MODEL VALIDATION FOR THE OEDOMETRIC RELAXATION TEST

MODELE VALIDATION POUR L'ESSAI OEDOMETRIQUE A RELAXATION

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SYNOPSIS : A rheological model was elaborated for the evaluation of the oedometric relaxation tests. It consisted of a one-dimensional coupled consolidation part-model and a phenomenological relaxation part-model connected by the principle of linear superposition (Imre, 1991). An analytic solution was developed in form of Fourier series. The initial condition was described by two parametric functions resulting in two model versions. The simulated system responses were linear with respect of two out of seven and eight parameters respectively. Soils with different plasticity were tested (Imre, 1990). The system response was simulated on the domain of the non-linearly dependent parameters, and it was compared with the measured system response. It was concluded, that the time dependency of the effective stress variable could not have been properly described without the relaxation part-model in case of plastic clays.

INTRODUCTION

The multistage oedometric relaxation test is the dual counterpart of the conventional multistage oedometric compression test in terms of loading condition: instead of the total stress the displacement is controlled on the upper surface of the sample. Although both tests provide similar results (compression curve, coefficient of consolidation) very few models for the oedometric relaxation test are known. The empirical model presented by Abelev et al (1981) agrees with data in a definite range of the time variable and soil parameters. Therefore a joined model was elaborated consisting of a coupled consolidation part-model added to a phenomenological relaxation part-model according to the principle of linear superposition (Imre, 1991).

The validation of the model comprised the following steps. (i) Soils with different plasticity were tested (Imre, 1990). Model responses were simulated and the measured and the calculated results were qualitatively compared. (ii) The model was fitted on measured data. Besides of the parameters (Imre et al, 1992) error estimates of the parameters and the statistical measure of goodness-of-fit were determined. The uniqueness of the solution and the identifiability of the parameters were checked.

Some results associated with the first step were presented in this paper. Two model versions were introduced differing in the parametric functions applied as initial condition. Model response was analyzed and simulated on the whole domain of the non-linearly dependent parameters. The measured data and the calculated system responses were qualitatively compared supporting the correctness of the suggested model. It was shown that relaxation had a key role in the case of plastic clays.

MODELLING

The system of differential equations of coupled consolidation was developed using the field equations for saturated soils (Hwang et al, 1971) by the assumption of linear strain state, and linear constitutive relation. Analytic solution in the form of Fourier series was developed (Imre, 1991). Fourier coefficients were determined for two parametric functions regarding the model-versions 'H' and 'E' respectively :

$$u\ (0,y) = A \cdot |y|^3 + B \cdot |y|^2 + C \cdot |y| \tag{1}$$

$$u(0,y\ ) = E \cdot (1 - e^{-\frac{|y|}{F}}) \tag{2}$$

Some features of the consolidation part-model were as follows. Its solution consisted of a transient (underlined) and steady-state (double underlined) part. The transient part is expressed in terms of the pore water pressure function in the following total stress and the effective stress solutions :

$$\sigma^c\ (t) = \underline{\frac{1}{H} \cdot \int_0^H u\ (t,y)\ dy} + \underline{\underline{E_{oed} \cdot \frac{v(0,0)}{H}}} \tag{3}$$

$$\sigma'^c\ (t,H) = \underline{\frac{1}{H} \cdot \int_0^H u\ (t,y)\ dy - u\ (t,H)} + \underline{\underline{E_{oed} \cdot \frac{v\ (0,0)}{H}}} \tag{4}$$

where H is the height of specimen, t time variable, y distance from the top of the specimen, v(0,0) is the displacement load, E_{oed} is the oedometric modulus, superscript c refers to the consolidation part-model solution.

According to the solution of the Laplace Equation under the given boundary conditions the pore water pressure is identically equal to zero at infinite time. It follows that consolidation part-model solutions reduce to their steady-state part that is they change by the initial value of their transient parts. If the parametric functions (1), (2) are characterized by alternative parameters (parameter 'I' can be factored out from both functions, the parameter 'D' is the average ordinate of the resulting dimensionless 'shape function' $\underline{u}$) :

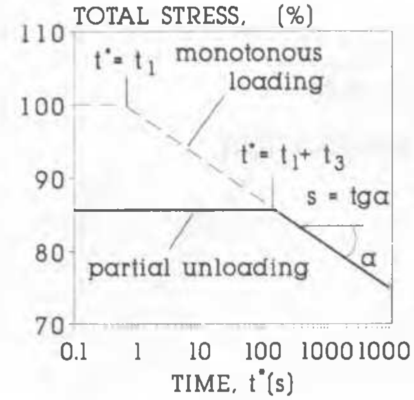

Fig. 1. The total stress-time relation given by the relaxation part-model

$$I = \frac{u(0,H)}{1.666}, \quad D = \frac{1}{H} \cdot \int_0^H u\,(0,y)\,dy \tag{5}$$

then it can be easily derived by the use of the foregoing Equations that decrease of I·D in the total stress and increase of I·(1.666-D) in the effective stress at the bottom occur. The steady state total stress equals the average effective stress due to consolidation for any value of the time variable following from the fact that the volume of the sample is constant. Being the steady-state part of solution identical to the final (constant strain) state it can be considered as the 'instant state' (Bjerrum, 1967).

The relaxation part-model was constructed on the basis of some experimental results including the so called partial unloading effect (Lacerda, 1975, 1977). It resulted in a monotonic total stress decrease for $t > t_3$ (Figure 1):

$$\Delta\sigma^r(t^*) = \begin{cases} -s \cdot \sigma(0) \cdot (1+q) \cdot \log(\frac{t+t_1}{t_1}) - q, & \text{if } t^* \geq t_3 \\ 0, & \text{if } t^* < t_3 \end{cases} \tag{6}$$

where superscript r refers to relaxation, s is the coefficient of relaxation, t_3 is the pause in relaxation, t_1 is the time delay of relaxation (see Fig 1) and $q = s \cdot b / (1 - s \cdot b)$, $b = \log[(t_1+t_3)/t_1]$.
The two part-models were superimposed using transformation of $t^* = t + t_1$ of the time variable of the relaxation part-model.

MATERIALS AND METHODS

Laboratory Testing

The oedometric relaxation test method suggested by Abelev et al (1985) was developed into a multistage procedure (Imre 1990). The load was increased by equal steps (i.e. it was not doubled) using loading strain rates greater than 10^{-4} %/s, and after applying the pre-described load increment the displacement was kept constant for about 10 minutes except the last stage that was longer than the 99% consolidation time.

Eleven multistage oedometric relaxation tests were performed. Geonor type automatic swelling pressure apparatus h-200 A was used in conjunction with a DAS getting 3.5 data/s. Pore water pressure at the bottom, total stress and displacement at the top of the sample were measured. The soil physical parameters of the samples are shown in Table 1.

Table 1. Soil physical properties of the tested soils

Sign	Plasticity index I_P [%]	Liquid Limit w_L [%]	Liquidity index I_L [%]	OCR [-]	Soil type
1	22.8	57.9	0.10	1.05	med. clay
2	17.0	41.7	0.10	1.05	lean clay
3	37.8	63.6	0.20	1.05	fat clay
4	29.9	63.0	0.00	1.05	fat clay
5	41.0	72.0	0.03	1.05	fat clay
6	37.0	64.1	0.00	3.60	fat clay
7	62.8	118.7	0.10	5.40	fat clay
8	6.2	28.5	0.04	3.20	Mo

Sign	Uniformity Coefficient C_U [%]	Dominant Grain Size d_m [mm]	Soil Type
10	2.00	0.12	fine sand
11	2.35	0.28	medium sand

Simulation

Models 'H', 'E' comprised 8, 7 parameters respectively on two out of them they depended linearly. The model response regarding the following stress variables were simulated: total stress, effective stress at the bottom of the sample, the pore water pressure at the bottom of the sample. The simulation was made in the frame of 3 series of numerical tests. Each of them comprised the systematic change of one of the following (group of) non-linearly dependent parameters within their range :

(1) average initial shape function ordinate, D [0.1, 2.5],
(2) coefficient of consolidation, c_v [10^{-1} , 10^{-10} m²/s], coefficient of relaxation, s [10^{-2}, 10^{-1}]
(3) pause in relaxation parameter, t_3 [0, 50 000 s]

The remainder parameters were kept as constant. In the first series the following values were used: initial condition parameter I = 195 kPa, c_v = 7·10^{-9} m²/s, steady-state total stress of 344 kPa, s = 0.08, t_3 = 600 s, t_1 = 168 s. In the second an third series the following values were used: initial condition parameters A = 1.88·10^3 kPa/m³, B = 1.70·10^6 kPa/m², C = 4.52·10^8 kPa/m, c_v = 7·10^{-9} m²/s, steady-state total stress of 344 kPa, s = 0.06, t_3 = 80 s, t_1 = 168 s.

RESULTS

Laboratory Test Results

Results can be seen in Figures 2, 3. The total stress variable refers to a value valid along the sample, the effective stress and the pore water pressure variables refer to values valid at the bottom of the sample in the following. In the beginning of the relaxation test the total stress began to decrease with time delay generally less than 1 s. Then some minute long fast decrease with constant rate occurred in every stress variable entailing temporary failure of the controlling system, resulting in partial unloading of the specimen. The partial unloading in terms of the displacement generally increased with the load. Then rebound in the total stress occurred in some cases.

The model concerned the subsequent so called time dependent relaxation period only. In the beginning of this period the total stress showed 'stagnant'

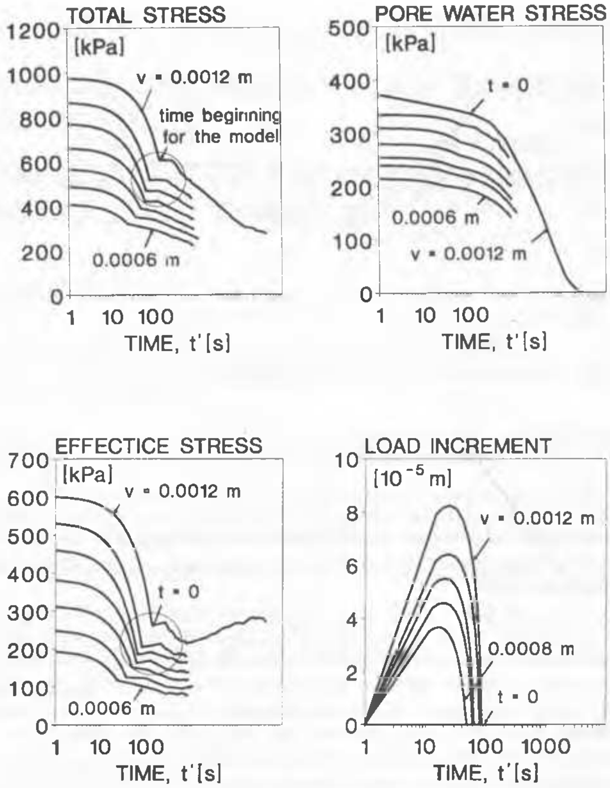

Fig. 2. Results of laboratory test 6

behavior, the pore water pressure decreased and the effective stress increased if partial unloading occurred previously (as indicated by circles on Fig 2). Otherwise this initial stagnant behavior of the total stress was not necessarily be encountered (Fig 3, v=0.6-0.7 mm). Then the total stress and the pore water pressure showed monotonic decrease with time until the end of the tests. Up to the 99% consolidation time the time dependency of the effective stress showed three different pattern: it was either increasing or initially decreasing then increasing or decreasing (Figs 2,3). The more plastic clays showed the more decreasing tendency. The decreasing tendency tended to vanish as the partial unloading in terms of the displacement increased.

In the case of samples 9-11 consolidation occurred during loading and during the time dependent relaxation period the total stress showed a steadily decreasing tendency with time.

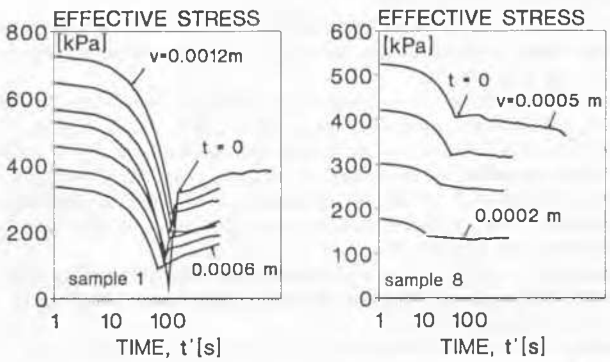

Fig. 3. Results of laboratory tests 1, 8

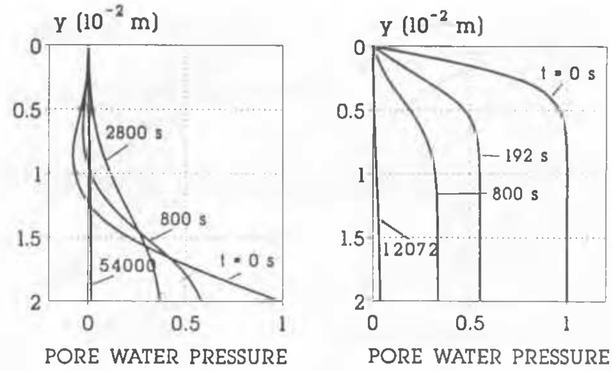

Fig. 4. Results of the first series of numerical tests

Results of Simulation

Results are shown in Figures 4-8. In the beginning of the numerical tests in the case of concave initial condition the pore water pressure increased with time at the upper part of the sample, in the case of convex shape functions decrease was experienced only (see Fig 4). Following from Equation (3), the time variation of the total stress showed a slight increase or stagnant behaviour in the case of concave shape functios in the beginning of the numerical test (Mandel-Cryer effect), however in the case of convex shape functions total stress decrease was encountered only (Figs see 5, 6). The pore water pressure valid at the bottom of the sample u(t,H) showed monotonic decrease with time for every value of D in accordance with the laboratory test results. Difference among the u(t,H) functions vanished at the 99% consolidation time (see Figs 5, 6).

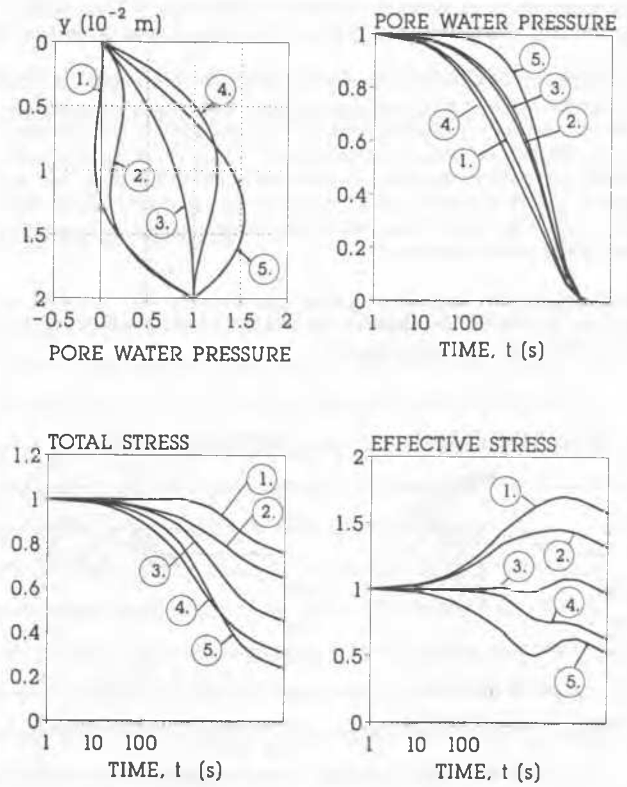

Fig. 5. Results of the first series of numerical tests (model 'H')

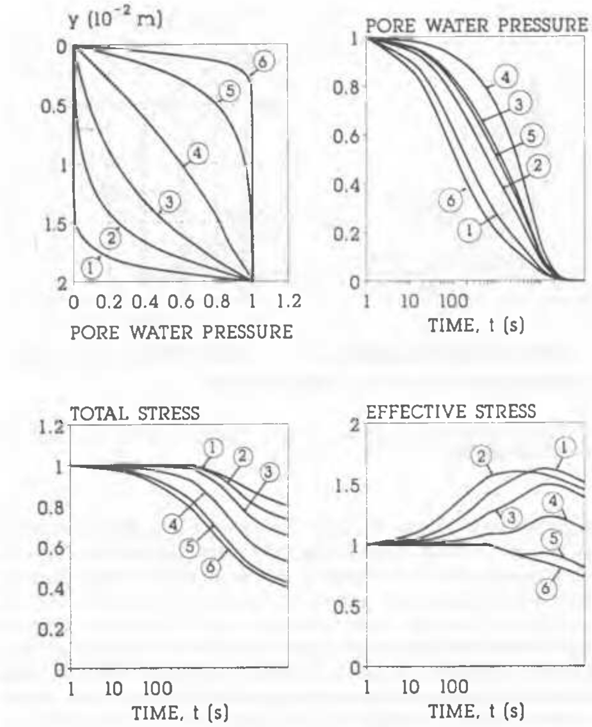

Fig. 6. Results of the first series of numerical tests (model 'E')

As far as the time variation of the simulated effective stress at the bottom of the sample the consolidation part-model resulted in monotonous increase that was temporarily or definitely suppressed by the monotonous stress decrease due to relaxation for some values of the varied parameters as follows.

Up to about the 99% consolidation time monotonous increase with time was experienced in the case of great values of the coefficient of consolidation. However temporary or definite decrease was encountered if the coefficient of consolidation was small (less than about 10^{-7} m^2/s), the initial pore water pressure distribution was convex (the difference 1.666-D was small) and, the pause in relaxation time variable of the relaxation part-model was less than 600 s (see Figs 5-8). These parameters concern plastic soils and no considerable partial unloading.

If value of the time variable was greater than about the 99% consolidation time, the decrease due to relaxation was prevalent in the solution regardless of the actual values of the parameters.

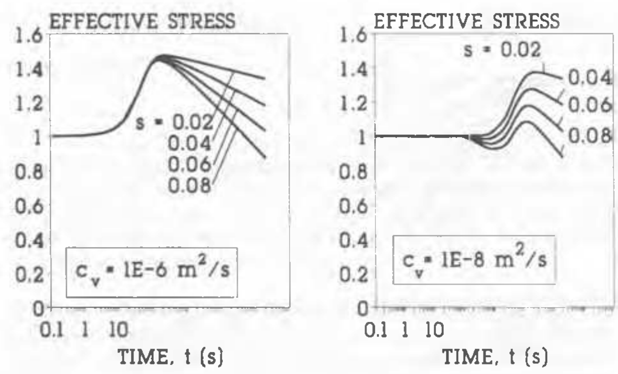

Fig. 7. Results of the second series of numerical tests

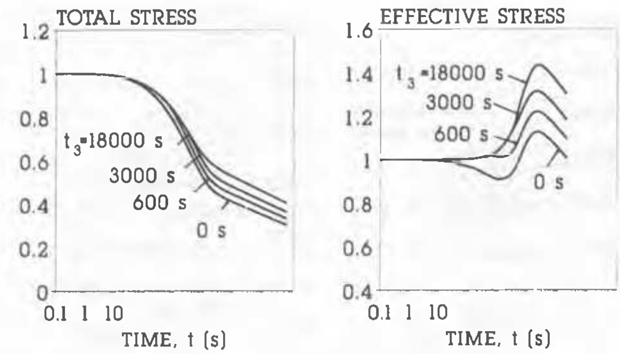

Fig.8. Results of the third series of numerical tests

DISCUSSION, CONCLUSION

In the beginning of the laboratory tests fast decrease in every stress variable was encountered. It can be considered as the dual counterpart of the so called initial compression experienced in the beginning of one-dimensional compression tests.

Time variation of the measured and calculated stress variables were in agreement with the following exception. After partial unloading some minute long stagnancy in the total stress were observed in every case. The simulated total stress showed some minute long stagnancy in case of concave initial condition only. It is very probable that this slight discrepancy can be attributed to the following reason. During unloading the pore water pressure distribution may become concave in the upper part of the sample regardless of the distribution pattern in its lower part, however the shape functions of both model versions are generally either concave or convex. Comparison between experimental and simulated data support the correctness of the suggested model.

Time variation of the effective stress at the bottom of the sample could not have been properly described without the inclusion of the relaxation part-model in the case of plastic clays. For these soils effect of simultaneous creep can probably not be disregarded in the modelling of the one-dimensional compression test.

References

Abelev, M.Y., Kubetski, V.L., Tsoi, A.V. and Meschyan, S.R. (1981).Investigation of Soils Creep. *Proc. of 11th Inter. Conf. on Soil. Mech. and. Found. Eng*. Stockholm, Vol.1, pp. 523-526.

Bjerrum, L. (1967): Engineering geology of normally consolidated marine clays as related to settlements of buildings. Seventh Rankine Lecture. *Geotechnique*,16(2):83-118.

Hwang, C.t., Morgenstern, N. R. and Murray, D. W. (1971). On solutions of Plane Strain Consolidation Problems by Finite Element Methods. *Canadian Geot. Jl.* 8(2):109-116.

Imre, E. (1990). Multistage oedometric relaxation test. *Proc. of the 9th Nat. Conf. on Soil Mech. and Found. Eng.*, Cracow, 1990. Vol. 1. pp. 171-179.

Imre, E. (1991). Evaluation of the oedometric relaxation test. *Thesis (M.Sc, Applied Mathematics)* Mech. Dep. Civ. Eng. Faculty, TU of Budapest

Imre, E. and Berzi, P. (1992). Identification of a soil model for rheological processes. *Proc. of the 2nd Czechoslocak Conf. on Num. Meth. in Geomech.* Vol.1.pp.198-203.

Lacerda, W.A. (1975). Undrained-Drained Stress Relaxation in Clays. *Proc. of 5th. Pan American Conf. on Soil. Mech. and. Found. Eng.*, Buenos Aires, Vol.1.pp.79-86.

Lacerda, W. A. (1977). Stress Relaxation after Partial Unloading. *Proc. of 9th. Conf. on Soil. Mech. and. Found. Eng.. Tokyo*, Vol.1. pp.117-179.