PROBABILISTIC METHODS FOR THE EVALUATION OF SHALLOW FOUNDATIONS BEARING CAPACITY

METHODES PROBABILISTES D'EVALUATION DE LA CAPACITE PORTANTE DES FONDATIONS SUPERFICIELLES

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SYNOPSIS: Many probabilistic methods for evaluating shallow foundation bearing capacity have been proposed. In this note are discussed the results obtained from many authors for cohesionless soils, only coesive soils and for soils characterized by cohesion and friction angle.

FOREWORD :

The following is an updated review of available computation methods and of the results achieved with a probabilistic evaluation of the allowable bearing capacity for shallow foundations in connection with the physico - mechanical parameters of the soil in which they have been laid.

SOLUTIONS

Available solutions are grouped here with respect to the types of mechanical parameters characterizing the soil, i.e.

- Cohesionless soil

A first solution, presented by Cherubini et al. (1988), provides an "exact" solution for the bearing capacity probability density curve as a function of the variability of Ø. However, this case necessarily requires using the computer.

A simpler, closed form solution, based on the transformation of variables (Hahn and Shapiro, 1967), has been presented by Cherubini (1990) starting from the approximation proposed by Krizek (1965) for the Ni coefficients of bearing capacity.

More recently, Cherubini et al. (1992) have reported a method which solves the problem arising from the application of Krizek's relations (non-optimal fitting for Ø >= 30°): this case too, however, requires computer processing to find a function of the type

$$q_{ult} = A \exp (B*\phi)$$

such that it will approximate the function qult = 0.50 γ B N + γ D Nq. This function represents exponentially the relation expressing bearing capacity with an excellent approximation and which, because it can be derived and inverted is well adapted for use with the tranformation of variables.

In all the above mentioned cases, one can determine the probability density curve by first assuming it to be normal, then by selecting, as the value of the allowable bearing capacity, the value associated to a given probability (e.g. 10^-4 according to Meyerhof, 1982).

In none of the reported methods did it appear necessary to investigate the influence of unit weight variability (Cherubini, 1991).

It is also appropriate to mention the work published by Ingra and Baecher (1983). By tests on models, the Authors evaluate, in addition to other parameters, the mean trend and the variability of the N_γ bearing capacity factor for cohesionless soil.

- Cohesive soil (undrained conditions)

A closed form solution has been provided by Easa (1991):

$$q_{ult} = K\ c_u + D\gamma$$

with K constant and with cu and γ normally distributed variables.

Given this condition, no complex computations are required and since the value qult is a linear combination of normally distributed variables, qult too is normally distributed. Moreover, the cumulated probability curve can be obtained directly from the tables of normal standardized variables published in any book on statistics and/or probability theory.

The writer admits thet in a case like this the variability of unit weight may affect the results. This effect may be especially strong if the unit weight variability is highest (Cherubini, 1991).

- Soil with friction and cohesion

This is the most general case and one of the utmost interest. Several methods are reported in literature on the geotechnical evaluation of probabilistically computed quantities. The different levels they are normally grouped into correspond to differences in the statistical information concerning the variables involved. Thus, Probability Density Function determination is defined with different degrees of accuracy which are a function of the initial input.
In addition to the transformation of variables, a technique that is also applicable to cases with two random variables (Hahn and Shapiro, 1967), the following are some of the most widely used probabilistic methods:

* The Point Estimation Method (P.E.M.), (Rosenblueth, 1975);

* Taylor's series expansion method (Benjamin and Cornell, 1970) and the second moment method (Hasofer and Lind, 1974) for the determination of the variability index;

* the Monte Carlo simulation method.

The first two techniques do not usually require sophisticated computation; instead Hasofer and Lind's method and the Monte Carlo simulation can only be handled with the use of computer codes.
Solution proposed for the most diversified cases, mostly regarding continuous foundations, are reported in text-books and in works by Harr (1977), Biarez and Favre (1981), Rethati (1988), Smith (1986), Singh (1971).

Of special interest are works proposing cases and/or methods capable of taking into account the correlation between friction angle and cohesion. The resulting correlation coefficient is usually negative and ranges between -0.24 and -0.70 (Harr, 1987).
Some of the above cited works compare probabilistic versus classical deterministic results.

CONSIDERATIONS ON THE RESULTS OBTAINED

A few general consideration can be draw from an examination of all the works listed above;

1) For low values of the coefficient of variation of the geotechnical parameters examined with the probabilistic approach, the values of allowable bearing capacity which has been computed directly by associating it to given probability (10^-4 according to Meyerhof) may be higher, and occasionally remarkably so, than the corresponding values obtained deterministically.

2) For high coefficients of variation of the basic parameters, the admissible bearing capacity obtained probabilistically is very much reduced, while at the same time the probabilities of failure associated to the deterministic values of qall are high and in contrast with the actual behaviour in situ. This is due essentially to the non-linearity of the Ni coefficients with respect to the friction angle, which causes an expansion of the variability of this strenght parameter.
Extremely high values of the coefficients of variation can be found with undrained cohesion (Kulhawy et al., 1991; Cherubini et al., 1992). Thus, whenever this parameter is present, special attention must be paid to the conclusion one can derive probabilistically, despite the considerations mentioned in the following paragraph.

2a) Whenever possible correlations are considered, the variability of qult is lower when the correlation is positive (as between ϕ and γ).

3) A remarkable, though not preponderant, share of "variability" of the geotechnical property being considered is not due to its own intrinsic variability, but rather to all the manipulations for taking the sample, when the soil is to be tested in the laboratory, as for the making of the specimen etc. Not many studies are available on this point. However, soil with undrained cohesion may appear considerably disturbed. Reference should be made to the data and comments presented by Rethati (1988). Very roughly and tentatively, this additional variability may be estimated to be in the order of some 20 - 30 % of overall variability.

4) Parallel to this, one should consider the "benefical" effect resulting into lower variability of the geotechnical characteristics considered in the probabilistic approach, of the spatial variability (Vanmarcke, 1977; Schultze and Pottharst, 1983) which is related to the evaluation of the fluctuation step. According to Li (1991) this fluctuation step may be estimated to be equal to 1.0 - 2.0 m in the vertical sense.
In a simplified model, one can introduce, according to Salembier (1979), a reduction of the variance in line with the relation $\sigma s^2 = \sigma e^2/K$, where σs^2 is the working variance and σe^2 is the variance measured in the tests. Mc Anally (1983) finds that K is equal to 4 in shallow foundations. With this approach, the working standard deviation of the strenght parameters considered is therefore equal to about half the one obtained in the laboratory test.
In this connection, Favre and Genevois (1987), using the finite elements method probabilistically, specifically examine the spatial variability of the friction angle for continuous foundations laid at field level. They consider two different fluctuation steps one vertical and one horizontal, and reach the conclusion that the dispersion of soil parameters produces important effects in extreme cases: strong dispersion of the parameters, strong (positive) correlation between them, an important fluctuation step in relation to

the dimensions of the foundation. The dispersion found by Favre and Genevois (1987) for bearing capacity are from 10 to 100 times lower than the actual values.

5) Still, it should be noted, in any case, that the solutions may contain possible systematic errors due to the fact that unsuitable computation models can be used. Vannucchi (1985) examines the best known defferences in the results obtained from methods used to calculate bearing capacity (Bowles, 1982). This kind of systematic error may be quite high.

6) Furthermore, in the above described cases and methods the variability of the Capacity (C) was evaluated without usually paying attention to the variability of the Demand (D) which is usually inserted by the structural engineer. A more realistic definition of failure probability can be expressed as a function of C and D and is connected to the computation of the reliability index β.

7) The problem can be rendered even more complicated by the additional variabilities brought about by the foundations' dimension and depth as well as by the fluctuations of the watertable, where there is one. Again Mc Anally (1983) suggest a possible coefficient of variation of about 2% for the width of the foundation bottom and of 5% for foundation depth. Bennet et al. (1989) add that, in any case, the effect of the variability of foundation dimensions upon their reliability is not significant.

CONCLUSIONS

Although it has been presented in a summary form, this general overview of studies published to date on probabilistic methods shows, in the writers' opinion, that we have proceeded half way towards a generalized use of probabilistic or semiprobabilistic methods. We have done much but we still have a long way to go before we arrive at a "reliable" use of probabilistic analysis methods. However, we shall necessarily have to be supported by an efficient system of standardization and execution of geotechnical tests both in the laboratory and in the field. Furthermore it is to be hoped that more detailed and numerous evaluations will be provided in the soils' geotechnical properties. Lastly, for an appropriate calibration of computation methods, data relative to real-size cases will hopefully be collected and processed.

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