SOME RESULTS OF THE COUPLED CONSOLIDATION PROBLEM OBTAINED WITH A PROGRAMME USING FINITE AND INFINITE ELEMENTS

QUELQUES RESULTATS POUR LE PROBLEME DE LA CONSOLIDATION LIE, OBTENUES AVEC UN PROGRAME USAND ELEMENTS FINIS ET INFINIS

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SYNOPSIS: This work presents some results obtained with a programme using finite and infinite elements which allows the analysis of the coupled consolidation process. Such programme was based on the theorectical formulation published by Ding and Naylor (1989). Two examples are presented comparing the performances between meshes using only finite elements and meshes using both finite and infinite elements.

1. INTRODUCTION

Since Terzaghi (1925) proposed a simple mecanism which supposed the soil as a porous material with a linear elastic skeleton, fully saturated with water and when loaded it deforms according to the water quantity drained by its pores, the consolidation phenomenon has been studied by many authors (Biot (1941) (1955), McNamee (1960, Gibson (1970)).
Among these authors a special reference is deserved to Biot who developed differential equations which allow to calculate at the same time stress, deformations and pore water pressures.
Based on his equations not only analytical solutions for problems with simple geometry and boundary conditions (McNamee (1960), Gibson (1970), Yamaguchi (1973)), but also numerical solutions based on the finite element method for more complicated problems (Sandhu (1969), Booker (1973), Krauss (1978), Kanok-Nukulchal (1982), Ding (1989)), have been developed.
As there are problems where the domains are considered infinite if one uses finite elements only it is necessary to truncate the mesh far from the analysis area.
To avoid this difficulty many authors (Simoni (1987), Schrefler (1987), Selvadurai (1989)) use infinite element meshes in order to continue the domains to infinite.
This work presents two examples concerned with the consolidation process and using both finite and infinite elements.

2. FORMULATION

The equations obtained by Ding and Naylor (1989) using finite elements in relation to space and finite diferences in relation to time are the following:

$$\begin{bmatrix} K & C \\ C^T & -H\alpha\Delta t \end{bmatrix} \begin{Bmatrix} r_u\ (t+\Delta t) \\ r_p\ (t+\Delta t) \end{Bmatrix} = \begin{bmatrix} o & o \\ C^T & H(1-\alpha)\Delta t \end{bmatrix} \begin{Bmatrix} r_u(t) \\ r_p(t) \end{Bmatrix} + \begin{Bmatrix} R_u \\ R_p \end{Bmatrix} \quad (1)$$

where
K = stiffness matrix of the solid phase;
C = coupling matrix;
H = flow matrix
Δt = time increment;
r_u = vector of nodal displacement;
r_p = vector of nodal pore pressures;
R_u = vector of nodal equivalent forces;
R_p = vector of nodal applied fluxes;
α = time integration parameter.

As the unit value for the α parameter gave good results in a previous work (Martins (1991)) in this paper we used the α=1.
Two types of finite elements are used:
i) standard quadrilateral elements of 8 nodes with quadratic interpolation not only for pore pressures but also for the displacements (8-8 type elements)
ii) "hybrid" elements with 8 nodes for the displacements (quadratic interpolation) and 4 nodes (linear interpolation) for the pore pressures (8-4 type elements).
The corresponding infinite elements of 5 nodes (Marques (1984) (5-5 type elements) using all of them for pore pressures and displacements for 8-8 type elements and infinite elements 5-2 type with 5 nodes for displacement and 2 nodes for pore pressure are used.

3. RESULTS

1st. Example
In this example is studied the consolidation of a semi-infinite medium loaded in a infinite strip with a permeable boundary

surface and an impermeable and smoth rigid bottom boundary (Fig. 1) at depth H.

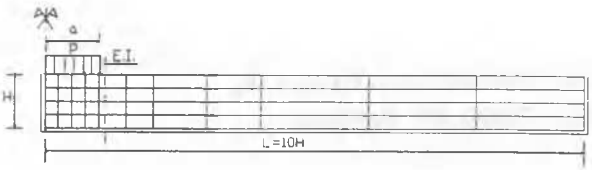

Fig. 1 Finite element network for the 1st. example

The parameters used are the following:

E = 15000 kN/m^2
ν = 0
k = 2 x 10^{-7} m/s
p = 1 kN/m^2
a = H = 1.6 m

$$\bar{c} = \frac{2\ GK}{\gamma_w} = 1.0714$$

$$T' = \frac{\bar{c}}{H^2} t = 0.4185\ t$$

One mesh of 44 finite elements (44F) using 8-8 or 8-4 type elements (44F) was used (Fig. 1). Another mesh of 16 finite elements and 4 infinite elements (16F4I) using either 8-8 and 5-5 type elements or 8-4 and 5-2 type elements was used (Fig. 1).

The results of the displacement of the point under the center of the loaded area against the time factor T' are given in Fig. 2.

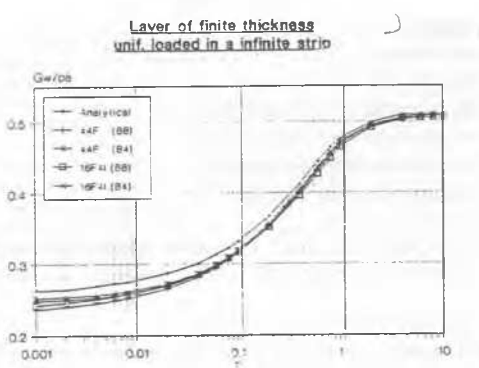

Fig. 2 Displacement of the point under de center of the loaded area against the time factor.

The results obtained with the two meshes using different kind of finite elements are not much different.

However, the best results were obtained using the mesh 16F4I (84). Comparing the CPU times obtained (Table 1) we see that it is exactly with the mesh that have smaller number of degrees of freedom and lower CPU time that we have the best results. This mesh use infinite elements and 8-4 finite elements.

Table 1. Used meshes and CPU times

1st. Example		2nd Example	
Mesh	CPU time (s)	Mesh	CPU time (s)
44F (8-8)	602	19F3I (8-4)	424
44F (8-4)	295	19F3I (8-8)	874
16F4I (8-8)	210	29F (8-4)	694
16F4I (8-4)	107	29F (8-8)	1443

2nd. Example

In this example is studied the consolidation of a half space loaded with a pressure equal to 1kN/m^2 applied in a infinite strip 2m wide. The meshes are represented in figure 3 and 4.

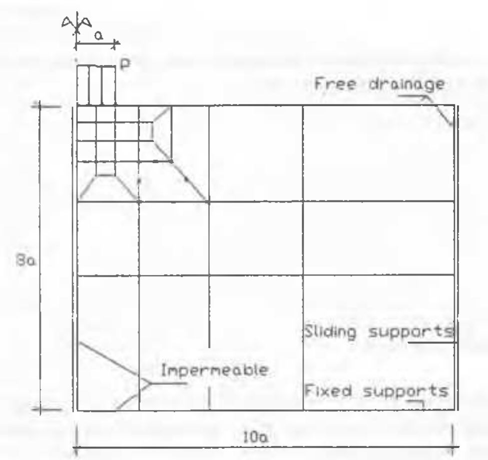

Fig. 3 Finite element netwok for the second example using only finite elements.

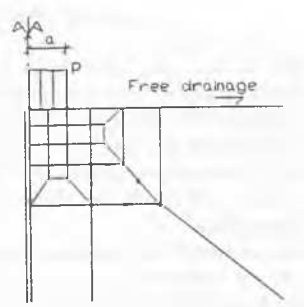

Fig. 4 Finite element network for the second example using both finite and infinite elements.

As we can see the mesh in figure 3 only uses finite elements (29) and the truncated space is symmetric in relation to a vertical plane and has the dimensions of 8m in depth and 10m wide. The mesh in figure 4 uses both finite (19) and infinite elements (3).

The soil parameters are as follows:

E = 15000 kN/m^2

ν = 0.0

k = 0.7 x 10^{-3} m/h

the results are given in figures 5 and 6.

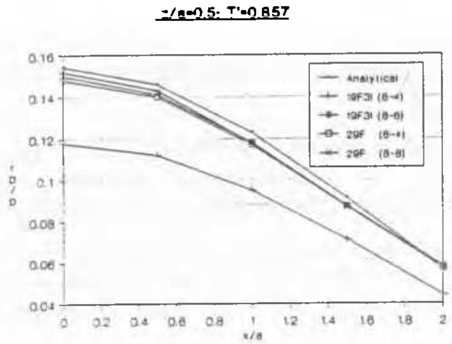

Fig. 5 Pore pressures on points situated on an horizontal line 0.5m under the loaded area for T' = 0.857.

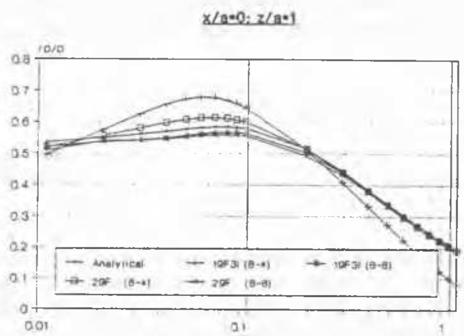

Fig. 6 Relationship between pore pressures and time factor in a point situated 1m below the central point of the distributed load.

In figure 5 the best results are obtained with the mesh 19F3I using 8-8 and 5-5 type elements and the worse results with the mesh 19F3I using 8-4 and 5-2 type elements. The same conclusion is obtained looking at the figure 6.

4. CONCLUSIONS

Considering the results of the two examples presented one can obtain the most satisfatory results using meshes with both 8-8 and 5-5 type elements. Such finite-infinite element combination also allows to save many CPU time in relation to meshes using only finite elements.

5. ACKNOWLEDGMENTS

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