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A PROBABILISTIC ANALYSIS OF SEISMICALLY INDUCED PORE PRESSURE IN EARTH DAMS

UNE ANALYSE PROBABILISTIQUE DES PRESSION DES PORES DANS UNE BARRAGE EN SOL

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SYNOPSIS: A probabilistic analysis for the seismic response of earth dams is presented. The progressive development of pore water pressure is the main focus of this study. The earthquake-induced ground motion is considered as a random process defined by Kanai-Tajimi spectral density function. A 2-D plane strain finite element idealization is used to model the earth dam. The dynamic response is formulated in frequency domain and the transfer functions for various parameters, viz, stress, strain, and acceleration, etc. are obtained. The power spectra and the extreme values of these parameters are then evaluated. Using the statistics of the dissipated shear energy, the pore water pressure response and the probability of liquefaction is computed. The effect of partial dissipation of pore water pressure is also included in an approximate way.

INTRODUCTION

The primary cause of soil failures during earthquakes is the progressive build up of pore water pressure and consequent loss of strength leading to large deformation or liquefaction. Significant advances have been made in understanding the nature of earthquake motion and the behavior of soils under seismic loading. Also, progress has been made towards the development of stochastic analyses for the seismic response of earth dams.

In this paper a stochastic analysis is developed to evaluate the seismic response of earth dams including the progressive build up of pore water pressure and the associated potential of liquefaction. The nonlinearity of stress-strain behavior of soils is handled by using the equivalent linear constitutive model and an iterative solution of the response. A 2-D plane strain finite element idealization is used to model the earth dam. The uncertainties associated with the input ground motion and the liquefaction resistance are formulated in the frame work of the theory of probability and random vibration. The response variables are evaluated in probabilistic terms. The random pore water pressure is evaluated from the statistics of the dissipated shear energy through an energy based model for the development of pore water pressure (Nemat-Nasser and Shokooh, 1979; Pires, et al. 1983). The liquefaction potential is evaluated in probabilistic terms.

RANDOM GROUND MOTIONS

In this study, for simplicity and ease of analysis, the earthquake motion is considered to be a stationary random process (with constant frequency content) truncated for a finite duration. An approach suggested by Vanmarcke and Lai (1977) is used, in which a chosen time history of recorded ground motion is considered as one sample of the entire ensemble representing the random process. The following spectral density function suggested by Kanai (1957) and Tajimi (1960) is used to characterize the frequency content of the strong earthquake ground motion,

$$S_a = \frac{S_0(1+4\zeta_g^2(\omega/\omega_g)^2)}{\left[1-(\omega/\omega_g)^2\right]^2+4\zeta_g^2(\omega/\omega g)^2}. \tag{1}$$

In the above, ω is component frequency, S_0 is the scale factor providing a measure of the energy content of the motion; parameters ω_g and ζ_g are the equivalent natural frequency and damping ratio of the ground, characterized by a single degree of freedom system. The three parameters S_0, ω_g, and ζ_g defining the power spectrum are estimated from the chosen sample time histories.

In order to approximate the nonstationarity of the ground motion, only a segment of the stationary motion, defined as equivalent stationary duration is used. This equivalent stationary duration, s is evaluated from the following:

$$s = \frac{\Pi}{\Omega}\exp\{a_{max}^2/(2\sigma_a^2)\}, \tag{2}$$

where

$$\sigma_a = \sqrt{\lambda_0}, \tag{3}$$

and

$$\Omega = \sqrt{\lambda_2/\lambda_1}, \tag{4}$$

in which the spectral moments λ_0, λ_1 and λ_2 are obtained from the following,

$$\lambda_i = \int \omega^i S_a(\omega)\, d\omega, \quad i = 0, 1, 2, . \tag{5}$$

In summary, the random ground motion is defined by (a) the Kanai-Tajimi spectrum with parameters S_0, ω_g, and ς_g, and (b) the equivalent stationary duration, s. Lai (1980) has estimated the above parameters of the model by processing 120 recorded motions each treated as a sample time history.

DYNAMIC RESPONSE

Equation of Motion

For dynamic analysis, a dam considered to be in the state of plane strain is discretized into a finite number of quadrilateral or triangular elements as required. The equation of motion for a dam discretized into finite elements and excited by ground motion at its base can be written as :

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = -[M]\{\delta\}a(t), \tag{6}$$

where [M], [C], and [K] are the mass, damping, and stiffness matrices, respectively. { δ } is the influence vector; $\delta_i = 1$ if the degree of freedom is the same as the direction of ground motion; otherwise $\delta_i = 0$. {u} is the displacement vector, and a(t) is the ground acceleration. The dot over a vector represents its time derivative. In eqn. (6), the damping and stiffness matrices are dependent on the level of strain in the finite elements. Considering the soils to be a Coulomb damped linear elastic material, the shear modulus, G, and damping ratio, β, can be lumped into a complex shear modulus, $G^* = G(1 + 2i\beta)$, which will then provide a complex stiffness matrix [K*].

Transfer Functions

The nodal displacements of the finite element model subjected to an unit amplitude ground acceleration $a[= \exp(i\omega_r t)]$ can be written as $\{u\} [= \{U\}_r \exp(i\omega_r t)]$ which after substitution in eqn.(6), provides:

$$([K^*] - \omega_r^2 [M])\{U\}_r = -\{m\}. \tag{7}$$

From this equation, the displacement amplitudes, $\{U\}_r$, may be obtained for each frequency, ω_r, of the input motion by Gaussian elimination. These displacement amplitudes are the complex transfer functions of displacement, $\{H_u(i\omega_r)\}$. Once the complex transfer functions for the displacement have been obtained, the complex transfer functions of absolute acceleration can be calculated from

$$\{H_{\ddot{u}}(i\omega_r)\} = -\omega_r^2 \{H_u(i\omega_r)\} + \{\delta\}. \tag{8}$$

The transfer functions, $\{H\gamma(i\omega_r)\}$, for the strain can be obtained by applying the strain-displacement relationship; and the transfer functions, $\{H\tau(i\omega_r)\}$, for the stress can also be calculated from the stress-strain relationship.

Spectral Density Functions for Response

For the linear system, with these transfer functions obtained from the preceding calculation, the power spectra for the various response variables can be readily obtained:

$$S_x(\omega) = |H_x(i\omega)|^2 S_a(\omega) \tag{9}$$

where $S_a(\omega)$ is the power spectra of input ground acceleration, and x represents any of the response variables, i.e. u, u, γ, and τ. The moments of the spectral density function $S_x(\omega)$ can be obtained by using eqns.(3) and (4), providing σ_x^2, the variance, and Ω_x, the central frequency. Another useful spectral parameter δ_x $[= \sqrt{1 - \lambda_{1,x}^2/\lambda_{0,x}\lambda_{2,x}}]$ representing the variability in the frequency content of the response variables may also be calculated.

Statistics of The Maxima

From the spectral density functions of the response variables, the extreme value (maximum value) within the duration, 's', of the process can be obtained. This is the so-called "First Passage problem" of random vibration theories. Many solutions have been proposed for this problem each solution being based on different assumptions and simplifications. The solutions are usually presented in the form of

$$E_x(s,p) = r_{s,p}\, \sigma_x, \tag{10}$$

where E_x is the extreme value of x which has the probability p of not being exceeded within the duration s. In general, $p = 60\%$ approximately corresponds to the mean of extreme values, and $p = 10\%$ and $p = 90\%$ are the lower bound and upper bound of extreme values, respectively. The parameter, $r_{s,p}$, is the peak factor, which can be expressed as (Vanmarcke, 1976):

$$r_{s,p} = [2 \ln\{ 2n [1 - \exp(-\delta_x \sqrt{\pi \ln 2n})]\}]^{1/2}, \tag{11}$$

in which

$$n = (\Omega_x s / 2\pi)(-\ln p)^{-1}. \tag{12}$$

The Equivalent Linear Model

The shear modulus and damping ratio of the soils depend on the level of strain induced. This nonlinearity of soil behavior is incorporated by solving the response iteratively. A set of initial shear modulus and damping ratio values are estimated for each soil element of the model. The dynamic responses of the system are then evaluated, and the maximum shear strain can be calculated in the time domain by using Mohr's circle,

$$\gamma_{max}(t) = \{ [\varepsilon_x(t) - \varepsilon_y(t)]^2 + \gamma_{xy}^2(t) \}^{1/2}, \tag{13}$$

where $\varepsilon_x(t)$ and $\varepsilon_y(t)$ are the strains in x- and y- directions, respectively, and $\gamma_{xy}(t)$ is the shear strain associated with $\varepsilon_x(t)$ and $\varepsilon_y(t)$. The extreme value $\hat{\gamma}_n(\gamma_{max})$ of γ_{max} may be estimated from the previous equations.

The mean square value of maximum shear strain may be calculated from the relation (Romo-Organista, 1977),

$$\sigma_{\gamma\, max}^2 = \int_0^\infty (|E|^2 + |\Gamma|^2) S_a(\omega)\, d\omega, \tag{14}$$

where E and Γ are the complex amplitudes of $(\varepsilon_x(t) - \varepsilon_y(t))$ and $\gamma_{xy}(t)$, respectively. The extreme values of the shear strains γ_{max} for each element are then evaluated by using the above mentioned formulations for the statistics of extreme. New values of G and β compatible with the effective shear strain, γ_{eff} (= .65 γ_{max}) are then obtained from the experimentally generated curves. These values of G and β are then used in the next iteration of all the above calculation steps, and the process is repeated until convergence is achieved (usually within 3 to 5 iterations).

SHEAR STRAIN ENERGY DISSIPATION STATISTICS

In this study an energy based model (Nemat-Nasser and Shokooh, 1979) is used for the evaluation of pore pressure response. According to this model the pore pressure due to cyclic loading is related to the dissipated shear strain energy.

The energy dissipated in unit volume of soil per cycle of uniform stress can be represented in terms of normalized shear stress amplitude, $\bar{\tau}$ (= $\tau_{oct,e}/\sigma'_{oct,o}$) as:

$$E_c(\bar{\tau}) = K\bar{\tau}^2 = \frac{2\pi\beta}{G(1 + 4\beta^2)} (\sigma_{oct,o})^2 \bar{\tau}^2. \tag{15}$$

For the earthquake-induced shear stresses which are random, the expected value and variance of energy dissipated per peak can be obtained:

$$E[E_c] = E[K\bar{\tau}^2] = K\, E[\bar{\tau}^2] = K \int \bar{\tau}^2 p(\bar{\tau})\, d\bar{\tau},$$

$$\mathrm{Var}[E_c] = E[E_c^2] - \{E[E_c]\}^2 = K^2 E[\bar{\tau}^4] - \{E[E_c]\}^2 = K^2 \int \bar{\tau}^4 p(\bar{\tau})\, d\bar{\tau} - \{E[E_c]\}^2. \tag{16}$$

In the above, the probability density function of normalized shear stress (considered as a non-narrowband Gaussian process) can readily be evaluated (Vanmarcke, 1976). Moreover, the statistics of the rate of energy dissipated can also be obtained; these are:

$$E[\dot{E}] = E[M_x] E[E_c],$$

$$Var[E] = E[\dot{E}^2] - \{E[\dot{E}]\}^2, \quad (17)$$

$$\sigma[\dot{E}] = \{Var[\dot{E}]\}^{1/2}.$$

Where $E[M_x]$ is the expected number of peaks per unit time, which is given by:

$$E[M_x] = \frac{1}{2\pi} \frac{\lambda_{4,x}}{\lambda_{2,x}}, \quad (18)$$

and the values $\lambda_{2,x}$ and $\lambda_{4,x}$ are defined according to eqn. (5).

PORE PRESSURE RESPONSE

According to the energy based model and the experimental observation of the pore pressure buildup (Seed, et al. 1976), the rate of the pore pressure development can be written as (Pires, et al. 1983):

$$\frac{dr_u}{dt} = \frac{1}{\theta\pi \sin^{2\theta-1}(\pi r_u' / 2)\cos(\pi r_u' / 2)} \frac{X(t)\dot{E}(t)}{\Delta W(r_u = 1)}. \quad (19)$$

In the above equation, θ is an empirical constant (an average value of θ = 0.7 is valid for a range of data); $\dot{E}(t)$ is the rate of energy dissipation; $X(t)$ is a function to be evaluated from cyclic resistance and $\Delta W(r_u = 1)$ is the total energy required to initiate liquefaction. The formula for $X(t)$ and $\Delta W(r_u = 1)$ can be found in Pires, et al. (1983). In the case of a stationary excitation, implementation is simple because $X(t)$ and $\dot{E}(t)$ are constants for any given loading.

Effect of Partial Drainage

The effect of partial drainage on the development of pore water pressure is included in the model in an approximate way. Umehara, et al. (1985) have proposed a method to evaluate the liquefaction resistances under partially drained condition. According to their laboratory studies, the strength increase due to partial drainage can be accounted for by the following equation :

$$(\tau_{oct,e}/\sigma_{oct,o}')_p = SR \cdot (\tau_{oct,e}/\sigma_{oct,o}')_u, \quad (20)$$

where $(\tau_{oct,e}/\sigma_{oct,o}')_p$ and $(\tau_{oct,e}/\sigma_{oct,o}')_u$ are the partially drained and undrained cyclic shear strength, i.e., the shear stress ratio required to cause initial liquefaction in the same number of cycles, N_l. The strength ratio, SR is found to be a function of permeability, k, drainage path, d, frequency, f, the relative density, D_r, and the number of cycles required to initiate liquefaction, N_l.

It should be noted that in Eq. 20 the dissipation of the pore pressure is being assumed to be one dimensional only in the lateral direction. The drainage path is the distance from element to the sloping face. On the basis of laboratory data, Umehara, et al. (1985), developed the empirical equations, relating the factor SR to a nondimensional parameter k/fd for various value of D_r, which are used in this study.

PROBABILITY OF LIQUEFACTION

The energy model for the pore pressure response presented earlier provides a unique relationship between the dissipated shear strain energy and the generated pore pressure ratio. The energy dissipated in time t, $\Delta W(t)$ can be considered as a damage parameter, while the energy required to generate a pore pressure ratio, r_u , =1 $\Delta W(r_u = 1)$, as the measure of liquefaction resistance. If we define a variable $Z = \Delta W(t)/\Delta W(r_u = 1)$, the probability of liquefaction, p (liquefaction) = $p(Z > 1)$. In the above $\Delta W(t)$, the energy dissipated till time t is given by

$$\Delta W(t) = \int_0^t X(s)\dot{E}(s)ds \quad (21)$$

Using the first order approximation the mean and variance of $\Delta W(t)$ can be calculated from

$$\mu_{\Delta W(t)} = tX(s)E[\dot{E}(s)], \quad (22)$$

and

$$\sigma^2_{\Delta W(t)} = t^2 X(s)^2 E[\dot{E}^2(s)] - \mu^2_{\Delta W(t)}. \quad (23)$$

The $\Delta W(t)$ is the quantity which may be described (Pires, et al., 1983) by a lognormal distribution $LN(\alpha_{\Delta W(t)}, \beta_{\Delta W(t)})$, where

$$\alpha_{\Delta W(t)} = \ln(\mu_{\Delta W(t)}) - \frac{1}{2}\beta^2_{\Delta W(t)}, \quad (24)$$

and

$$\beta_{\Delta W(t)} = \sqrt{\ln(1 + \sigma^2_{\Delta W(t)}/\mu^2_{\Delta W(t)})}. \quad (25)$$

Similarly, $\Delta W(r_u = 1) = LN(\alpha_{\Delta W(r_u = 1)}, \beta_{\Delta W(r_u = 1)})$. The parameter Z follows a lognormal distribution, $Z = LN(\alpha_Z, \beta_Z)$, with the following parameters:

$$\alpha_Z = \alpha_{\Delta W(t)} - \alpha_{\Delta W(r_u = 1)}, \quad (26)$$

and

$$\beta_Z = \sqrt{\beta^2_{\Delta W(t)} + \beta^2_{\Delta W(r_u = 1)}}. \quad (27)$$

Thus, the probability of liquefaction is given by

$$p\,(\text{liquefaction}) = 1/2[\,1 + \text{erf}(\alpha_Z/\beta_Z)\,] \quad (28)$$

NUMERICAL RESULTS

All the elements of a probabilistic analysis of the seismic response of an earth dam have been presented in the earlier sections. The evaluation of the response including the liquefaction potential involves the following steps: (1) determination of the initial stresses in the earth dam before earthquake; (2) determination of the characteristic of the input motion in base rock underlying the earth dam and the development of a random model for the ground motion; (3) evaluation of the response of earth dam to base rock excitation and calculation of the dynamic stress and strain induced in each element in the earth dam; (4) evaluation of the statistics of dissipated shear strain energy; (5) evaluation of the pore water pressure response, and liquefaction potential in probabilistic terms.

The initial stresses can be evaluated by a static finite element procedures. In this study, the initial stress were evaluated (step 1) the computer program FEADAM (Duncan et al., 1984). Steps (2) to Steps (5) are based on the formulations presented here and which have been used to develop a computer program PROSE Probabilistic Response of Soils during Earthquake (Hwang and Rahman, 1989).

Lower San Fernado dam --- A case study

The primary cause of the upstream slide in the Lower San Fernado Dam during February 9, 1971 earthquake of magnitude 6.6 was the development of very high pore water pressure in an extensive zone of hydraulic fill near the base of the embankment and upstream of clay core so that much of this soil was in a liquefied or very low strength condition (Seed, et al. 1973). In this study, a modified form of the record obtained at the nearby Pacoima Dam scale to a peak acceleration of 0.5 g with a duration of 15 seconds was used as a reasonable estimate of rock motions at the San Fernado Dam sites. The following excitation parameters were found: s = 15 secs; σ_a^2 = 33.59 ft^2/sec^4; S_0 = 0.545 ft^2/sec^3; ω_g =20.73 rads/sec; ζ_g = 0.41. The power spectrum of the input motion was obtained and the response of embankment and the dynamic stress were then evaluated. The same soil properties as reported by Seed et. al. (1973) were used for the present analysis.

The computed maximum acceleration at the crest ranges from 0.48g to 0.70g with a mean value of 0.6g, which is close agreement with the acceleration (≈ 0.6g) recorded by the seismoscope and the acceleration (≈ 0.6g) computed by the deterministic method (Seed et. al., 1973).

Using the formulation presented earlier, the liquefaction probabilities are computed, which are presented in Fig. 1. These are shown only for the liquefiable hydraulic fill materials after 5 seconds, and 15 seconds of the earthquake shaking. The zone of liquefaction as reconstructed by Seed et. al. (1973) from the field investigation is also shown on the figure showing the liquefaction probability at t = 15 seconds (end of the earthquake). This zone is in the zone of the maximum calculated probability of liquefaction of 0.1. This moderately high value of probability of liquefaction is indicative of the observed field behavior.

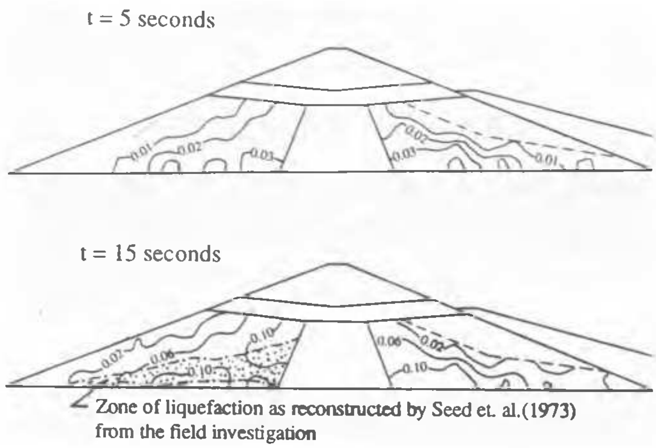

FIG. 1 Contours of probability of liquefaction in hydraulic fill after 5 sec, 10 sec, and 15 sec earthquake shaking (a_{max} = 0.5g; k=0.01cm/sec) -- lower San Fernado dam

A few additional points should be noted here: (i) the downstream side of the dam has higher probability of liquefaction than upstream side, in spite of the lower values of normalized shear stresses. This is because of their lower initial shear stresses and higher initial normal stresses, whose effects on the liquefaction resistance have been considered in the analysis, (ii) the higher probability of liquefaction does not necessarily imply the higher probability of the sliding failure, as evidenced from the fact that no slides were observed in the downstream slope of the dam, and (iii) some zones must have liquefied in the downstream side of the dam also; however, the additional resistance provided by the berm might have prevented the sliding failure.

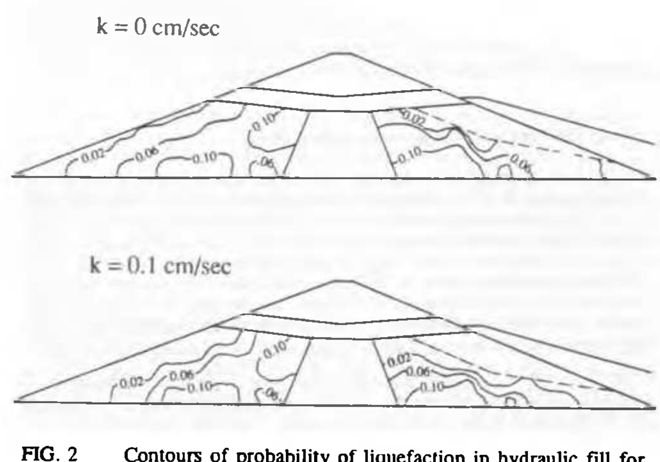

FIG. 2 Contours of probability of liquefaction in hydraulic fill for different permeability conditions (a_{max} = 0.5g; t = 15 sec) -- lower San Fernado dam

Fig. 2 shows the probability of liquefaction for undrained condition (k = 0) and for a condition of partially drainage (k = 0.1 cm/sec). As seen from the figure, the probability of liquefaction along the clay core of upstream side is about the same. This is because that the drainage path is too long to make drainage effect significant. The influence of drainage effects are significant in the boundary zone on both upstream and downstream. It should be noted that the effect of drainage has been included in the analysis in an approximate way. The dissipation only in the horizontal direction has been considered and each point has been considered to be hydraulically isolated without any consideration of redistribution of the pore water pressure. The effect of these approximations should yield a conservative estimate of probability of liquefaction.

SUMMARY

The formulation for a probabilistic analysis of seismic response of earth dam was presented. The uncertainties in (a) specifying the random seismic environment, (b) the liquefaction resistance parameters, and (c) evaluation of the seismic response were treated explicitly in the frame work of the theories of stochastic processes, probability, and random vibration. One of the limitation in this study however is that the stiffness and damping properties of the materials have been considered as deterministic, which can be eliminated by random field description of those properties in conjunction with the stochastic finite element method. The analysis procedure presented in this paper may be used to evaluate pore pressure response within an earth dam in a probabilistic term. The evaluated spatial distribution of liquefaction probability may help identify the zone with higher risk of liquefaction. Such information may be useful in mitigating the seismic hazard associated with the liquefaction failure of an existing dam through some remedial measures.

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