SURFACE SUBSIDENCE INDUCED BY EXTRACTION OF GROUND WATER

L'AFFAISSEMENT DE SURFACE PRODUIT PAR L'EXTRACTION D'EAU SOUTERRAINE

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SYNOPSIS: A fully coupled numerical method is presented for the analysis of surface subsidence due to extraction of ground water, where the effect of drawdown of the water table has been taken into consideration. The prediction of soil deformation due to changes in pore pressure is based on consolidation theory, and the transient drawdown position of the water table accompanying the extraction of ground water is evaluated by use of the Residual Flow Procedure. Examples are given here comparing the numerical solutions of surface subsidence for the cases with and without drawdown of the water table. Extensive parametric studies have been carried out to provide non-dimensionalised design charts for the surface subsidence and the drawdown of the water table due to extraction of ground water.

INTRODUCTION

Extraction of ground water is often accompanied by surface subsidence (e.g. Delfranche 1978; Premchitt 1979). The reason for this is that when the groundwater is withdrawn, the pore water pressure reduces, and in addition, the water table is lowered and so there is an increase in effective stress causing consolidation of the layer of soil.

There have been some solutions obtained for the prediction of how the water table will fall as a result of extraction of ground water (e.g. Taylor and Luthin 1969; Neuman and Witherspoon 1971). However these solutions do not include predictions of the surface deformation. Some approaches have also been developed for predicting surface subsidence due to extraction of ground water. These approaches consider only the effect of reductions in pore pressure in the soil for the case where the phreatic surface remains at the surface (Small and Booker 1984; Booker and Carter 1987). These solutions have generally been obtained by analytic or semi-analytic techniques. Because the extraction of ground water will often draw down the water table, significantly affecting the distribution of the pore water pressure, these analyses of subsidence may not be satisfactory in practice.

In this paper, a fully coupled numerical solution has been obtained. This method takes into account both the effects of the pore pressure changes on the deformation of the soil and the effects of the deformation of the soil on the pore pressures, as well as the drawdown of the water table. Good predictions of surface subsidence are expected with this method because the effects of extraction of ground water are more accurately simulated.

Comparisons have been made for the prediction of surface settlements due to extraction of ground water with conditions where drawdown and no drawdown occur. Extensive parametric studies have produced non-dimensionalised design charts, which allow an estimation of ground surface deformation and drawdown of the water table due to pumping for a wide range of cases of practical interest.

THEORY

The fully coupled analysis of soil consolidation involving movements of the ground water table has been presented recently by Hsi and Small (1992a, 1992b) and Hsi (1992). The analysis for a consolidating soil is based on Biot's equations (Biot 1941, 1956) and the procedure used to deal with drawdown of the water table is the same as that proposed by Desai and Li (1983) and Bathe *et al.* (1982). The concept of this procedure is that when the water table falls from a previous position to the present one, the water initially stored in the pores of the soil in the zone between these two positions is released, and this amount of water needs to be imposed as a flow across the present free surface. When this is done, a new location of the free surface can be determined.

As the water table drops, the soil above the free surface becomes unsaturated and so reduced flow occurs in this region. To model this small flow, the permeability of the soil above the free surface is reduced. To avoid numerical instability, a permeability – pore pressure relationship (Bouwer 1964; Desai and Li 1983) which may be taken as linear as shown in Figure 1 (but which may have some other shape) needs to be adopted in the analysis. A limiting value of negative pore pressure

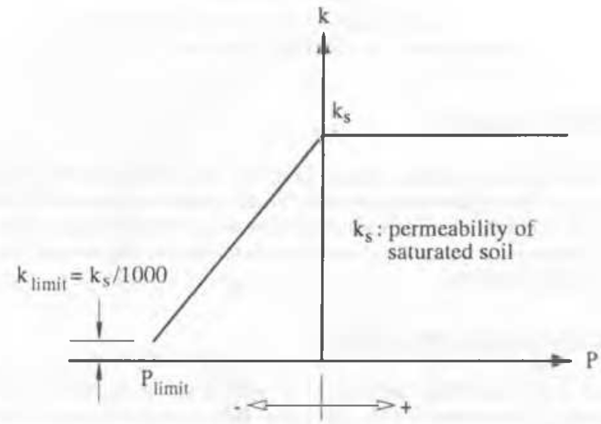

Fig. 1. Permeability – pore pressure relationship

P_{limit} is assigned and the permeability at this point is given a very small value k_{limit}.

The governing equations for this transient problem may be solved approximately using the finite element technique and details of the method can be found in Hsi (1992), Hsi and Small (1992a) and Hsi *et al.* (1992). A time marching scheme, employing the α integration rule proposed by Small *et al.* (1976), is used together with an iterative solution procedure. At each time step, the approximate equations are solved for nodal displacements and total pore water heads.

EXAMPLES

Two examples of the extraction of ground water are presented here. One considers that the position of the water table remains constant; whereas the other allows for drawdown of the water table. The numerical solution to the example where no drawdown is considered was firstly verified by an analytic solution (Booker and Carter 1987), and then the effect of drawdown of the water table on surface subsidence was investigated by comparing the two numerical solutions.

The problem is considered to consist of a point sink located at depth h within a porous soil deposit of thickness H, as shown in Figure 2. The soil parameters chosen for the comparative study were: Young's modulus $E' = 1 \times 10^5\ kPa$, Poisson's ratio $\nu' = 0.25$, permeability $k = 0.01\ m/day$ and the unit weight of water $\gamma_w = 10\ kN/m^3$. The pumping rate Q was specified as $2\ m^3/day$. The depth h was selected to be $10\ m$, and the thickness H was assumed to be $150\ m$ in the numerical analysis.

Based on the study by Booker and Carter (1987), the non-dimensional time factor T_v for transient problems is given by

$$T_v = \frac{c\ t}{h^2} \tag{1}$$

where $c = (\lambda + 2G)\ k/\gamma_w$. G is the elastic shear modulus, $G = E'/2(1 + \nu')$, λ is the Lamé constant, $\lambda = 2G\nu'/(1 - 2\nu')$, and t is time.

For the case where no drawdown occurs, good agreement between the numerical solution and the analytic solution for the surface subsidence and excess pore pressure was obtained (Hsi *et al.* 1992). The numerical solution for this surface subsidence ds is presented nondimensionally on the left hand side of Figure 3.

The second example presented here allows for the drawdown of the water table. In this example the specific yield S_y, which determines the amount of water released by the soil when it passes from being saturated to unsaturated, needs to be specified in the analysis for the consideration of drawdown of the water table and it was chosen to be 0.00833. The ultimate negative pore pressure P_{limit} above the free surface was specified as $-20\ kPa$. The predicted subsidence is much greater than the case where drawdown of the water table is not included and this is also shown nondimensionally in Figure 3. It is found that the final surface settlement with drawdown on the vertical axis through the pump ($r = 0$) is about 5 times greater than without drawdown.

DESIGN CHARTS

In an attempt to establish design charts for the problem of extraction of ground water, extensive parametric studies have been carried out to find the major factors which affect the drawdown of the water table and the surface subsidence. The results of these studies are presented in non-dimensional form.

Drawdown of the Water Table

It has been found that the ultimate drawdown dh^* of the water table is closely proportional to $1/k$, $1/h$ (when H/h is constant), and Q assuming k is not affected by stress. The transient positions of the water table were found to be affected by the value of S_y used in the analysis; however the steady-state position is not affected by this parameter. The

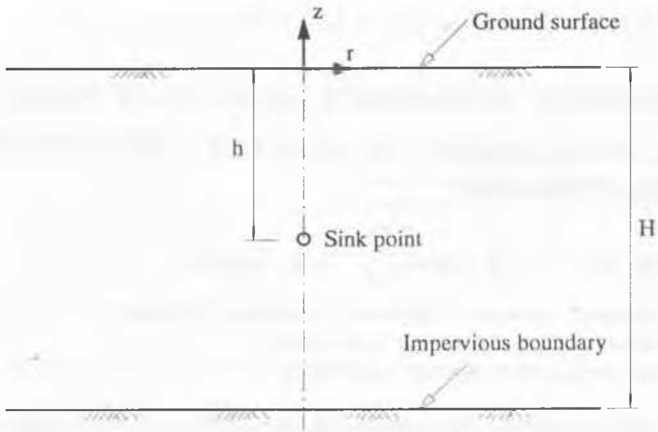

Fig. 2. Problem definition

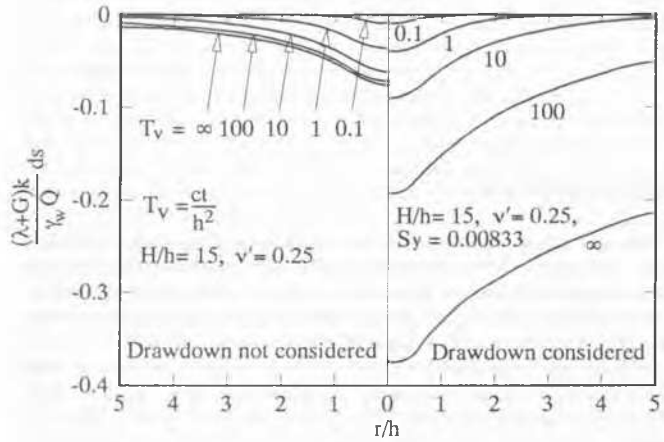

Fig. 3. Surface subsidence profiles

Young's modulus E' and the Poisson's ratio ν' of the soil skeleton have a slight influence on the drawdown behaviour. The ratio H/h affects the drawdown solution significantly and this will be discussed later. For all these numerical studies, the outer boundary was always maintained at a distance r_{max} where $r_{max}/h = 50$, so that this boundary will not have had a great effect on the solution. Total head is assumed constant along this boundary.

Transient and steady-state solutions for drawdown of the water table are presented nondimensionally in Figure 4, for which $T_v/T_d = 10$ (this will be discussed later) and $H/h = 15$. In this plot, the non-dimensional time factor T_d was chosen to be

$$T_d = \frac{k\ t}{S_y\ h} \tag{2}$$

As this numerical method is fully coupled, the drawdown of the water table can significantly affect the surface settlement and *vice versa*. Therefore the time factor T_d used in non-dimensional plots of the drawdown of the water table should be used in association with the time factor T_v (see Equation 1) used previously in non-dimensional plots of the surface subsidence. It has been found that almost unique non-dimensionalised plots can be obtained when the ratio T_v/T_d remains constant. However it is only the transient solutions which depend on this ratio and not the steady-state solutions. The transient drawdown solutions for the position of the water table for various T_v/T_d ratios are presented in the paper by Hsi *et al.* (1992).

It should be noted that it is not possible to obtain a unique plot for the drawdown of the water from the coupled numerical analysis as the drawdown is influenced marginally by other factors such as the elastic properties of the soil. However Figure 4 provides a satisfactory non-dimensionalisation of the results (with only very small variations) for a wide range of soil parameters (e.g. E' varied from 1×10^3 to 1×10^5 kPa; k varied from 0.001 to 100 m/day; S_y varied from 0.001 to 0.1 etc.).

It is also of interest to see how the ratio H/h, i.e. the ratio of layer thickness to sink depth, affects the drawdown behaviour. Steady-state solutions for drawdown of the water table for various values of H/h are presented in Figure 5. It is seen that the smaller the ratio H/h the larger the drawdown. It is also shown that when H/h is smaller than 5, the finite layer thickness has a much greater influence on the drawdown behaviour.

Surface Subsidence

The surface subsidence ds has been found to be closely proportional to $1/E'$, $1/k$ and Q. The non-dimensional time factor T_v (as defined in Equation 1) was previously chosen for use in non-dimensional plots of surface subsidence when no drawdown occurred. As was mentioned previously, the solutions for the surface settlement are closely related to the drawn down position of the free surface. Unique non-dimensionalised plots can be obtained when the ratio T_v/T_d remains constant. However only transient solutions are affected by this ratio.

The transient and steady-state solutions for surface subsidence are presented nondimensionally in Figure 6 for the cases where $T_v/T_d = 10$, $H/h = 15$ and $\nu' = 0.3$. The transient solutions for various T_v/T_d ratios were given by Hsi *et al.* (1992).

The steady-state surface subsidence profiles for various values of H/h are shown in Figure 7. The influence of the finite layer thickness is more pronounced close to the axis $r = 0$. It is also shown that when H/h is smaller than 5, this boundary has a greater influence on the surface subsidence.

The Poisson's ratio ν' was found to have some influence on the surface subsidence. The steady-state surface subsidence profiles for various Poisson's ratios are presented in Figure 8, where it is seen that the predictions fall into a narrow band.

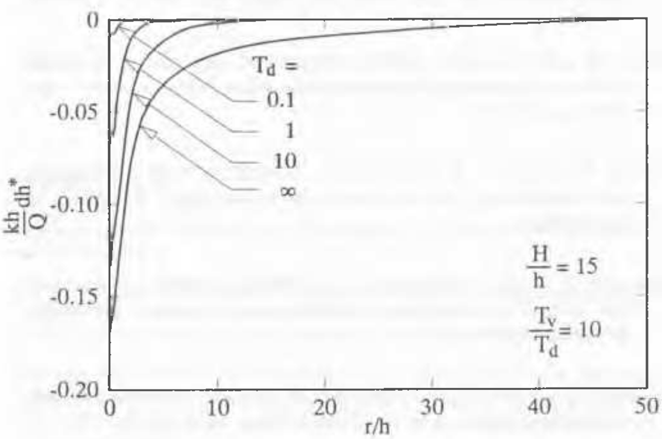

Fig. 4. Transient and steady-state drawdown profiles of the water table ($T_v/T_d = 10$)

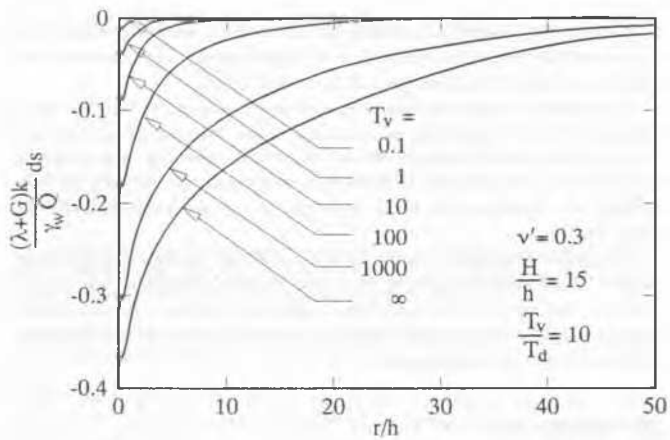

Fig. 6. Transient and steady-state solutions for surface subsidence profiles ($T_v/T_d = 10$)

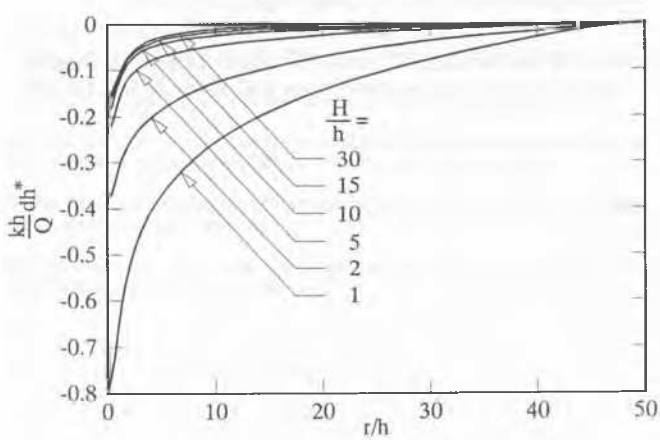

Fig. 5. Influence of H/h on steady-state drawdown profile

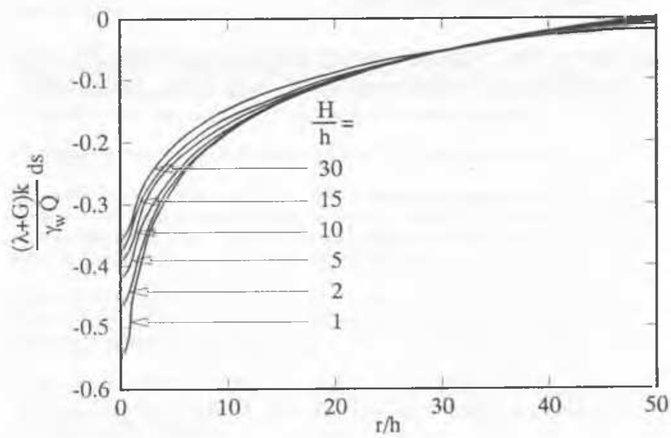

Fig. 7. Influence of H/h on steady-state surface subsidence profile

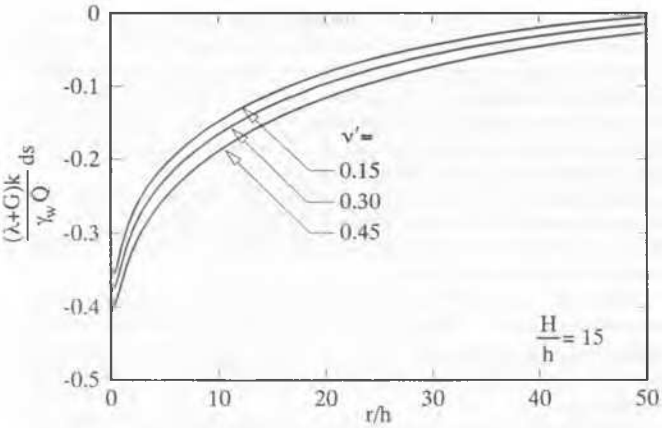

Fig. 8. Steady-state surface subsidence profiles for various Poisson's ratios

CONCLUSIONS

A fully coupled numerical method has been developed for the analysis of soil subsidence due to extraction of groundwater. This analysis can take into account the drawdown of the water table.

Examples of the extraction of ground water were given for the cases with and without drawdown of the water table. The numerical analysis has indicated that the drawdown of the water table has a significant effect on the pore pressure distribution within the soil and the surface subsidence. Ignoring this effect could lead to gross underestimates of the subsidence.

Extensive parametric studies have allowed the preparation of design charts. Nondimensionalised plots of the surface subsidence and drawdown of the water table have been presented in these charts, which may be used to obtain rapid solutions to problems involving pumping of groundwater in uniform soils.

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