# SOLDIER PILE ANALYSIS USING NONLINEAR BEAM-FOUNDATION THEORY

# ANALYSE D'UNE BARRETTE EN UTILISANT LA THEORIE DE POUTRE ET FONDATION NON-LINEAIRE

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SYNOPSIS : Existing theory for the behaviour of an elastic beam on elastic Winkler foundation is extended to allow for inelastic response of both the soil and the beam. A method, which takes account of plate load tests and the stress-strain characteristic for the beam, is incorporated in a computer program allowing the analysis of beam-soil problems, in particular those of soldier piles used in excavation support systems.

The soldier pile analysis requires further modification to the horizontal beam-soil theory to include a two-layered soil foundation system which correctly models the in-situ soil and the backfill material around the soldier.

Analysis of laboratory and field tests shows the method to be sufficiently accurate in predicting interaction between the soil and beam (or pile) to warrant further field tests after which the method could be adopted for routine design.

## INTRODUCTION

In recent years, a number of deep basements have been excavated in residual soils in the Johannesburg area. Many of the shallower basements (less than 8m deep) are supported using soil nails whereas multilevel anchored soldier pile walls are used for deeper excavations (10m - 20m). In most cases, the residual soils have sufficient cohesion for the face to be left unsupported between the soldier piles with timber lagging or shotcrete cladding being provided over only the upper 2 - 3m of the face. The anchors thus bear directly on the soldier piles which distribute these loads to the face of the retained soil.

In designing the support system, the required anchor force is usually calculated by analyzing the stability of the active wedge and the length of the anchors is governed by overall stability considerations. However, the methods used for the design of soldier piles are widely divergent. The application of Terzaghi and Peck's (1967) earth pressure diagrams results in uneconomical, even wasteful pile sections. This has lead many designers to use beam on elastic foundation methods to assess bending moments and shear forces in the piles. In recent years, further savings have been achieved by using plastic design methods.

In the endeavour to rationalise the design of soldier piles, a new design method is being researched which combines the subgrade reaction method with principles of the ultimate limit state (plastic) design. The validity of this design approach has been investigated both in the field and in the laboratory.

This paper describes the analysis procedure and presents the results of some analyses comparing experimental observations with predicted solutions.

## THEORETICAL ANALYSIS

### Beam On Elastic (Winkler) Foundation

Basic analysis of steel or concrete structures supported by an elastic foundation (or subgrade) comprises the determination of stresses and displacements resulting from the imposition of loads upon the structure. Winkler (1867) proposed that the soil behaviour be modelled as a spring, resulting in a linear relationship between the pressure applied to the soil and the consequent displacement of the subgrade at the point of load application.

Using this analogy, a continuous elastic foundation can be described as consisting of closely spaced, independent linear springs uniformly distributed along the length of the beam. Friction between the beam and soil is not considered, and the spring model is still assumed to apply even when the beam lifts off and separates from the soil.

In the Winkler foundation model, the compressibility of the subgrade is characterised by the subgrade modulus, $k_0$ ($F.L^{-3}$), the pressure which, when applied to the surface of the subgrade, will cause a unit displacement.

Many researchers have proposed related models for describing the beam-soil interaction. Pasternak (1954) extended the Winkler model by assuming the existence of shear interactions between the springs. Previously, Filonenko-Borodich (1940) assumed the top of the springs were connected to an elastic membrane and subjected to a constant tension field.

Winkler's model is widely used in the analysis of linear foundations and numerous investigators (Hayashi, 1921; Hetenyi, 1946 and others) have presented numerical solutions to the differential equation for the deflection of a beam on elastic subgrade with a variety of boundary conditions

$$EI\frac{\delta^4 y}{\delta x^4} + ky = q \tag{1}$$

where $k$ is $k_0$ multiplied by the width of the beam.

## Nonlinear Foundations

Based on the general solution of equation (1), Eisenberger et al (1985) developed terms for a stiffness matrix to enable analysis of beam-soil interaction problems using a matrix method for structural analysis (Gere et al, 1965). This method requires the beam to be defined as of a number of finite segments (elements).

To model nonlinear soil behaviour using the stiffness matrix method, Yankelevsky et al (1989) proposed an iterative solution which utilises a piecewise linear force-displacement ($F.L^{-1}$) curve for both tensile and compressive behaviour of the beam-soil interaction system. The need for inclusion of a tensile contact pressure arises from the fundamental assumption that the beam and soil remain in contact even if the beam lifts off the soil.

An initial solution (first iteration) is performed assuming a constant subgrade modulus (stiffness) for each element. The resulting deflections identify the portions of the foundation in tension or compression. The appropriate force-displacement relationship is then used in assessing whether such portions require modification of their stiffness values. If there is a region where the beam has displaced by an amount incompatible with the stiffness assigned to it, that region is subdivided at the points of incompatibility (transition points) and each subdivision assigned an appropriate stiffness value. A new stiffness matrix and load vector are formed and another solution obtained. The iterative process continues until the locations of transition points do not change by more than a predetermined amount.

While this nonlinear, iterative approach of evaluating the foundation modulus relates to a rather theoretical force-displacement relationship for the soil, a more direct assessment of the subgrade reaction upon loading can be incorporated which greatly simplifies the analytical procedure. Making use of the relationship between soil pressure and displacement as established using the plate load test (a common field test to determine the compressibility of the soil), the relevant stiffness can be determined directly for any given displacement without the need for a piecewise linear approximation.

For a given pressure ($q$), and displacement ($d$) the stiffness or foundation modulus is

$$k = \frac{q}{d} \times b \tag{2}$$

where b is the width of the beam.

Adopting the stiffness matrix derived by Eisenberger et al (1985), a computer program was developed to analyze elastic beams on inelastic (non-linear) soil foundations as a first step towards a more specific design procedure for soldier pile walls (Howie, 1991). In the program, the beam is modelled as a planar structure with two degrees of freedom only, namely the deflection, $\delta_z$, in the direction of support, and rotation, $\theta_y$, normal to the X-Z plane.

## Plastic Beam Behaviour

The direct stiffness approach can be extended to include yielding of the beam material and subsequent formation of plastic hinges. During each iteration, in addition to consideration of inelastic soil response beneath the beam elements, the bending stresses at the ends of each element are evaluated. Reference is made to the stress-strain characteristic for the beam material to determine whether the yield stress in the beam flange has been exceeded. Where such a condition has occurred, plastic deformation has taken place and a plastic hinge is inserted at the appropriate end of the associated element for subsequent iterations.

As with the handling of nonlinear soil behaviour, the stiffness of the beam material can be adjusted for each element to reflect nonlinear behaviour of the beam material by altering the Young's modulus in the stiffness matrix of the appropriate element. This is done by means of a piecewise linear approximation of the stress-strain characteristics of the beam material.

The stress-strain relationship for structural steel is generally known and the pressure-displacement relationship for the supporting soil is established in the field. Thus, the input parameters for the analysis can be readily determined.

## Soldier Piles: Beam On Vertical Foundation

Soldier piles, in the context of a lateral support system, can be considered as the equivalent of a horizontal beam on a soil foundation rotated through 90°. The analytical model, however, requires the following modifications:

- Due to an increase in overburden stress with depth, an increase in the soil modulus along the beam needs to be considered.
- The soldier piles, usually comprising one or more steel sections, are often installed in predrilled holes which are then backfilled with a low strength soil-cement mixture. The soil foundation thus consists of a two-layer system (backfill and soil), each layer of different properties.

In applying the design method to the soldier pile case, an increase of the soil modulus with depth was not considered.

In-situ loading tests of the backfill material to establish a representative stiffness are regarded as impractical and an engineering estimate is needed. For the purposes of this analysis, the backfill material was considered to be more compressible than the natural soil. The stiffness of the natural soil behind the backfill can be readily determined by means of horizontal plate load tests in which the load is applied normal to the excavation face.

To integrate both the measured pressure-displacement relationship for the in-situ material and the estimated response for the backfill material into the analysis method, the following conceptual relationship was applied: The horizontal stress at the interface of a backfill-only support medium was evaluated for a strip (beam) load assuming linear elastic half-space conditions. In an iterative scheme, the load was steadily increased to obtain a stress, at a distance representing the surface of the in-situ material, sufficient to cause a noticeable displacement (within test accuracy, here taken as 0.5mm) in the in-situ material if it were present. This load in turn allowed the calculation of the displacement of the low-strength backfill material, denoted as $z_i$.

Once the deflection of the pile into the backfill material resulting from an applied load exceeds $z_i$, the in-situ soil will begin to resist further pile deflection. This reaction is determined according to the relevant pressure-deflection relationship between the backfill and in-situ material.

For analysis, both pressure-displacement curves are simply combined into a single curve with a point of discontinuity where the relationships join together. In Fig. 1, backfill and in-situ material response curves are shown in the global co-ordinate system (q,d), while the plate load data of the original in-situ soil is local to (q',d'). In this case, the backfill was assumed to be more compressible than the natural soil. In cases where the backfill is significantly stiffer, it may be sufficient to base the analysis on the stiffness of the natural soil alone and merely to modify the width of the beam to that of the backfill around the soldier pile.

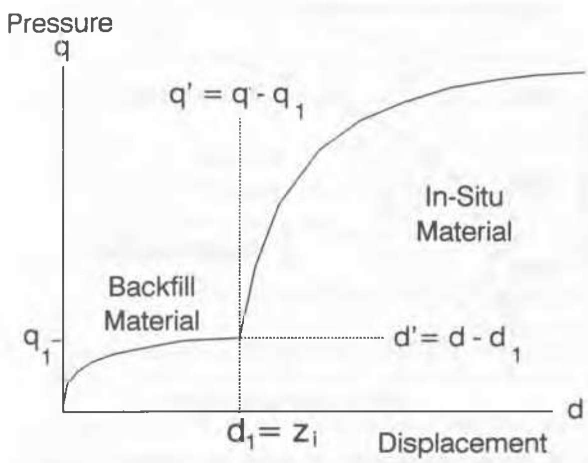


**Fig. 1.** Conceptual backfill and in-situ pressure-displacement response

Loss of contact or even separation of soldier pile and excavation face was ruled out. The bottom end of the pile is generally embedded below the excavation level and the free top end of the pile resists the lateral earth pressure. Only in cohesive materials can this end theoretically deflect independently of the soil. A tensile pressure-displacement relationship was therefore not considered in the nonlinear beam-foundation analysis.

## LABORATORY AND FIELD VERIFICATION

### Laboratory Tests: Beam On Horizontal Foundation

A series of load tests of horizontal beams was performed on beams placed on a soil foundation in the laboratory. The results of one such test are presented below.

The test was conducted using a 2,7m I-beam (100mm x 50mm, $I = 0,32 \times 10^{-6} m^4$) with free end conditions resting on a uniform sand which was compacted to a uniform dry density $\gamma = 15,2 kN.m^{-3}$. The natural moisture content of the soil was 6,75%. A vertical plate load test conducted on the soil using a 300mm plate gave the pressure-displacement response shown in Fig. 2 with an ultimate soil bearing capacity of 294kPa at 2,3mm displacement.

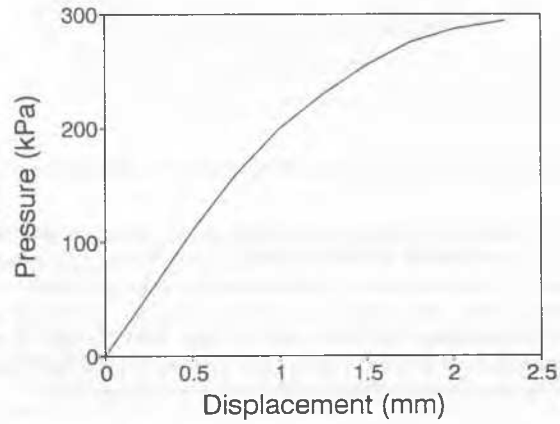


**Fig. 2.** Pressure-displacement relationship of laboratory sand

The steel beam used had a yield stress of 300MPa (Grade 300W steel SABS 0162-1, 1992). The approximate bilinear stress-strain relationship is shown in Fig. 3.

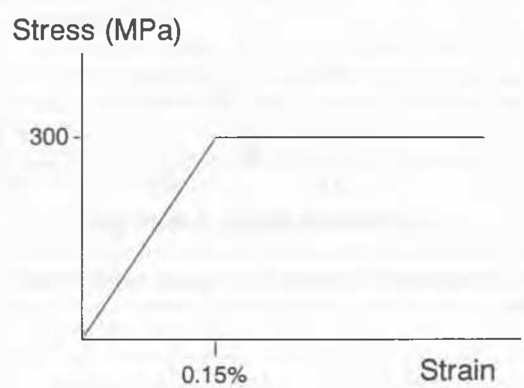


**Fig. 3.** Stress-strain relationship of grade 300W steel

Figure 4 shows the steel beam on the sand foundation including dimensions, locations of the linear variable displacement transducer (LVDT), as well as the position of the jack used to apply a vertical load to the beam.
The load, applied at about mid-span, was increased continuously by means of an electronically controlled hydraulic jack. The highest load measured before the bearing capacity of the soil was exceeded was 36,8kN. The displacements at ultimate bearing capacity were relatively small, insufficient to cause formation of a plastic hinge within the beam.

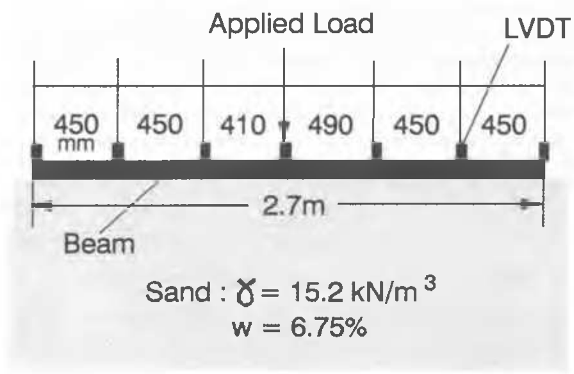


**Fig. 4.** Schematic of beam on soil foundation in laboratory test

The above beam-soil system was analyzed at the maximum recorded load using the numerical method referred to above to verify the beam deflections. The comparison of observed deflections compared with those predicted by the analysis is given in Fig. 5. Although the observed displacements are small and may have been affected by the beam 'bedding in' to the sand foundation, a reasonably good agreement between measured and predicted beam deflections was obtained.

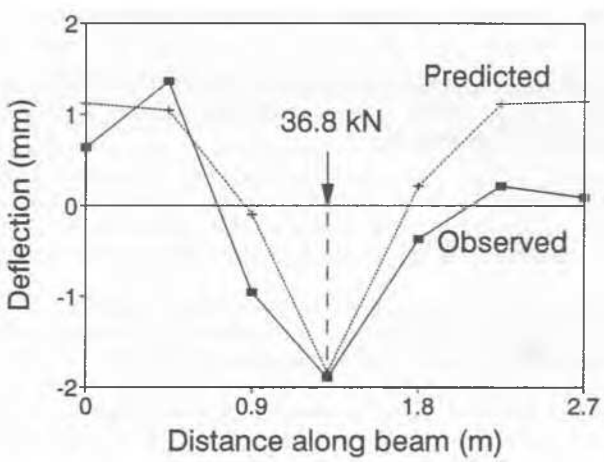


**Fig. 5.** Deflections of test beam at ultimate applied load of 36.8kN

**Field Tests: Soldier Piles On Excavation Face**

In order to investigate the performance of flexible soldier piles, a series of tests were carried out on working soldier piles on the perimeter of a 13m deep excavation in the Johannesburg city centre. Two slender piles, with moments of inertia of less than half of that of the adjacent soldiers, were installed in non-critical locations. Each pile was supported by five prestressed ground anchors with capacities of 450 or 600kN. During the field tests, the load on the middle anchor was increased progressively while the deflections of the pile were monitored. The results of the test on one of the soldiers is described in detail below.

The soldier pile concerned had a total length of 13m and consisted of 2 I-beams (160mm x 82mm, I = $8.7 \times 10^{-6}$m$^4$) placed side-by-side and connected at both flanges with tie-plates. The soldier pile was installed in a 600mm diameter auger hole which was subsequently backfilled. The test soldier pile penetrated 2m below the bottom of the excavation.

The in-situ material into which the pile was installed was a decomposed residual andesite in the form of a moist, red brown, clayey silt with a firm to stiff consistency. The residual soil has a density of 13.10 kN.m$^{-3}$, a moisture content 31.7%, and plasticity index 12.6%. The average effective angle of internal friction was 27° and the effective cohesion 20 - 40kPa. (In view of the jointed nature of the material, the cohesion is generally ignored in the design of the lateral support.) A plate load test of the in-situ material produced the pressure-displacement response curve shown in Fig. 6. The backfill material response curve, also shown in Fig. 6, was estimated.

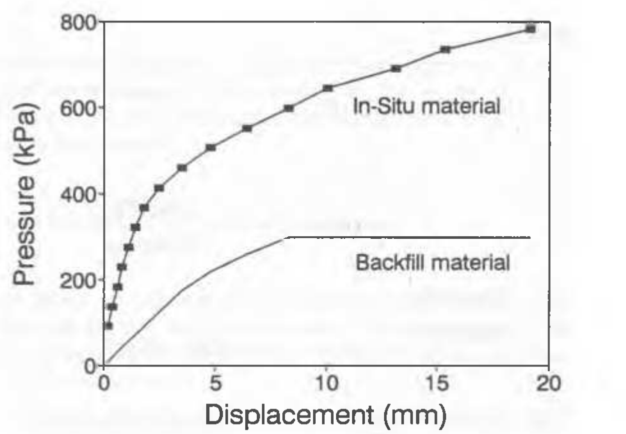


**Fig. 6.** Pressure-displacement relationship of backfill and in-situ material

Figure 7 shows a simplified section through the anchored wall. The soldier pile was tied back at five levels using pressure grouted, prestressed ground anchors with working loads of 450 - 600kN. All

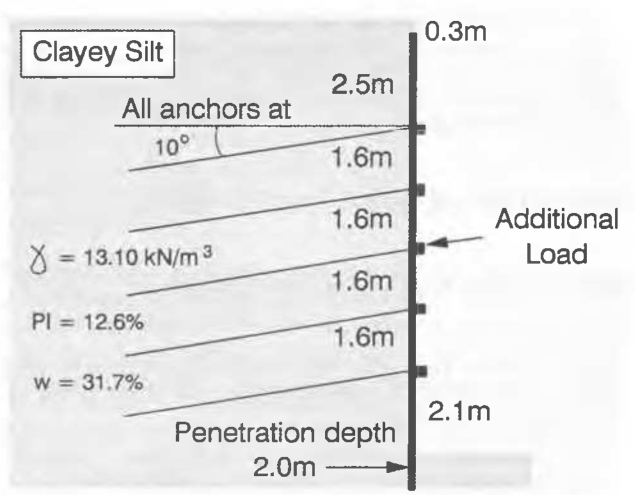


**Fig. 7.** Schematic of the soldier pile support system

anchors were inclined at 10° to the horizontal. At the commencement of the test all anchors had been stressed to the point where the soldier started bedding into the retained soil. The load of the middle anchor was 417kN at the commencement of the test. Movements of the wall were not recorded during excavation or initial stressing of the anchors.

In the experiment the load of the central anchor (as indicated in Fig. 7) was increased in increments to 615kN, 668kN, and 719kN (giving additional loads of 198, 271, and 302kN, respectively) while the deflections were monitored along the total length of the pile. The observed deflections resulting from the application of the additional load are shown in Fig. 8. The deflections were measured relative to a stable datum and are accurate to the nearest millimetre.

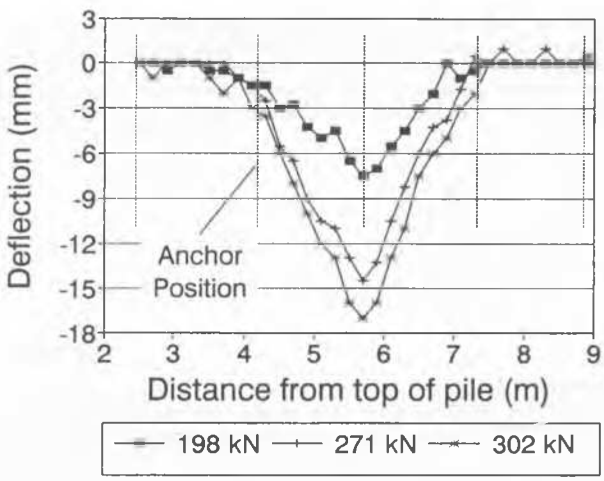


Fig. 8. Observed soldier pile deflections from field test

From these observations, it appears that the load increment of 198kN was insufficient to result in the formation of a plastic hinge. However, hinging appears to have occurred for the subsequent load increments of 271 and 302kN.

In the computer analysis of the soldier pile experiment (Howie, 1993), the thickness of the backfill layer was calculated as 160mm and the maximum displacement of the backfill prior to activating any significant response in the in-situ material was assumed to be 8mm.

With regard to the pile, the base of the pile was modelled as a fixed end condition, the top as a free end. The pile was discretised into 150 elements of equal length. The pressure-displacement relationships of the foundation materials used were those shown in Fig. 6. The stress-strain characteristics of the steel soldier sections were in accordance with those of Fig. 3.

The predicted pile deflections for the three additional load increments are presented in Fig. 9.

The computer solution predicted hinging of the beam (the maximum fibre stress of the steel section reached the yield stress) occurring for displacements greater than or equal to 10 mm. Thus, the 198kN load increment did not cause a hinge in the pile. Hinging clearly developed for both additional loads of 271kN and 302kN as the deflected shapes of the soldier pile at these load levels indicate.

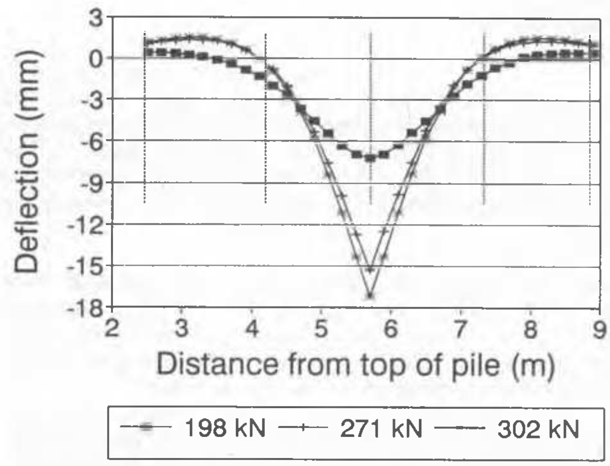


Fig. 9. Predicted soldier pile displacements using direct stiffness method

Another test pile in a similar configuration with the same boundary conditions was also loaded to plastic hinging. However, due to eccentric loading of the soldier pile one I-beam hinged independently causing the loading jack to shift out of alignment. Only the first load steps could be recorded. The backpredictions using the computer program were equally successful.

## DISCUSSION OF RESULTS

The back-analyses of both the laboratory and field tests gave good agreement between the observed and predicted deflections providing confidence in the proposed method.

In the laboratory tests, the beam section used was the smallest commercially available. In this case the applied load was incapable of causing a hinge within the beam due to a classical bearing capacity failure in the sand foundation. In spite of the low loads applied and consequent small deflections, accurate predictions of the elastic beam behaviour were obtained.

The field tests in which plastic hinging was achieved provided a better opportunity for verifying the nonlinear analysis routines on which the proposed method is based. In this case, the theoretical analysis of the soldier piles produced predicted peak deflections very close to those observed in the field for all three load increments. Minor differences in deflections are apparent further from the point of load application. The computer analysis predicts small deflections away from the excavation face which appear unrealistic due to the restraining action of the other anchors along the beam. Although no measurements were taken of change in loading of the other anchors, these changes are likely to have been small as a result of the limited movements observed at these points and the significant free length of the anchors used. The restraining effect of the other anchors was therefore ignored in the analyses.

With the exception of a slight stiffening near the base of the pile, the influence of end conditions had negligible effect on the pile deflection. This is due to the length of the pile, and the very localised nature of the deflection in the vicinity of the applied load.

## CONCLUSIONS

The computer analysis based on the exact matrix method with provisions for inelastic soil and beam behaviour is shown to have the potential of becoming a powerful tool in foundation design. In two examples, the predictions made by the propósed numerical method show satisfactory comparison with observations in the laboratory and the field.

In soldier pile design, the procedure allows the evaluation of the distribution of deflections, moments and shear forces in a continuous beam of given configuration, based on the conventional limit equilibrium approach. Realistic response will be accounted for since in-situ data is a key component in the analysis method.

Adequate safety against plastic hinging may now be provided when designing soldier piles, and thus a more balanced relationship between safety and economy can be achieved. It is hoped that more field experiments will be conducted to provide further verification of the method and to pave the way for general acceptance of this design tool.

## ACKNOWLEDGEMENTS

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