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POST-LIQUEFACTION BEHAVIOUR OF SAND
COMPORTEMENT POST-LIQUEFACTION D’UN SABLE

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SYNOPSIS
Static undrained behaviour of a saturated sand is shown to be dilative in triaxial compression even in the loosest deposited state. The behaviour in triaxial extension, however, is contractive for relative densities of up to 60%, implying a profound anisotropy of response to undrained loading. On monotonic loading, following liquefaction, the sand always responded in a dilative manner even though it was contractive under static loading. The post-liquefaction response represents continuously stiffening behaviour and no approach to any residual strength is noted regardless of the density or effective stress conditions prior to cyclic loading.

INTRODUCTION
Until recent past the major concern during earthquake loading of saturated sands has been to safeguard against the occurrence of liquefaction. If liquefaction was a possibility depending on the initial stress and density state of the sand together with the characteristics of cyclic stresses imposed by the earthquake, remedial measures in the form of densification were specified. No attempt was generally made to estimate earthquake induced displacements. During the last few years, however, several researchers have emphasized a great need for estimation of such displacements. As a consequence some empirical and analytical techniques towards this goal have been proposed (Finn et al., 1986; Hamada et al., 1987; Byrne, 1990; Bartlett and Youd, 1992).

One of the key information required in estimating earthquake induced displacement is the post-liquefaction stress-strain response of sand. Specifically, the response of sand when it undergoes excursions through states of zero effective stress, is needed when modelling spacial progress of liquefaction in a given earth structure. Little research has been carried out on this aspect of sand behaviour. If the sand is contractive under static loading, it has been assumed that its steady state (or residual) strength remains unaltered on monotonic loading following liquefaction induced by cyclic loading (Byrne et al., 1992). No experimental evidence exists in support of such a contention.

This paper presents an experimental study of post-liquefaction behaviour of a sand in the triaxial test. Clearly, this requires a comprehensive investigation of the static and cyclic behaviour which takes the sand to the liquefied state prior to assessment of its post-cyclic behaviour.

The study encompasses static undrained behaviour over a range of deposition densities, from loosest to dense, and a range of confining stresses. Both triaxial compression and extension loading paths are included, in order to assess path dependence of behaviour. Cyclic loading leading to liquefaction is studied at specific targeted densities from loose to dense, and again a range of confining stress levels is used. Finally post-liquefaction monotonic undrained response is studied as it is influenced by factors such as the maximum shear strain due to cyclic loading, relative density, mode of loading and the level of confining stress prior to cyclic loading.

Possible similarity of post-cyclic behaviour between sand brought to the liquefied state by cyclic loading and by static load/unload cycles is also investigated.

EXPERIMENTATION
Tests were performed on Fraser River sand that underlies highly seismic populated Fraser Delta in British Columbia, Canada. The sand used has grain sizes ranging from 0.074 to 0.6 mm and comprises about 98% of the original material dredged from the river. Maximum and minimum void ratios in accordance with ASTM D2049 are 1.00 and 0.68, respectively, and $D_{50} = 0.3$ mm, $C_u = 1.5$. The average mineral composition is 40% quartz, 11% feldspar, 45% unaltered rock fragments and 4% other minerals.

Triaxial specimens were 63 mm diameter x 126 mm high. They were reconstituted saturated by water pluviation. This reconstituting technique yields homogeneous replicable specimens (Vaid and Negussey, 1988) that possess fabric similar to those of water deposited natural or tailings sands (Oda, 1972). Specimens were prepared in the loosest state and then densified, as needed, by low energy high frequency vibrations.

TEST RESULTS AND DISCUSSION
Monotonic Loading Behaviour

Behaviour of loosest deposited sand

Figure 1 shows compression and extension response of isotropically consolidated sand at several levels of confining pressure. Deviator stress is defined as $\sigma_v - \sigma_H$ to facilitate distinction between compression and
extension loading modes. Compression behaviour may be seen to be dilative except under the highest consolidation stress $\sigma_{3c} = 1200$ kPa when it manifests a slightly contractive response. In contrast, the behaviour in extension at each confining stress is contractive. At lower levels of $\sigma_{3c}$ including 200 kPa, the response is of the true liquefaction type with steady state conditions realized at axial strain of 2 to 3%. At higher $\sigma_{3c}$, the behaviour changes to the limited liquefaction type (Castro, 1969). The material did not show any tendency towards necking even after loaded to a strain level of up to 9% in extension.

The test results shown in Fig. 1 indicate the effect of confining stress on response in compression is opposite to that in extension. In compression increasing $\sigma_{3c}$ results in a less dilative behaviour, eventually turning into contractive response at the highest $\sigma_{3c}$ used, whereas the effect of increasing $\sigma_{3c}$ in extension is to cause a less contractive response. The effect of increasing $\sigma_{3c}$ at a given placement density $D_{rc}$ is to densify the sand. Increased density promotes less and increased $\sigma_{3c}$ more contractive response. The results in Fig. 1 thus suggest that whereas the effect of increasing $\sigma_{3c}$ offsets the effect of densification in compression, the opposite is true during extension loading.

That pluviated sand is contractive in extension over a much larger range of consolidated $D_{rc}$ than in compression has been demonstrated by several investigators (Bishop et al., 1965; Miura and Toki, 1982; Kuerbis and Vaid, 1989). Loosest Fraser River sand is dilative in compression except under very high confining stress. Inherent anisotropy in pluviated sand has been considered responsible for the differences in compression and extension behaviour for a given initial state.

### Behaviour at fixed density states

The undrained static response of the sand at three fixed consolidated relative densities, $D_{rc}$ of 19%, 40% and 59% was assessed. This enables isolation of the effect of confining stress alone on stress-strain response. A clear evidence of lesser dilative behaviour was noted in compression with increasing confining stress at each density state. In extension, however, the contractive response of the true liquefaction type at lower $\sigma_{3c}$ changes to limited liquefaction type with increase in confining stress. This amounts to a decreased contractive tendency not commonly associated with increasing confining stresses. The contractive behaviour is exhibited by Fraser River sand over a large range of (loosest to more than 50%) placement density in extension loading. But in compression loading, the contractive response was associated with only the loosest depositional density.

### Steady state and phase transformation (PT) state

Effective stress conditions at steady state or phase transformation (PT) state were found to lie on unique straight lines passing through the origin regardless of relative density, initial stress state ($\sigma_{3c}$ and $K_0$), type of response (contractive or dilative) and mode of loading. These lines have equal slope implying that the friction angle at steady state and PT state for both contractive and dilative response are equal (at 32°) and independent of the mode loading as was earlier observed by Vaid and Chern (1985) and Vaid et al. (1990) for other sands.

Figure 2 shows the relationship between undrained shear strength $S_{PT}$ (= $\sigma_d$/2) at PT or steady state and consolidated void ratio for a range of initial confining stresses for contractive response in extension loading. At a given $\varepsilon_c$, the shear strength at PT or steady state, $S_{PT}$ or $S_{SS}$, increases with initial confining stress. This is contrary to the common belief based on compression loading that this shear strength is a function only of the consolidated void ratio. For Fraser River sand, no such unique relationship between $S_{PT}$ or $S_{SS}$ and $\varepsilon_c$ exists, but different relationships exist, each characteristic to a given $\sigma_{3c}$. For a given $\sigma_{3c}$ the initial static shear stress however does not influence $S_{PT}$ or $S_{SS}$ versus $\varepsilon_c$ relationship.

### Static unloading behaviour

Figure 3 shows static undrained loading-unloading behaviour of sand at $D_{rc}$ = 19% and $\sigma_{3c} = 100$ kPa. Its response is dilative in compression and contractive of the true liquefaction type in extension. It may be noted that on unloading of the shear stress both specimens liquefy, i.e.
end up in a state of $\sigma'_3 = 0$. This type of behaviour will later be referred to as liquefaction induced by a static load/unload cycle as opposed to liquefaction induced by cyclic loading.

A static loading/unloading cycle did not always result in a state of $\sigma'_3 = 0$, upon unloading. This situation occurred in dense sands ($D_{rc} = 59\%$) consolidated to $\sigma'_3c = 400$ kPa and higher in the compression mode of loading and was apparently due to an insufficient level of strain prior to unloading. It was found that the straining must surpass a minimum level beyond PT state before a state of zero effective stress state is realized on unloading.

**Cyclic Loading Behaviour**

This behaviour was assessed for three targeted relative densities, and at three levels of confining stress $\sigma'_3c$ on isotropically consolidated sand. Cyclic loading resistance (or resistance to liquefaction) is defined as the cyclic stress ratio $\sigma_{cyc}/2\sigma'_3c$ that causes $\varepsilon^\prime = 2^{1/2}\%$ single amplitude axial strain, $\varepsilon_{ax}$, in 10 cycles. Strain development due to cyclic loading occurred as a consequence of either contractive deformation or cyclic mobility depending upon the initial state $D_{rc}$, $\sigma'_3c$ and the associated $S_{PT}$ together with the amplitude of cyclic stress $\sigma_{cyc} = \sigma_{cyc}/2$ (Castro et al., 1982; Vaid and Chern, 1985).

The $K_0$ value (the ratio of cyclic resistance at $\sigma'_3c$ divided by this resistance at $\sigma'_3c = 100$ kPa) for Fraser River sand is shown in Fig. 4 together with data from the literature on other sands (Seed and Harder, 1990). The correction factor $K_0$ may be seen to be a function of both confining stress and relative density. The resistance to liquefaction reduces by a maximum of about 20% at the highest confining stress 1200 kPa for the dense 59% relative density. Most of the reduction occurs between $\sigma'_3c = 100$ to about 600 kPa, and it seems that not much further drop in $K_0$ is likely for confining stresses in excess of 600 kPa.

There is a wide range of reported $K_0$ values at a given confining stress (Fig. 4). Lumping data without regard to relative density state may contribute to this large $K_0$ range. For granular materials for which relative density has been specified, such as Sacramento River sand, a tailing sand and Ottawa sand (Vaid et al., 1985), a clear decrease in $K_0$ with relative density may be seen.

It may be pointed out that for loose sands which have the largest susceptibility to liquefaction, $K_0$, is approximately unity regardless of the confining stress level. Adoption of lower values in design based on some average value using the body of data in Fig. 4 would thus lead to a conservative design.

**Residual condition at the conclusion of cyclic loading**

When cyclic loading was terminated, a state of zero effective stress ($\sigma'_3c = 0$ or 100% pore pressure ratio) was realized in most cases except for specimens at medium and dense relative densities (40% and 59%) under confining stresses of 400 kPa and larger. This state of $\sigma'_3c = 0$ occurred for the first time following conclusion of the last loading cycle in which the specified $\varepsilon_{ax} \geq 2.5\%$ developed. A typical example is shown in Fig. 5. The last half cycle of extension loading may be seen to cause liquefaction by contractive deformation and the unloading phase brings the specimen to the $\sigma'_3 = 0$ state. Thus excursion through a transient state of $\sigma'_3 = 0$ did not take place before the specified level of strain development defined as liquefaction occurred. Even for cyclic tests in which residual finite effective stress remained at the conclusion of cyclic loading, the strain development as a result of cyclic loading was not due to any excursion through a transient $\sigma'_3 = 0$ state. Such states would have been achieved only if further loading cycles were applied. In that case, $\varepsilon_{ax}$ much larger than the specified 2.5%, would have developed.

A state of zero effective stress was also not realized by a static load/unload cycle in medium and dense sand unless strain during loading exceeded a certain minimum value. At the conclusion of cyclic loading, dense and medium dense specimens at $\sigma'_3c \geq 400$ kPa developed a strain level of only 2.5% to 5%. Static load/unload tests indicated that a minimum strain level of about 7% was needed during loading in order to achieve a $\sigma'_3 = 0$ state on unloading.
Post-Cyclic Undrained Monotonic Behaviour

The residual strain at the end of cyclic loading was always extensional. This was because the specified strain \( \varepsilon_a \geq 2.5\% \) during the last cycle developed on the extension side at the instant of peak stress amplitude or contractive deformation in extension, and little strain recovery occurred when the shear stress was unloaded to zero. Post-liquefaction behaviour of test samples that ended with residual states \( \varepsilon_a = 0 \) at the conclusion of cyclic loading is presented first.

Stress strain response

Figure 5 showed that the response during the last loading cycle also shows the post-cyclic monotonic stress-strain response of loose sand under \( \sigma'_{3c} = 100 \text{ kPa} \) prior to cyclic loading. Axial strains shown are based on the sample configuration prior to cyclic loading. During post-liquefaction undrained loading, the sand deformed at virtually zero stiffness over a large range of axial strain (almost 20%). With further straining, the stiffness increases with increase in strain. This stress-strain response in which the modulus increases with increase in axial strain is opposite to the usual response of soil. The unusual stress-strain response of the liquefied soil results from the fact that, upon shearing the soil dilates all the way causing the effective stress to increase. The deformation progresses at a mobilized friction angle (36°) that equals the angle of maximum obliquity under static loading. The stress-strain curve after some axial strain becomes essentially linear and there is no tendency towards approaching a residual strength even after a post-liquefaction strain of about 32%. Under static loading, this sand would behave contractive in extension with a residual (steady state) strength of only 5 kPa. In compression, however, it would be dilative.

Dependence on relative density

Figure 6 shows post-liquefaction monotonic response at three relative densities for a fixed \( \sigma'_{3c} = 100 \text{ kPa} \) prior to cyclic loading. To facilitate comparison the response of dense sand is taken as the reference curve and stress-strain curves at other relative densities are horizontally shifted so as to match \( \sigma_d = 5 \text{ kPa} \) point on each curve. It may be noted that stiffness increase beyond this 5 kPa point increases as the relative density increases and the axial strain at which the curves become practically linear decreases with increase in relative density. The post-liquefaction deformation proceeds along the average line of maximum obliquity observed under static loading \( (\phi' = 36°) \) regardless of the relative density state. The region of strain over which the stiffness is close to zero is less for dense than for loose sand.

Comparison with behaviour following \( \sigma'_{3} = 0 \) induced by static loading and unloading

In Fig. 7, typical response of the sand following the state of \( \sigma'_{3} = 0 \) induced by a static load/unload cycle is compared with that of sand which was brought to a state of \( \sigma'_{3} = 0 \) by cyclic loading. As in the previous section, the response of cyclically loaded sample is taken as the reference curve and the response curve of statically loaded samples were horizontally shifted so as to match \( \sigma_d = 5 \text{ kPa} \) point on each curve. The post-liquefaction responses may be noted to be essentially similar at each relative density regardless of the manner by which the state \( \sigma'_{3} = 0 \) was brought about. Thus a convenient way of obtaining post-liquefaction response would be to use static loading and unloading instead of cyclic loading to induce liquefaction \( (\sigma'_{3} = 0) \).

Comparison of pre-cyclic and post-cyclic behaviour

Pre-liquefaction and post-liquefaction stress-strain response of the sand at \( D_{rc} = 40\% \) and \( \sigma'_{3c} = 100 \text{ kPa} \) is compared in Fig. 8. The stiffness of the sand decreases with increase in strain until the phase transformation state for the sand subjected to pre-liquefaction static loading, but it continuously increases with strain during post-liquefaction monotonic loading. The contractive behaviour during pre-cyclic loading has been eliminated as a consequence of cyclic loading. The post-cyclic stress-strain response is always dilative, the stiffness is very small during the initial phase of loading, but with increase in strain level, the stiffness essentially becomes equal to the pre-cyclic value in the post-PT region. No indication of any residual strength condition on post-cyclic loading is apparent regardless of density state or the mode of loading.

Range of post-liquefaction behaviour

The post-liquefaction undrained stress-strain curve can be characterized into three distinct regions (Fig. 5). Region 1 spans from the state at which \( \sigma'_{3} = 0 \) at the end of cyclic loading until a measurable \( \sigma_d = 5 \text{ kPa} \) develops on some shearing. This is the region with very small stiffness. The size of this region decreases with increase in relative density. Region 2 commences from \( \sigma_d = 5 \text{ kPa} \) and can be approximated as a parabolic curve representing continuously increasing...
stiffness with axial strain. Region three corresponds to the linear segment of the stress-strain curve, and thus a constant modulus.

An approximate length of region 1, may be taken as the axial strain increment \( \Delta e \) from the point at which \( \varepsilon_{\text{max}} \) developed in bringing about liquefaction until the point at which a measurable \( \sigma_d = 5 \) kPa is recorded. As shown in Fig. 9, \( \Delta e \) is a function of the maximum strain amplitude \( \varepsilon_{\text{max}} \) developed during cyclic loading which decreases with increasing density. Average values of \( \Delta e \) for the sand tested are respectively 20%, 3.5% and 2% for 19%, 40% and 59% relative density states.

Figure 10 shows the range of post-liquefaction compression stress-strain response at relative densities of 19%, 40% and 59%. Only the curved portions of the response that start at \( \sigma_d = 5 \) kPa and extend to the beginning of the linear segments are shown. As before, all curves are translated horizontally so as to merge at \( \sigma_d = 5 \) kPa. It may be noted that there seems a definite trend for the response to become stiffer with increase in confining stress for the sand at loose relative density. But as the relative density increases, the effect of confining stress is not so apparent.

The stress-strain response in region 3 is a straight line in all cases. Its slope increases with increase in relative density. For the loose sand the slope of the line is somewhat larger for higher confining stress, but no such dependence seems to exist at higher relative densities.
Figure 10 also shows the range of post-liquefaction monotonic behaviour in extension. Both deviator stress and axial strain are taken positive for direct comparison with compression behaviour. Unlike the compression behaviour, loose sand does not appear to show specific dependence of behaviour with the level of confining stress prior to liquefaction. The spread of stress strain curves at each relative density is of the same order as observed under compression loading. It may, however, be noted that at each relative density, extensional response is substantially softer than compressional response, implying stress path dependent post-liquefaction behaviour.

Behaviour if residual state $\sigma'_3 \neq 0$

Clearly, the post-cyclic behaviour for a given $\sigma'_3c$ prior to cyclic loading will depend on the magnitude of residual $\sigma'_3$ remaining after cyclic loading. This is illustrated in Fig. 11 for $D_{rc} = 40\%$, at $\sigma'_3c = 400$ kPa. For post-cyclic residual states of $\sigma'_3 = 8$, 25 and 45 kPa, the sand had liquefied (developed axial strain between 3.5 to 3.7%). However, the maximum strain developed for the residual $\sigma'_3$ states of 105 and 175 kPa was less than 0.4% and hence, according to the definition the sand did not liquefy. An exploded view of the stress strain response in the earlier stage is shown in the inset.

It may be noted in Fig. 11 that regardless of the level of residual $\sigma'_3$, the post-cyclic stress-strain response is similar to the pre-cyclic, in that the modulus first decreases with strain before it starts to increase following the initiation of dilation corresponding to the PT state. The stress strain curves move progressively higher and axial strain until phase transformation state decreases as the level of residual $\sigma'_3$ increases. Thus the undrained stress strain behaviour of sand at deformation levels typical of concern during earthquakes does not correspond to moduli that continually degrade with strain as assumed by some researchers while carrying out effective stress analysis of earthquake problems (Finn et al., 1986). Depending on the residual effective stress state, the modulus on loading does decrease initially, but it starts increasing once dilation commences.

CONCLUSIONS

Fraser River sand a medium uniform sand, responds in static undrained loading in a dilative manner even in the loosest deposited state under confining stresses up to 1000 kPa. In extension, however, the sand behaves contractive over a range of densities - from loose to just under 60% relative density, implying a strong direction dependent behaviour.

On post-liquefaction monotonic loading, the sand that developed 100% pore pressure ratio, deformed initially with essentially zero stiffness which then increased with the level of strain, until at some strain level it became constant. The rate of build up of deviator stress with strain increased with relative density, and the strain at which stiffness became essentially constant decreased with increase in relative density. The pre-cyclic contractive response was eliminated as a consequence of cyclic loading phase and thus no indication of any residual strength condition on post-liquefaction loading was apparent regardless of density state or the mode of loading.

A convenient way to assess post-liquefaction behaviour is to liquefy sand by a static load/unload cycle instead of cyclic loading.

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