displacements of a tailings dam. The procedure extends the simple Newmark method from a single-degree-of-freedom rigid plastic to a multi-degree-of-freedom flexible system using post-liquefaction stress-strain relations and energy concepts. The method has been validated by comparison with field experience during earthquakes, including the response of the San Fernando Dam. The method is described and applied here to the proposed Kensington tailings dam in Alaska and the results compared with conventional Newmark analyses.

### INTRODUCTION

The Kensington Project is a proposed underground gold mine located 40 miles north of Juneau, Alaska, on the east side of the Lynn Canal. The mine will require construction of an 89 m high dam to contain the tailings from the mining operations. The dam is to be constructed in stages using compacted earthfill and rockfill and a modified centreline arrangement, which differs from conventional centreline construction in that the upstream contact between the compacted fill and the tailings is inclined slightly upstream. The project is located in an area of high potential seismicity and earthquake induced liquefaction of the tailings is possible. The stability of the top portion of the dam and the potential displacements resulting from earthquake loading are therefore of extreme importance. A number of cases of liquefaction induced failure of tailings dams built using the upstream construction have been reported in the literature, e.g. two Chilean tailings dams (Dobry and Alvarez, 1967), and Mochikoshi tailings dam in Japan (Marcuson, 1979; Ishihara, 1984).

Conventional limit equilibrium and Newmark type analyses which included hydrodynamic loading from the liquefied tailings, indicated that the embankment is stable and deformations would be very small. However, because the Newmark analysis is restricted to modelling soil as a single-degree-of-freedom rigid plastic system and does not include liquefaction effects, a more detailed analysis was considered appropriate.

Deformation analyses were carried out using a pseudo-dynamic finite element procedure which allows both the inertia forces from the earthquake as well as the softening effect of the liquefied soil to be considered. The method is essentially an extension of Newmark's procedure. The procedure together with the results are presented in this paper.

### KENSINGTON TAILINGS DAM

The proposed tailings dam is a compacted earth/rockfill structure with an ultimate height of 89 m. The initial 50 m of embankment will be constructed as a conventional earthfill dam with a very broad crest, a glacial

till core and upstream and downstream sand and gravel shell zones. Ongoing raising of the dam will be carried out as the level of the stored tailings rises, using mine waste rock and earthfill with intermediate raises placed partially onto the tailings beach, as shown on Fig. 1. The resulting embankment section is referred to as a modified centreline embankment in that the upstream contact between the compacted fill and the tailings is inclined slightly upstream. It provides a cost-effective method of ongoing construction and ongoing reclamation of the downstream slope to reduce the visual impact of the embankment. The section differs from the upstream construction method as it does not rely on the strength of the tailings or slime for stability. The concern here is that liquefaction of the slimes together with inertia effects in both the slimes and the dam could lead to large deformations and failure of the impoundment during a major earthquake.

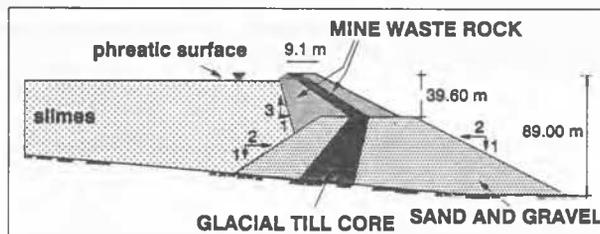


Fig. 1. Geometry and soil types of the tailings dam.

### ANALYSES PROCEDURE

Prediction of earthquake induced movements of earth structures is a difficult problem. Complex effective stress dynamic analyses procedures have been proposed (Finn et al., 1988; Prevost, 1981) but are essentially research tools and not generally appropriate for analysis of most dam structures.

The simplest deformation analysis procedure is that proposed by Newmark (1965) in which a potential slide block is modelled as a single-degree-of-freedom rigid plastic system. Any prescribed time history of acceleration can then be applied at the base and the resulting displacements computed by numerical integration. Newmark also found that the maximum displacement at the end of the shaking period could be estimated from simple formulae by considering the earthquake to be approximated by a number of pulses.

There are two concerns when applying Newmark's simple procedure to an earth structure such as a tailings dam: (1) the soil, particularly in zones where liquefaction is triggered is not rigid plastic and does not take into account the softening effect of the liquefied soils; and (2) the single-degree-of-freedom model does not allow the pattern of displacements to be computed. Byrne (1991) and Byrne et al. (1991) discuss this and show a way of allowing for a general stress-strain relation as well as extending Newmark's approach to a multi-degree-of-freedom system. Basically a pseudo-dynamic finite element procedure is used in which earthquake induced displacements which satisfy energy considerations are achieved by use of a seismic coefficient. The appropriate seismic coefficient is the one which satisfies the work-energy equation and is found by trial-and-error as described by Byrne et al. (1991). This approach is briefly described here. It is first applied to the Newmark problem and then extended to a general stress-strain and multi-degree-of-freedom system.

Newmark's simplified model is that of a block of mass  $M$  resting on an inclined plane of slope  $\alpha$ , and subjected to a velocity pulse,  $V$ , relative to the base (Fig. 2a). The resulting displacement is given by

$$d = 6V^2/2gN \quad (1)$$

where  $d$  = maximum displacement,  $V$  = the velocity pulse which Newmark took as the peak ground velocity,  $N$  = the yield acceleration, i.e., the acceleration as a fraction of "g" required to initiate yield and sliding, and  $g$  = the acceleration of gravity. The number 6 in his formula comes from considering 6 pulses of velocity  $V$  which Newmark found gave agreement with the integrated records when the ratio  $N/A < 0.13$ , where  $A$  is the peak ground acceleration. This is usually the condition for most practical cases of concern.

His model will now be developed in terms of work-energy and this will allow its extension to a general formulation.

The work-energy theorem states that the work done by the external forces minus the work done by the internal forces or stresses must equal the change in kinetic energy of the system, namely,

$$W_{EXT} - W_{INT} = \frac{1}{2}M(V_f^2 - V^2) = -\frac{1}{2}MV^2 \quad (2)$$

where  $V_f$  is the final resting velocity taken to be zero, and  $V$  is the specified initial velocity.

The external force that does work is the gravity driving force,  $Mg \sin \alpha$ , and in this case is constant with displacement as shown in Fig. 2b. The work done is the area beneath the driving force line. The work done by the internal forces depends on the stress-strain relations of the material and since Newmark assumed the material to be rigid plastic, the internal force or resistance is constant with displacement (Fig. 2b). The work done is the area beneath the soil resistance line. The net work done is the difference between the two areas as shown in Fig. 2b, namely the shaded area and this must equal the loss in kinetic energy,  $1/2 MV^2$ .

$V$  = velocity  
 $M$  = mass of the block  
 $d$  = displacement

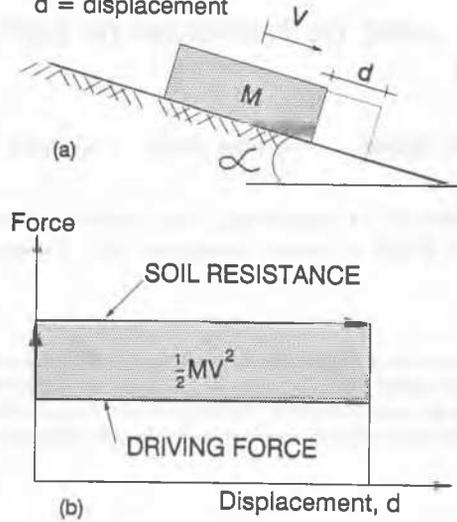


Fig. 2. (a) Block on inclined plane subjected to velocity pulse  $V$ ; and (b) work-energy, Newmark method.

Now  $W_{EXT} = (Mg \sin \alpha)d$ , and  $W_{INT} = (s.L.b)d$ , where  $s$  = the shear strength of the soil and  $L$  is the length of the slide block,  $b$  = the width of the slide block. Thus equation 2 reduces to

$$d(Mg \sin \alpha - s.L.b) = -\frac{1}{2}MV^2 \quad (3)$$

from which

$$d = \frac{\frac{1}{2}V^2}{gN} \quad (3a)$$

where the yield acceleration,  $N$ , is given by

$$N = \frac{s.L.b - Mg \sin \alpha}{Mg} \quad (4)$$

Equation (3a) is for a single velocity pulse, and when 6 pulses are considered the result is identical to Newmark, equation (1).

Soil when triggered to liquefy will not behave in a rigid plastic manner and this is now examined. Loose saturated sandy soils under undrained conditions can be triggered to liquefy by either monotonic or cyclic loading and this is depicted in Fig. 3a. The element is initially consolidated to the stress state,  $A(\sigma', \tau)$  depicted in Fig. 3a, in which  $\sigma'$  is the normal effective stress, and  $\tau$  is the shear stress, and then tested under constant volume undrained conditions. For monotonic loading the peak strength occurs when the stress state reaches the critical stress ratio (CSR) line, and thereafter the strength drops to its residual value as the pore pressure rises and the stress point reaches the constant volume friction angle,  $\phi'_{cv}$  line. Under load controlled conditions such as might occur in the field, large deformations and flow would occur when the stress point reached the CSR line, and this condition as has been termed

"true liquefaction" by Castro (1969). It really refers to a condition where the applied stress exceeds the strength, and can happen for any material which is strain softening. The associated characteristic stress-curve is shown in Fig. 3b.

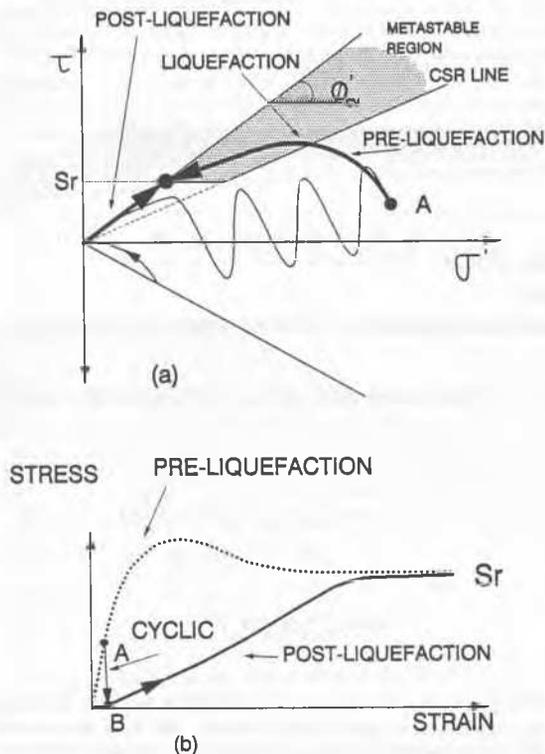


Fig. 3. (a) Characteristic of liquefaction behaviour; and (b) post-liquefaction monotonic stress-strain curves.

If starting from the stress state, A, the element is first subjected to cyclic loading, pore pressure rise will occur causing the stress point to move to the left as shown in Fig. 3a. In an element in the field, the static bias will also be gradually lost as the static stress will shift to those stiffer elements that have suffered less pore pressure rise. Under these conditions, 100% pore pressure rise can occur causing the stress point to move to the origin or zero effective stress state. The strains required to induce this state are generally less than 1%. Thus cyclic triggering of liquefaction is a small strain phenomenon (Byrne, 1991). If at this stage the element is loaded monotonically, it has a very low stiffness because its modulus depends on effective stress, which is zero, and it behaves essentially as a liquid. This state would conform to the Oxford dictionary definition of liquefaction "bring into liquid condition". As the liquefied sample is sheared, it dilates causing a drop in pore pressure and an increase in effective stress and stiffness until it reaches the residual state point. The associated monotonic post-liquefaction curve is shown in Fig. 3b.

The strain to trigger liquefaction in Fig. 3b is depicted to be essentially zero and this is reasonable when the static bias is small or zero. If the static bias is significant, then larger strains may be required to "liquefy" the element and the post-liquefaction behaviour would be stiffer.

The characteristic stress-strain curves of Fig. 3b will now be incorporated in an energy balance approach similar to that used by Newmark. Upon liquefaction the stress in the soil drops from its static value A to B as shown in Fig. 4. Its resistance then increases with strain to a residual value,  $s_r$ . The driving force from the ground slope remains constant, so that the system accelerates and deforms. When the strain reaches point C the material has hardened so that the stress developed is now sufficient to balance the driving stress. However, the system has a velocity at this point and the stress continues to increase until point D is reached where the net energy ( $W_{EXT} - W_{INT}$ ) is zero, which is equivalent to equating the area ABC to CDE. If the system also had an initial velocity at the time liquefaction was triggered, it would carry on to point E.

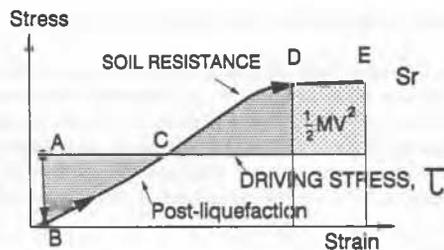


Fig. 4. Work-energy, extended Newmark.

Comparing the rigid plastic Newmark approach with the extension to a general stress-strain relation (Fig. 2b and Fig. 4) it may be seen that Newmark neglects the displacement from A to D. This could be a very considerable displacement since strains of 20 to 50% are commonly required to mobilize the residual strength,  $s_r$ . It should be noted that Newmark derived his equation for rigid plastic soils, and it is therefore not applicable without correction to liquefied soils that are very flexible in shear.

Newmark simulated the effect of an earthquake by applying a series of velocity pulses, where the number of pulses depended on the N/A ratio but could go as high as 6. The magnitude of the velocity pulse  $V$  was taken to be equal to the peak ground velocity. In carrying out analyses where liquefaction is triggered, only one pulse is considered. This is felt to be justified because the displacements that occur due to pulses prior to liquefaction will, in general, be small compared to those that occur upon liquefaction. Once liquefaction is triggered it is considered that no further major pulses would occur.

For a single-degree-of-freedom system, the displacement can be computed directly from the energy equation 2 and this is described in detail by Byrne (1991).

For a multi-degree-of-freedom system a finite element approach can be used. The displacements can be computed from the solution of

$$[K]\{\Delta\} = \{F + \Delta F\} \quad (5)$$

where  $[K]$  is the global stiffness matrix of the system,  $\{\Delta\}$  is the vector of nodal displacements,  $\{F\}$  is the static load vector acting on the system (gravity plus boundary loads), and  $\{\Delta F\}$  is an additional load vector applied to produce displacements to satisfy the energy balance of equation 2. If  $\{\Delta F\} = 0$ , then for the single-degree-of-freedom system, a displacement corresponding to C (Fig. 4) would be predicted. An additional force is required to balance the energy and predict points D or E. This additional force can be applied using a seismic coefficient,  $k$ , such that  $\{\Delta F\} = \{kw\}$ . However,  $k$  is not related to the peak ground

acceleration but is selected by an iterative procedure so as to balance the energy in accordance with equation 2.

Displacements occur due to softening caused by liquefaction and from kinetic energy due to the velocity pulse. We have found that displacements due to softening are best computed using a  $\{\Delta F\}$  based on a vertical seismic coefficient as it is essentially the vertical gravitational force that causes the displacement. The displacements due to the velocity pulse are best computed using a  $\{\Delta F\}$  based on a horizontal seismic coefficient as the velocity is assumed to be horizontal.

The horizontal seismic coefficient was also adjusted to reflect the relative stiffness of elements. Dynamic analyses indicated that a seismic coefficient proportional to the square root of the element stiffness gave best results. Thus the appropriate force distribution to balance the energy depends on both the mass and stiffness of the element.

For the multi-degree-of-freedom system  $W_{INT}$  equals the work done by the element stresses and strains, and  $W_{EXT}$  equals the work done by the static load vector  $=\{F\} \cdot \{\Delta\}^T$ . The additional force  $\{\Delta F\}$  is adjusted to give displacements  $\{\Delta\}$  so as to balance the energy and satisfy Eq. (2). This additional force is not included when computing the work done by the static forces as it is merely an artifact to obtain the appropriate displacements.

The procedure has been incorporated into the finite element computer code SOILSTRESS (Byrne and Janzen, 1981) and found to give exact agreement with Newmark when the assumptions made correspond to a single-degree-of-freedom rigid plastic system. It gives good agreement with liquefaction induced field observations reported by Hamada et al. (1987). The procedure predicts the failure of the Lower San Fernando dam, and gives displacement predictions for the Upper San Fernando dam that are in good agreement with the measurements in terms of both the magnitude as well as the pattern of deformations (Byrne et al., 1992). The method was used to predict possible liquefaction induced displacement of the intake structure at the John Hart Dam (Byrne et al., 1991), and is currently being used by BC Hydro to estimate possible liquefaction induced displacements at Duncan Dam.

## CASES ANALYSED

The variables in the analyses were essentially the level of the earthquake excitation and the location of the water table within the slimes. Peak horizontal ground accelerations ranging from 0.2g to 0.6g were considered with corresponding peak ground velocities of 0.2 and 0.6 m/second. Tailings to the full height were examined with the water table at the surface as well as several lower levels. Only two water table conditions will be shown here: *Case 1* - Water table at the surface of the tailings; and *Case 2* - the lowest water table considered. The finite element mesh, the soil types, and the water table for these two conditions are shown in Fig. 5. It is assumed that tailings above the water table will not liquefy and that tailings below the water table will liquefy to the base of the impoundment. The zones of liquefaction are shown in Fig. 5 for both cases.

## SOIL PARAMETERS USED IN THE ANALYSES

Soil parameters used were based on the hyperbolic model, and were obtained from laboratory data following the method described by Duncan et al. (1980) and Byrne et al. (1987). The properties of liquefied slimes were obtained from post-cyclic monotonic triaxial test.

The soil is treated in the analysis as equivalent isotropic elastic using secant shear and bulk moduli that vary with stress level as follows:

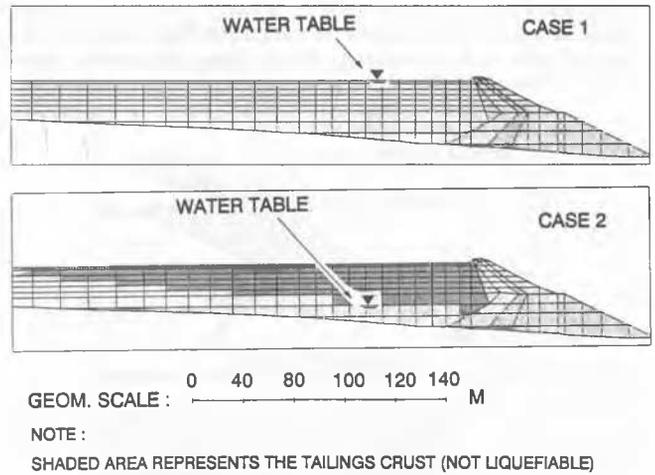


Fig. 5. Finite element mesh, soil types and groundwater conditions.

$$G = k_g P_a (\sigma'_m / P_a)^n (1 - \frac{\tau \cdot R_f}{\tau_f}) \quad (6)$$

and

$$B = k_b P_a (\sigma'_m / P_a)^m \quad (7)$$

in which  $k_g$  and  $k_b$  are shear and bulk modulus numbers,  $n$  and  $m$  are modulus exponents,  $\tau_f$  is the failure strength, and  $R_f$  is the ratio of the strength at failure to the ultimate strength from the best fit hyperbola,  $\sigma'_m$  is the mean normal stress,  $P_a$  is atmospheric pressure, and  $\tau$  is the mobilized shear stress. The soil parameters used in the analyses are listed in Table 1.

TABLE 1: Soil Parameters Used in the Analyses

	Slimes		Sand/Gravel	Till
$k_g$	540	(1.5)	650	1025
$n$	0.50	(0)	0.50	0.50
$k_b$	1490		1085	1700
$m$	0.25		0.25	0.25
$\phi$ (deg)	36.5	(0)	39.0	42.0
$\Delta\phi$ (deg)	0	(0)	0	0
$c$ (kPa)	0	(50)	0	0
$R_f$	0.60	(0.10)	0.60	0.60
$\gamma_s$ (kN/m <sup>3</sup> )	14.1		19.5	19.5

Note: a) Brackets indicate the properties after liquefaction  
 b) Pre- and post-earthquake properties for sand, gravel and compacted till are assumed identical since these soils were assumed not to liquefy.

The shear stiffness values for the sand/gravel and till represent first time loading values. Unload-reload values generally considered appropriate for dynamic loading could be four times stiffer.

## RESULTS

**Case 1** - The predicted deformations for Case 1 with the water table at the surface of the slimes and a peak ground acceleration of  $A = 0.6g$  are shown in Fig. 6. For this site an  $A/V = 1$  ratio was considered appropriate, and so a velocity of  $V = 0.6$  m/sec was applied uniformly in the downstream direction to all the mass including the dam and tailings. This figure shows the deformed finite element mesh magnified by a factor of 10. The liquefied slimes are predicted to undergo large horizontal movements in the "free-field", away from the face of the dam. They are constrained by the dam and move upward turning some of their kinetic energy into potential energy.

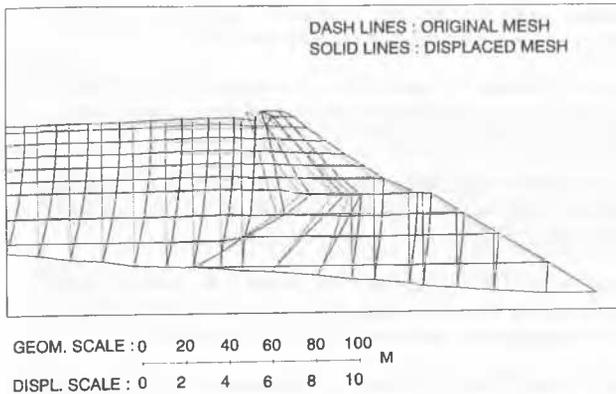


Fig. 6. Displacement pattern of the dam, Case 1.

The predicted upward movement of the tailings adjacent to the dam is 0.21 m. Such a movement is not likely to be of concern with regard to overtopping.

The predicted peak displacements of the crest of the dam are 0.88 m horizontal, and -0.03 m vertical. Analyses were also run using stiffer moduli reflecting the unload-reload response of the dam, but the resulting deflections were very similar. This is so because the earthquake loading causes both elastic and plastic deformation, and it is the larger plastic response rather than the initial elastic response that controls displacement.

The maximum movement of the dam predicted from the Newmark analysis using the same soil strengths was 0.18 m. In the Newmark analysis the hydrodynamic effect of the slimes was accounted for using the Westergaard approach, considering the slimes as a heavy liquid. The Newmark value is significantly lower than the 0.88 m predicted in the proposed method. Predicted displacements of the dam crest for peak accelerations of 0.2g and 0.35g are listed in Table 2.

The lower displacement value predicted by Newmark is mainly due to the fact that he assumed the block above the slide plane to be rigid, so that no deformations occur within the block itself - only along the slide plane. The extended procedure presented here considers both elastic and plastic deformations within the block. In addition, the effect of the slimes on the dam is treated differently.

**Case 2** - The predicted deformations for Case 2 with the water table at a low level and the same initial velocity ( $V = 0.6$  m/sec) are shown in Fig. 7. In this case only a relatively thin zone of slimes near the base of the impoundment was assumed to liquefy during earthquake shaking.

TABLE 2: Soil Movements

Case	Peak Accel. m/s <sup>2</sup>	Newmark Dam X-Displ. (m)	Proposed Energy Method		Rise in Slime Level Adjacent to the dam (m)
			Dam Crest (m)		
			X-Displ.	Y-Displ.	
1	0.20	0.00	0.26	0.06	0.23
1	0.35	0.01	0.64	- 0.01*	0.23
1	0.60	0.18	0.88	- 0.03*	0.21
2	0.20	-	0.25	0.05	0.09
2	0.35	-	0.26	0.06	0.10
2	0.60	-	0.42	0.09	0.16

\*Downward

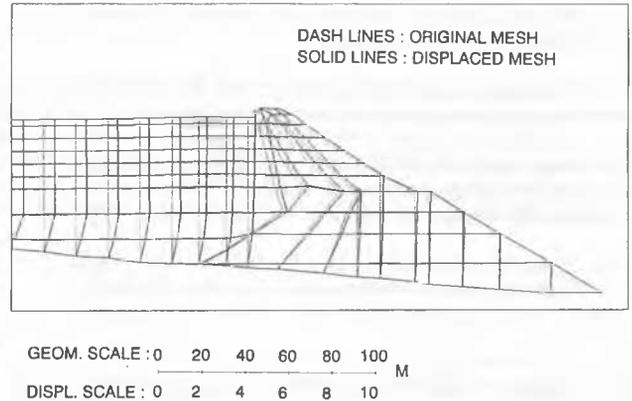


Fig. 7. Displacement pattern of the dam, Case 2.

The predicted displacements of the crest are 0.42 m horizontal and 0.09 m vertical. The crusted tailings essentially move horizontally with only a small surface rise.

The Newmark analysis is difficult to apply to this condition as the predicted displacement depends upon the mass of crust considered as part of the slide block. It was therefore not applied.

Comparing Figs. 6 and 7, the predicted horizontal displacement values are generally smaller for the lower groundwater condition in the tailings (Case 2). The smaller values result from the fact that much more energy is dissipated in the tailings crust which is stronger than liquefied slimes. Since the total kinetic energy to be dissipated is the same for both cases, the energy dissipated in the dam is less for Case 2 which in turn results in smaller displacements.

A summary of important soil movements for Cases 1 and 2 is given in Table 2.

## SUMMARY

A simple analysis procedure is presented for predicting earthquake induced displacements of a tailings dam. The procedure is an extension of the simple Newmark method from a single to a multi-degree-of-freedom system taking into account the softened stress-strain response of liquefied soils and is based on energy principles. The method has been verified by application to field case histories including the San Fernando Dams. Herein, the method was applied to the proposed Kensington tailings dam in Alaska and gives the pattern of displacement as opposed

to a single displacement obtained from Newmark. The displacements are larger than computed from the standard Newmark method and arise because additional displacements due to the flexible nature of soil are included. The formation of a crust reduces the predicted displacements.

#### ACKNOWLEDGEMENTS

The authors are grateful to NSERC for its financial support and to Ms. Kelly Lamb for her typing and presentation of the paper.

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