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A PROCEDURE FOR THE SEISMIC AND POST-SEISMIC ANALYSIS OF NATURAL SLOPES

UN PROCEDE POUR L'ANALYSE SISMIQUE ET POST SISMIQUE DES PENTES NATURELLES

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SYNOPSIS: A procedure based on displacement analysis of natural clayey slopes during and after earthquakes is presented. This procedure takes account in a simplified way of the following factors: 1) complex morphological, stratigraphic and geotechnical conditions 2) variations of the shear strength, and consequently of the critical acceleration, during and after the earthquake due to the loading rate and the cyclic degradation 3) the amplification effects and the different seismic histories of the various zones of the mass in movement 4) the uncertainties of the geotechnical properties. The procedure is developed in different phases with regard to the definition of the following factors: 1) the geometry of the mass in movement 2) the critical acceleration 3) the amplification effects 4) the cumulated displacements 5) the post-seismic conditions. The probabilistic analysis includes determining the distribution of the maximum displacement of the slope, considering the uncertainty of the critical acceleration and of the initial shear modulus, G_o, and the distribution of the safety factor after the earthquake. Thus is calculated the probability of the maximum displacement exceeding a specified value and the probability that the safety factor after the earthquake would be less one.

INTRODUCTION

The stability of a natural slope in seismic conditions depends on the morphological, stratigraphic and geotechnical conditions and on the characteristics of the ground motion at the site. Such factors are generally complex, variable and uncertain. So, an appropriate compromise between theory and practice must be sought. The pseudo-static methods which reduce the 'seismic loading' to a single number (the seismic coefficient), underrate the importance thereof. The classic sliding block models underestimate the importance of the morphological, stratigraphic and geotechnical conditions. The finite element methods require a knowledge of detailed information which is generally impossible for natural slopes. From these considerations the authors proposed a procedure based on the evaluation of displacements (Crespellani et al., 1990; Crespellani et al.,1992) which, in a simplified way, accounts for a) complex morphological, stratigraphic and geotechnical conditions b) shear strength variations (and consequently of the critical acceleration) due to the effects of the loading rate and the cyclic degradation c) the amplification effects with different accelerograms in the various zones of the mass in movement d) post-seismic conditions. The major assumptions concerned with points from a) to d) are briefly described in the following section. The other important aspects of the problem specifically considered in the present work are: e) the determination of the seismic histories applicable to each homogeneous zone of the slope, and f) the estimation of the probability of failure during and after the earthquake.

MAJOR ASPECTS OF THE PROCEDURE

a) Morphological, stratigraphic and geotechnical conditions. The classic displacement methods determine the critical acceleration and analyse the motion of a rigid block sliding on a vibrating plane surface (plane failure mechanism) or on a logarithmic spiral surface (logspiral failure mechanism) (Chang et al., 1984). With the proposed procedure the inverse pseudo-static seismic analysis of the slope is carried out with the global method or by slices method more suitable. The procedure determines 1) the critical slip surface 2) the initial critical acceleration, $K_{c,o}$ 3) the weight and centre of gravity of the potentially sliding mass 4) the plane or the logarithmic spiral surface approximating to the critical surface.

b) The critical acceleration. The sliding block models presuppose a constant critical acceleration. With the proposed method and with reference to the seismic tensional or deformative history of characteristic points of the critical slip surface, the variations of the critical acceleration, $K_c(t)$, during and after the earthquake are determined. This is done in the following way:

$$K_c(t) = K_{c,o}\, R(t)\, D(t) \qquad (1)$$

where R(t) is a loading rate coefficient and D(t) is a cyclic degradation index.
R(t) is determined, with reference to a history of seismic strains, as follows:

$$R(t) = 1 + F_R \log \left[\frac{r(t)}{r_r}\right] \qquad (2)$$

in which F_R is the rate factor (assumed to be $F_R=0.1$), $r(t)=d\tau(t)/dt$ is the actual seismic loading rate, and r_r is the reference loading rate (for cohesive soils we can assume that $r_r=q_u/t_f$ where q_u is the unconfined compression strength and t_f is the time in which the failure is produced).
D(t) is determined by reference to the deformative seismic history in the following way: in the equivalent strain cycle (n+1) it is

$$D_{n+1} = D_n \left(1 + D_n^{\frac{1}{k_n}}\right)^{-k_n} \qquad \text{or} \qquad D_{n+1} = D_n \qquad (3)$$

(in which k_n is the degradation parameter) respectively if the cycle (n+1) does or does not exceed the value of the threshold strain.

c) Amplification effects. In the classic displacement methods a single accelerogram is considered, the inertia force is applied to the centre of gravity of the sliding mass and the amplification effects are omitted. The proposed method is a 'hybrid' procedure, since it maintains the rigid block model, but takes account of the amplification effects and of the instantaneous position of the seismic force. To this end, it is first necessary to define the zones of the slope where the acceleration is uniform and for each zone: to calculate the mass and centre of gravity, to determine the reference accelerogram, to calculate the instantaneous seismic force. Then, the modulus and position of the resulting instantaneous seismic force, and, finally the displacements of the block during the earthquake, are calculated.

d) Post-seismic analysis. Static analysis of the slope in undrained conditions is carried out, assuming the shear strength throughout to be reduced by the final value of the cyclic degradation index.

DETERMINATION OF THE SEISMIC LOADING HISTORY IN DIFFERENT ZONES OF THE SLOPE

The determination of the site effects on the seismic motion, either superficial or at any other point in the subsoil, is notoriously a problem of great theoretical and practical complexity. The problem is simplified if the boundary conditions permit a one-dimensional model to be used. SHAKE (Schnabel et al, 1972) is the most common and reliable code for calculating local seismic response in soil deposits on undefined homogeneous horizontal strata lying on horizontal bedrock. This programme is theoretically usable only if the hypotheses on geometric conditions are verified, but it has been demonstrated that reliable results are also obtained in less restricted conditions (Elton et al. 1991; Hayashi and Ang, 1992).

Thus, it is proposed to use, for the seismic history of each zone of uniform acceleration within the slope, that obtained by SHAKE, modified in order to empirically take account of the effects of the topography. In particular, for each zone of uniform acceleration the column of soil corresponding to the centre of gravity of the mass is considered. The reference accelerogram on hard soil is applied at the base of the column. The accelerogram calculated by SHAKE in the centre of gravity of the zone of uniform acceleration, is multiplied by a topographic amplification factor, $\alpha>1$. Practical suggestions for an empirical quantification of the effects of local topography are contained in the recommendations of the French Association of Earthquake Engineering (AFPS.90, 1990) in which the topographic amplification factor is defined[1]. The coefficient α at a point P of the surface is conventionally determined on the basis of some characteristics of the profile of the steepest gradient passing through P. The profile is schematically broken down into sections of equal gradient. In figure 1, C indicates the outline of the ridge, (I) the tangent of the angle of slope downstream and (i) the tangent of the angle of slope upstream, H the height of the slope. If $H \geq 10$ m and $i \leq I/3$, the amplification factor at the surface takes on the values:

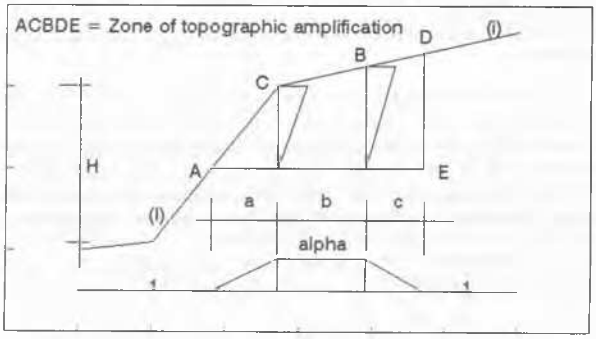

Fig. 1. Topographic amplification effect

$$\alpha=1 \qquad \text{if } (I - i) \leq 0.4$$
$$\alpha=1 + 0.8\,(I - i - 0.4) \qquad \text{if } 0.4 \leq (I - i) \leq 0.9$$
$$\alpha=1.40 \qquad \text{if } (I - i) \geq 0.9$$

In the two sections AC and BD, α varies linearly from the maximum value to the value of 1. In the sections downstream of A and upstream of D is $\alpha=1$. The lengths a, b, c are: $a=H/3$, b=the lowest value between $(20I)$ and $(H+10)/4$, $c=H/4$. From below the topographic surface between the points A and D it is assumed that α decreases linearly until it reaches the value of 1 at the AE level.

The representative accelerogram for the calculation of the force of inertia associated with each slope zone of uniform acceleration is obtained by multiplying the accelerogram derived from SHAKE in correspondence to the centre of gravity, by the average value of the topographic amplification factor of that zone.

PROBABILISTIC ANALYSIS

As has been seen, the stability of a slope depends on a large number of seismic, geometric and geotechnical parameters. Such parameters are characterized by different levels of uncertainty and can have effects of varying importance upon the results. The uncertainty of the seismic parameters is undoubtely the greatest of these and is the one which most affects the results, such that, until now it has been this uncertainty to which researchers have directed their attention (Lin and Whitman, 1986; Lin, 1990; Yegian et al., 1991). Since there has not yet been sufficient experimentation on the influence of the geotechnical parameters upon the seismic response of natural slopes, evaluated with account taken of the multiplicity of effects which occur during and after earthquakes, in the present work the influence of the variability of geotechnical parameters, has been examined. The uncertainty of geometric parameters has not been introduced because the consideration of such effects would not have allowed a full understanding of those due to the uncertainty of geotechnical parameters.

The geotechnical parameters whose uncertainty most influence the seismic response in the slope are: the shearing resistance parameters (cohesion, c, and the angle of shearing resistance, ϕ) and the initial shear modulus, G_o. The other geotechnical parameters and decay laws have a markedly less relevant influence and, for simplicity, can be treated deterministically. The dependent random variables considered in the present work are the critical acceleration, the maximum displacement of the slope at the end of the earthquake and the safety factor in the slope after the earthquake. In order to adapt the probabilistic analysis to the level of the general complexity of the proposed procedure, the point estimate method (PEM) has been used (Rosenblueth, 1975) for estimating the distribution of the dependent random variables. The probabilistic analysis of the seismic response in the slope is expounded in several phases: firstly the statistical parameters of the distribution of the critical acceleration, $K_{c,o}$, are calculated, having accounted for the intrinsic variability of the shear strength parameters, c and ϕ, for each zone considered. In the second place the amplification effects along the slope, associated to the variability of the shear modulus, G_o, are determined. As a third step the expected value and the coefficient of variation of the maximum displacement of the slope are calculated. Then follows the post-seismic analysis, evaluating the distribution of the safety factor. The probabilistic analysis concludes with an analysis of reliability, which consists of calculating the probability that the maximum displacement u_{max}, exceeds a specified value u_a, and that the safety factor, SF, after the earthquake, is less than 1.

NUMERICAL EXAMPLE

A numerical example is described to highlight the different steps and characteristics of the proposed procedure. Although the example refers to an imaginary slope, the geometric, stratigrafic and geotechnical conditions (fig. 2 and table 1) are fairly common in the central southern Apennines. The design accelerogram is the Sturno N-S record from the earthquake on November 23, 1980 (fig. 2). The slope is made up of two strata with clearly different characteristics. The upper stratum is a fractured sandstone, which along the joints between the blocks has only frictional resistance. The lower stratum is a normally consolidated and unsaturated clay. Its shear strength in terms of total stress and its stiffness, both vary linearly with consolidation pressure and therefore depth.

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[1] In the AFPS.90 recommendations the symbol τ is used to indicate the topographic amplification factor. To avoid confusion, the present paper uses the symbol α

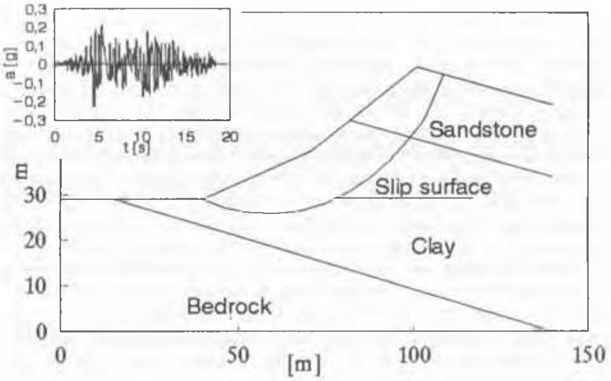

Fig. 2. Geometric and stratigraphic data, and the design earthquake for the example

The bedrock has the same slope as the contact surface between the two strata. The simplified Bishop method has been used to perform the inverse pseudo-static seismic analysis in order to determine the critical circular slip surface (fig. 2) and the initial critical acceleration, $K_{c,o}$. The stratigraphic and geo-technical conditions of the slope suggest that the zone delimited by the potential critical slip surface and by the topographic surface be divided into three zones having an uniform field of acceleration (for which a single accelerogram can be assumed) (fig. 3). For each zone a representative stratigraphic column has been selected. In these columns two characteristic points have been identified: the point S corresponding to the critical slip surface, and the point G at the depth of the centre of gravity. For the two soils considered typical decay laws have been assumed. With the SHAKE program a seismic response analysis was carried out for each stratigraphic profile, and, in particular, the accelerograms corresponding to points S and G were calculated. For points S in columns II and III, in the clay, the loading rate coefficient, R(t), the cyclic degradation index, D(t), and the critical acceleration, $K_c(t) = K_{c,o}\,R(t)\,D(t)$, were evaluated. The two functions $K_c(t)$ for the two points S result in very similar values, consequently one of them has been assumed as characteristic of the whole slope (fig. 4). The accelerograms calculated at the points G of the three zones of uniform acceleration have been multiplied by the respective average topo-graphic amplification coefficients: in particular, for zone I, that nearest the ridge, the maximum value of the topographic amplification coefficient is α_{max}=1.27 and the average value is α=1.04; for zones II and III the topographic amplification effect is negligible (α=1). The final accelerograms for the three zones of the slope are used to determine the corresponding instantaneous seismic forces applied at the three centres of gravity. The resulting seismic force is variable during the earthquake in modulus and position, thus at the centre of gravity of the rigid block, a force F(t) and a momentum M(t) are instantaneously applied (fig. 4).

Table 1. Geotechnical properties and their statistical distribution parameters

Soil		γ [kN/m^3]	c_u [kPa]	ϕ_u [°]	ϕ' [°]	G_o [MPa]	D_{min} [%]
Sandstone	M.	21.5	-	-	42	2000	1.13
	S.D.	-	-	-	4	-	-
Clay	M.	19.5	3z+30	7	-	6z+60	2.30
	S.D.	-	0.7z+7	0.14		1z+10	-
Bedrock	M.	21.5	-	-	-	5000	-

M = Mean value, S.D. = Standard Deviation

Before the earthquake, the safety factor and the initial critical acceleration result:

$$SF(before) = 1.284 \qquad K_{c,o} = 0.117\ g$$

The cumulated displacement during the earthquake, the critical acceleration at the end of the earthquake, and the safety factor after the earthquake are respectively:

$$u_{max} = 13.3\ cm \qquad K_c(after) = 0.105\ g \qquad SF(after) = 1.252$$

The probabilistic analysis has been developed by means of the PEM assuming as normally distributed random variables the shear strength parameters of the clay and of the sandstone, and the initial shear modulus of the clay (table 1). The shear strength parameters of the clay have been supposed to be weakly correlated (r=-0.6). In the first place, the statistical parameters of the distribution of the pre-seismic safety factor and of the initial critical acceleration $K_{c,o}$ have been calculated (table 2). The variability of the shear modulus determines the variability of the deformative seismic history and, therefore, of the degradation index, D(t), of the instantaneous critical acceleration, $K_c(t)$, and of the post-seismic safety factor. Assuming as independent random variables the instantaneous critical acceleration and the initial shear modulus, the distribution parameters of the cumulated displacement of the slope during the earthquake and of the post-seismic safety factor have been calculated. The cumulated displacement has been assumed lognormally distributed (Yegian et al., 1991). Therefore the espected value and the median have different values. The expected value of the cumulated displacement is 39.3 cm and the median is 13.3 cm. This last value equals the deterministic one. The results are shown in the table 2 and in fig.5.

As critical displacement, u_a, for the slope, a value of 10 cm was adopted. The values of the probability of failure before, during and after the earthquake are shown in the table 3.

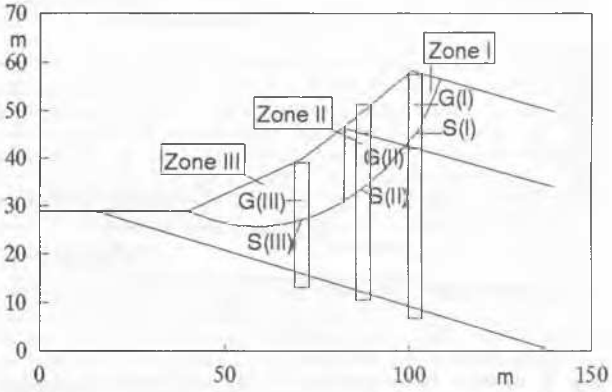

Fig. 3. Zones with uniform acceleration field and columns analyzed by SHAKE

Table 2. Statistics of the major parameters controlling slope stability before and after the earthquake

	Before		After	
	SF	$K_{c,o}$ [g]	SF	$K_{c,o}$ [g]
Expected value	1.284	0.117	1.252	0.105
St. Deviation	0.163	0.067	0.168	0.069

SF = Safety Factor

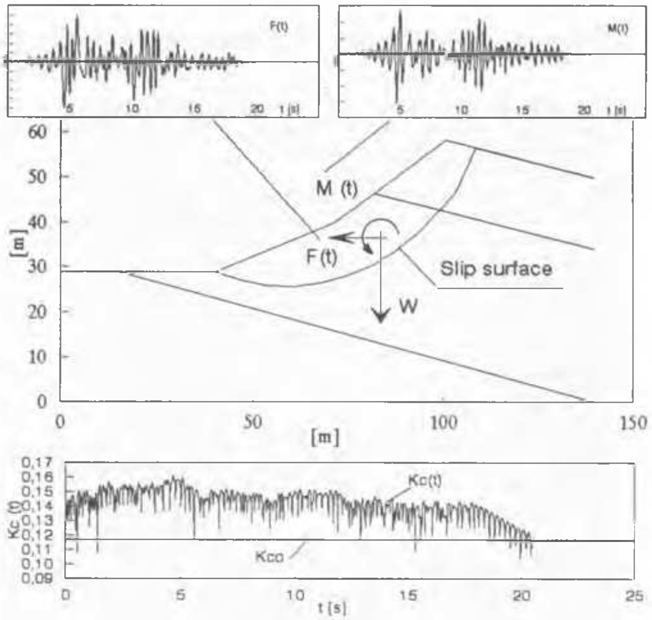

Fig. 4. Resulting instantaneous force and momentum during the earthquake and variations of the critical acceleration

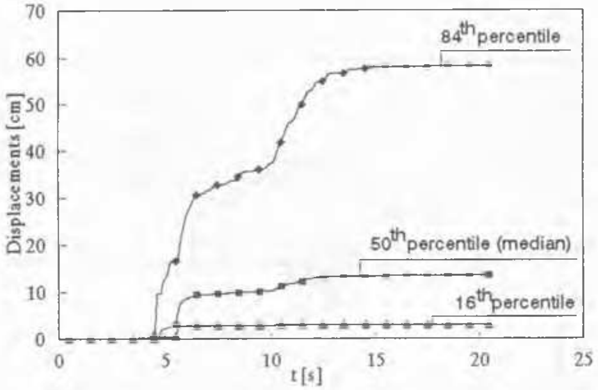

Fig. 5. Median and percentiles of the cumulated displacements assumed lognormally distributed

Table 3. Probability of failure before, during and after the earthquake

Before	During	After
P[SF<1]	P[u_{max}>u_a]	P[SF<1]
0.04	0.42	0.07

CONCLUSION

By calculating the displacements and the post-seismic safety factor, the present procedure attempts to take account, in a simplified way, of some effects produced by earthquakes in natural slopes. These effects (amplification of seismic response, effects of the loading rate, decay of stiffness and shear strength with the number of loading cycles) are generally neglected in classic displacement methods. The procedure can be applied even in complex morphological, stratigraphic and geotechnical conditions. The major assumption introduced is the possibility of applying a one-dimensional model of seismic response to the different homogeneous zones that can be identified in the slope. It allows the resulting instantaneous force and seismic momentum at the centre of gravity of the block to be defined in more realistic way. The procedure also allows examination of the sensitivity of the displacement and of the post-seismic safety factor to the various geotechnical parameters and to estimate the probability of exceeding pre-fixed values for the displacement and of the failure of the slope.

The illustrative example, which has been chosen only in order to describe the steps of the procedure, shows clearly that the inclusion of the uncertainties and of the various seismic effects can notably influence the displacement, and it confirms, as has already been highlighted by other authors, that the uncertainty of the shear strength in static conditions is the factor, after the earthquake characteristics, which most affects the results.

REFERENCES

AFPS.90 (1990). Recommandations pour la redaction de regles relatives aux ouvrages et installations a realiser dans les regions sujettes aux seismes. *Association Française du Genie Parasismique.*

Chang, C.J., Chen, W.F. and Yao, J.T.P (1984). Seismic displacements in slopes by Limit Analysis. *Journal of Geot. Eng.*, ASCE, Vol. 110, No. 7

Crespellani, T., Ghinelli, A., Madiai, C. and Vannucchi, G. (1990) Analisi della stabilita` dei pendii naturali in condizioni sismiche. *Riv. Ital. di Geotecnica*, No. 2

Crespellani, T., Madiai, C. and Vannucchi, G. (1992). Seismic stability analysis of slopes including amplification effects. *Proc. of French-Italian Conference on Slope Stability in Seismic Areas*, Bordighera, Italy

Elton, D.J., Shie, C.F. and Hadj-Hamou, T. (1991). One- and two-dimensional analysis of earth dams. *Proc. of 2^{nd} Int. Conf. on Recent Advanced in Geotechnical Earthquake Engineering and Soil Dynamics*, St. Louis, Missouri, paper No. 7.16

Hayashi, H. and Ang, A. H-S. (1992). Reliability of cut slopes under seismic loading. *Soils and Foundations*, vol. 32, No. 2

Lin, J.S. (1990). Regional seismic slope failure probability matrices. *Earth. Eng. and Struct. Dyn.*, Vol. 19

Lin, J.S. and Whitman, R.V. (1986). Earthquake induced displacement of sliding blocks. *Journal of Geot. Eng.*, ASCE, Vol. 112, No. 1

Rosenblueth, E. (1975). Point estimates for probability moments. *Proc. Nat. Acad. Sci. U.S.A.*, Vol. 72, No. 10

Schnabel, P.B., Lysmer, J. and Seed, H.B. (1972). SHAKE - A computer program for earthquake response analysis of horizontally layered sites. *Earth. Eng. Res. Center*, Rep. EERC 72-12, University of California, Berkeley, U.S.A.

Yegian, M.K., Marciano, E.A. and Ghahraman, V.G. (1991). Earthquake-induced permanent deformations: probabilistic approach. *Journal of Geot. Eng.*, ASCE, Vol. 117, No. 1