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# A LABORATORY INTRODUCTION TO CRITICAL STATE SOIL MECHANICS

## UNE INTRODUCTION EN LABORATOIRE A L'ETAT CRITIQUE EN MECANIQUE DES SOLS

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**SYNOPSIS:** Following a brief reference to some of the difficulties associated with the teaching of the shear strength of soil, an approach to introductory undergraduate laboratory work is described. The primary aim of the laboratory programme is to provide clear experimental evidence of important basic concepts. Detailed attention is focussed on the use of the direct shear test as a means of observing the relevance of the concept of a critical state to the shearing behaviour of a real sand. It is shown that the systematic application of the saw-blade dilatancy model to the experimental data for a range of tests on the sand leads to a unified picture of the response of the sand to shear. The analysis is capable of clarifying problems associated with test data, which initially might appear to be less than satisfactory.

### INTRODUCTION

#### Soil Mechanics Triangle

Burland (1987) has emphasized the crucial importance of education in geotechnical engineering and has highlighted the real difficulties in communicating the subject to undergraduate students. His references to past discussions among eminent geotechnical engineers have shown that long recognized difficulties are still with us.

Not only did Burland (1987) discuss the issues, but he proposed a way forward through a clear structured pedagogical approach, based on the 'soil mechanics triangle'. This pedagogical model highlights the need to develop expertise in the corners of the triangle, namely, Applied Mechanics, Soil Behaviour and Ground Profile, and to establish linkages between them. It is the synthesis of these strands of the knowledge base and, in particular, the reconciliation of the conflicts between them that lead to sound and progressive empiricism in geotechnical engineering. The soil mechanics laboratory is central to activities in the 'Soil Behaviour' corner of the triangle.

The pedagogical basis provided by Burland's soil mechanics triangle for the teaching of geotechnics is worthy of widespread application throughout the teaching of civil engineering and beyond, by generalizing the three corners to theory, laboratory and practical reality. The concept of the engineer in the middle, drawing on the different resources in order to solve problems in the real world, emphasizes the central role of the engineer and provides a role model, to which the student may aspire.

#### Shear Strength of Soil

The shear strength of soil has always been a central, and perhaps dominant, theme in the geotechnical engineering curriculum. It focusses attention on aspects of soil behaviour, which differentiate soil from other widely used man-made construction materials. A clear understanding of the essential elements of the shear strength of soil is a prerequisite to any real progress in the study of soil mechanics.

The traditional approach to shear strength through the concepts of 'cohesion' and 'friction' has had some undesirable, though probably unintended, consequences. In the minds of many practising civil engineers, the concept of

clay as 'cohesive' is twinned with the concept of sand as 'frictional', leading to a presumption that they are fundamentally different. This terminology may sometimes be found reversed, with sands being described as 'cohesionless' and clays being utterly fallaciously described as 'frictionless' or 'apparently frictionless', admittedly under the special conditions of undrained loading. Given the fact that there are very clear visual and tactile differences between sand and clay, it is easy to understand how the soils may come to be regarded as fundamentally very different. While the new generation of textbooks have done much to remove such confusion, the visual and tactile differences still exist and students remain to be convinced that sands and clays are more alike than the 'uninitiated' might believe.

It is important to put sensory and emotional perceptions of students to good use in our attempts to convey a rational approach to soil mechanics, lest they act to undermine our best efforts. The laboratory has an important role to play in encouraging students to develop a physical sense of the importance of fundamental concepts in soil mechanics.

#### Critical State Soil Mechanics

Critical state soil mechanics represents more than just a class of elegant theoretical models for the analysis of the deformation and yielding of soils. The link between the deformation and shear strength of soil embodied in critical state soil mechanics provides the key to understanding the behaviour of soils under load. Thus, progress in the fundamental understanding of the deformation and shear strength of real soils must be based on the concepts of critical state soil mechanics.

Students usually come to the study of soil mechanics from a background of isotropic elasticity, where shear and volumetric behaviour are uncoupled. Any previous exposure to plasticity and yielding has probably been confined to rigid perfectly plasticity materials. The acceptance by the student of significant coupling between shearing and volume change should not be taken for granted. The experience of observing such coupling in a simple laboratory experiment is perhaps the most effective means of communicating its importance to the student. If essential concepts are to impinge truly on the consciousness of the engineering student, they must be taken beyond the lecture theatre to the soil mechanics laboratory, where they can be seen, and even felt, by the student to apply to real soil.

## UNDERGRADUATE LABORATORY WORK

Burland (1987) has expressed reservations about the instructional role of routine laboratory testing in the undergraduate curriculum. Such reservations are shared by the writer, who has organized a programme of laboratory work, influenced by the recommendations of Burland (1987). In bringing the student into contact with real soil in the laboratory, it must be recognized that this laboratory is competing for the attention of the student with many other laboratories, where teaching orientated experiments have a strong immediate visual impact on the student. Successful experiments must be both instructive in the context of soil mechanics and appealing to the students.

During the introductory course in soil mechanics at Cork, each student attends three working sessions in the soil mechanics laboratory. Each session includes a number of clearly defined tasks, focussed on a particular aspect of the course. The primary purpose of these sessions is to provide first hand experimental support for important matters treated during lectures. It is fundamental to the success of such experimental work that it brings clarity, rather than confusion, to the subject matter.

During the first session, which is devoted to the description and classification of soils, the students, working in pairs, determine initially the liquid and plastic limits of a fine soil. The soils in use by the different groups vary from low plasticity silts through to extremely high plasticity clays. This is followed by a visual and tactile assessment of a range of disturbed soils, for which classification data are available. The students are then required to describe formally, using the MCCSSOOW methodology of Burland (1987), a number of disturbed and undisturbed samples of soil. This session brings the student into physical contact with real soils and affords an opportunity of experiencing the essential differences between the processes of describing and classifying soils.

Another session is devoted to the flow of water through soil. Constant head and falling head tests are used to estimate the coefficients of permeability of a coarse sand and a silt, respectively. This highlights the use of both steady state and transient flow conditions for the measurement of permeability and enables the student to assess at first hand the widely differing permeabilities of the two soils. In another task, artesian pressure conditions are introduced gradually into a cylindrical column of saturated fine sand, on the surface of which rests a small brass weight. The students observe the effect of the increasing artesian pressure on the stability of the model foundation and estimate also the critical hydraulic gradient at which mass instability of the soil is initiated. By varying the rate at which the artesian pressure is increased, both boiling and plug failure mechanisms may be observed.

The third session, which is devoted to the shear strength of sand and centred on the direct shear test, is the main subject of this paper and is discussed in detail later.

The shear strength and seepage sessions are linked by the observation over the two sessions of the performance of a model earth dam, comprising fine sand, in a glass tank 0.6m high x 2m long x 0.12m wide. The dam is built initially by pouring the sand loosely to stand at its angle of repose in the tank. When water is introduced to one side of the sand mound to mid height of the slope and maintained at this level, an extensive seepage instability develops gradually on the downstream face. The slope of the seepage surface is seen to correspond closely to the calculated slope for seepage flow parallel to the surface of an infinite slope. Immediately above the uppermost point of the seepage surface, a steep escarpment is observed, highlighting the sand castle effect, i.e., the effect of suction in the capillary zone above the water table. Above this escarpment, the dry sand rests at its angle of repose, oblivious to the events below. At the following session, a changed situation is evident - the water is being retained almost to full height of the mound and yet there is neither a seepage surface nor a downstream instability. A small gravel drain, wrapped in an environmentally friendly geotextile, has been placed under part of the downstream slope. The writer is of the view that this two part 'observe and discuss' session, which is an aside from the main thrust of the laboratory activity, does much to bring together in a vivid and practical way a number of

important basic concepts.

This summary of the complementary laboratory sessions serves to place in context the session discussed in detail in this paper.

## LABORATORY INTRODUCTION TO THE SHEAR STRENGTH OF SAND

### Direct Shear Test

Given the many disadvantages usually attributed to the direct shear test in undergraduate textbooks, students could be forgiven for concluding that the test is a source of confusion rather than of clarity. The essential simplicity of the test is often clouded in allusions to non-uniform states of stress, rotations of principal stresses, progressive failure from the edges, decreasing area of shearing and lack of control of drainage. Such criticisms tend to detract from the general suitability of the test as a means of quickly and easily making useful experimental observations on the shearing behaviour of sand. In contrast to the triaxial test apparatus, which demands considerable user skill, the shear box is immediately accessible to the student.

### Friction

The essentially frictional nature of the shear strength of soil is at the heart of both classical and critical state soil mechanics. Students need little convincing that sand is a frictional material, given that it is rough and often gritty to the touch. The friction in a soft lump of high plasticity clay is not explicitly evident - it is sticky to touch and feels soapy when rubbed in the fingers. The concept of friction applied to the shear strength of sand must be teased out on its own, uncluttered by references to unnecessary complicating factors, in order that the student may develop a strong physical sense of its importance. A chapter, simply entitled 'Friction', in the textbook by Bolton (1979) is a good example of a convincing introduction to the friction concept applied to soil. His treatment of the subject is clear, concise and yet sufficiently complete to provide a student with one of Burland's 'sheet anchor points'. Bolton's detailed consideration, right from the beginning, of the importance of deformation sets the subject firmly in the context of critical state soil mechanics. The laboratory session was designed to provide the student with personal experimental experience of many points raised by Bolton (1979).

### Summary of Activities

While the experiment is based on the use of the direct shear, or shear box, apparatus to measure the shear strength of sand, further activities have been added to focus attention on the concept of critical void ratio and to relate the experimental results to the stability of a sand slope.

#### Task No. 1 - Direct shear tests

In the present approach to the experiment, it is accepted from the outset that the sand is frictional and thus attention is restricted to a single value of normal load. Four tests are done on loose dry, compact dry, loose saturated and compact saturated, specimens of a well graded sand, to investigate the effect of initial soil density and the presence of water on the shearing behaviour of the sand.

#### Task No. 2 - Estimation of initial void ratios

Direct estimation of the initial void ratio of the sand in the shear box is not an easy task. A rapid estimation of the initial void ratios of both loose and compact sand may be obtained by partly filling a graduated cylinder with the dry sand. A maximum initial void ratio, corresponding to the sand being spooned slowly into the shear box, is obtained by inverting the graduated cylinder, containing the dry sand, and then tilting it slowly back into the upright position. Mass and volume readings, together with an assumed specific gravity of 2.65 for the sand particles, are used to estimate the initial void ratio of the loose dry sand. The loose sand in the cylinder is then compacted by vibration. During this process, the student may actually feel the increase in shear strength of the sand by using a 3mm welding rod as a poker

to assist compaction. A further volume reading provides an estimate of the initial void ratio of the compact sand.

### Task No. 3 - Angle of repose of dry sand slope

The construction of a conical mound of the loose dry sand facilitates the direct measurement of the angle of repose of the sand. Repeated undermining of the slope, by making a small cut at its toe, highlights that the slope is in a state of critical equilibrium and demonstrates the failure mechanism, a shallow near surface slide, typical of slope failures in natural slopes.

### Direct Shear Test - Typical Experimental Results

The experimental results, shown in Figure 1, were obtained by a group of students during a routine laboratory class and may be regarded as typical. The contrast between the dilatant behaviour of the initially compact dry sand and the contractant behaviour of initially loose dry sand was very clear. These results were regarded as 'good' by the student, because they conformed very well with textbook diagrams and were thus 'expected' results. Critical states of shearing at constant volume were attained in both cases.

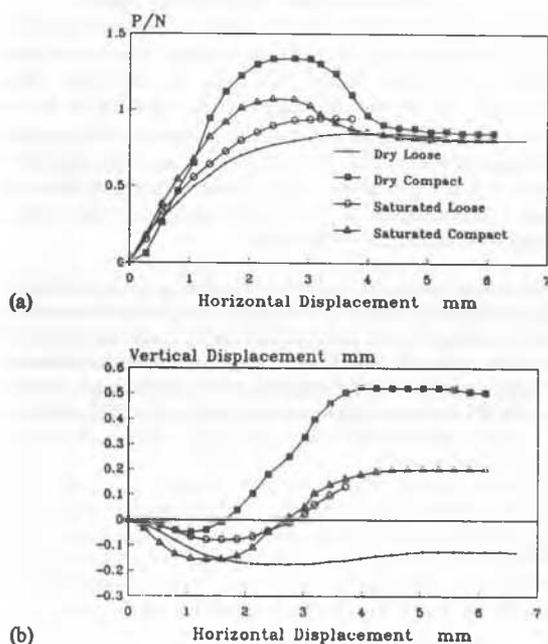


Figure 1. Direct shear test results

The tests on the saturated loose and compact sand presented a less convincing picture. The saturated compact sand exhibited a smaller peak shear than the dry compact sand. This smaller peak was consistent with the greater contraction initially and later the smaller rate and ultimate magnitude of dilation of the saturated compact sand. It also suggested that the saturated sand was less compact than the dry sand, despite the students' best efforts to compact them equally. The test on the saturated loose sand did not show 'expected' behaviour. The shearing load v. horizontal displacement curve looked good in that it rose to a plateau, admittedly at a level above that for the ultimate load in the other three tests. While the sand contracted initially, it was seen to dilate strongly at failure. Such strong dilation at a constant ultimate shearing load would appear to confound the notion that the sand had reached a critical state. Did something go wrong during the experiment? Was the apparatus to blame? Did somebody misbehave? Should the test result be dismissed as a rogue result and the experiment repeated? Did the test on the saturated loose sand serve to confuse rather than to clarify the emerging picture?

While it may be argued that the effect of the water is self evident, it is important that the student discovers at first hand that the mere presence of water does not have a lubricating effect on the frictional contact between the sand particles. On the basis of the test results in Figure 1, it could have been argued that the mere presence of water had little effect on the qualitative aspects of the response of the sand to shearing. On the other hand, the student could be forgiven for suspecting that the presence of water had made the compact sand weaker and the loose sand stronger. Was the principle of effective stress being undermined? Would it have been reasonable to invoke 'experimental error' as the culprit?

For the results shown in Figure 1, the  $\phi'$  values for ultimate failure lay in the range  $39^\circ$  to  $43^\circ$ , with the saturated loose sand giving the highest result. In three of the four cases, the vertical movement of the loading platen had virtually ceased, indicating that the sand was shearing at constant volume. These observations, taken together with the contrasting dilatant and contractant behaviour of the compact and loose sand, respectively, provided a reasonably strong indication that the sand had reached an essentially unique critical state of shearing. Had the fourth test not gone 'off the rails' the case would have been quite convincing. This series of tests represents probably the minimum amount of direct evidence required by the student to support the notion of the existence of such a critical state. The peak  $\phi'$  values of  $53^\circ$  and  $47^\circ$  for the dry and saturated compact sand, respectively, showed that the difference between peak and ultimate  $\phi'$  values could be related to the rate of dilation.

The direct evidence of Figure 1, which is a quite typical outcome of an undergraduate laboratory session, is overall less than convincing in presenting a unified view of the shearing behaviour of sand.

### Saw blade dilatancy model

Bolton (1979) used the saw blade dilatancy model, shown in Figure 2, as a simple, yet powerful, means of understanding the link between the peak  $\phi'$  and the angle of dilation. The basis of the model is that, while the macro-plane of shearing is the horizontal plane, the sand is actually shearing along a number of parallel micro-planes, inclined at the instantaneous angle of dilation to the horizontal. These micro-planes are connected by vertical planes, along which there is no significant interaction between the sand particles. Hence the zone of shearing activity appears conceptually as a saw blade. In the words of Bolton (1979):

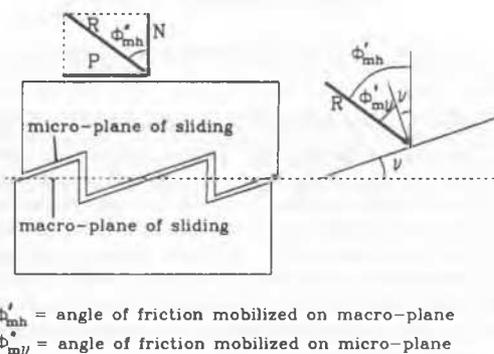


Figure 2. Saw-blade dilatancy model (Bolton, 1979)

"This dilatancy model predicts that the maximum or peak angle of friction is mobilised when the angle of dilation,  $\nu$ , is greatest, and that

$$\phi'_{\max} = \phi'_c + \nu_{\max}$$

Careful laboratory work substantiates the assertion that the peak is controlled by the angle of dilation, but also shows that the equation somewhat overestimates its contribution."

Thus, the saw blade dilatancy model is seen by Bolton (1979) and others as a "crude visualisation of dilatancy".

### Further analysis of direct shear test results

If the saw-blade dilatancy model does indeed provide a useful insight into the mechanism of shearing, it is reasonable that it should be used to interpret not only the peak shearing behaviour for compact sand but the shearing behaviour throughout each of the four tests. In this regard, it is assumed that shearing throughout the test duration is taking place, not along the horizontal macro-plane, but rather along the micro-planes, inclined at the instantaneous angle of dilation to the horizontal. Thus, the inclination of the instantaneous micro-planes of shearing is assumed to change continuously throughout the test.

The practical implementation of this analysis is really quite easy. The instantaneous angle of dilation is estimated for each data point in the plot of vertical displacement of the loading platen  $v$ , relative horizontal displacement of the two halves of the shear box. This is done directly from the recorded data, by calculating the slope of the line joining the data points on either side of the point of interest. The instantaneous angle of dilation,  $v$ , is  $\tan^{-1}(\text{slope})$ . The conventional mobilized angle of friction on the horizontal macro-plane is denoted by  $\phi'_{mh}$ .  $\tan^{-1}(\phi'_{mh})$  is plotted already as the ordinate in Figure 1(a). The angle of friction,  $\phi'_{mv}$ , mobilized on the instantaneous micro-planes of shearing is calculated as

$$\phi'_{mv} = \phi'_{mh} - v$$

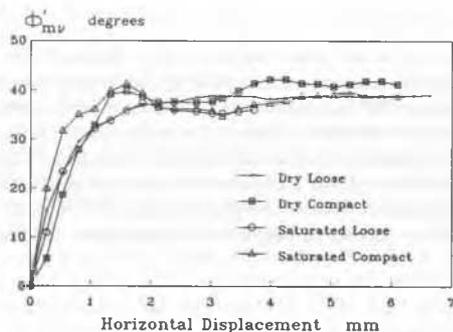


Figure 3. Mobilization of friction along micro-planes of shearing

This  $\phi'_{mv}$  is plotted against horizontal displacement of the box in Figure 3, for all four tests. Each of the four curves follows a similar pattern, with the mobilized angle of friction rising rapidly to a general plateau, with little further change as shearing continues. The initial downward movement of the platen in all four tests implies that the direction of shearing on the micro-planes is initially in a downward direction with respect to the horizontal macro-plane and that the mobilization of friction on these micro-planes is taking place more rapidly than the conventional plots in Figure 1 would suggest. To allow for the slight differences in the measured ultimate  $\phi'_{mv}$  values in each of the four tests, Figure 4 shows a plot  $v$ , horizontal displacement of  $\phi'_{mv}$ , normalized with respect to the ultimate  $\phi'_{mv}$  value in each test. Figures 3 and 4 present a relatively unified picture of the mobilization of friction within the sand during the four tests.

If the saw-blade dilatancy model is more than a 'crude visualisation of dilatancy' and is indeed a reasonably accurate global representation of the kinematics of the response of sand to shearing, it may be inferred that the fundamental development of frictional resistance within the sand is essentially independent of initial density. The peak shear strength associated with initially compact sand may be attributed substantially to the kinematic constraints on

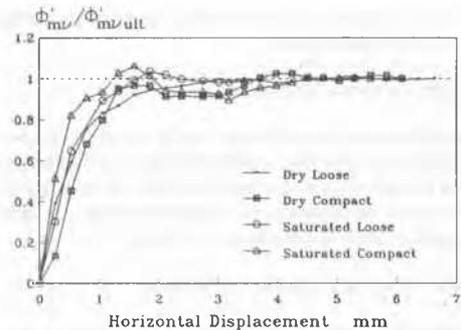


Figure 4. Normalized mobilization of friction

the direction of shearing imposed by the close packing of essentially rigid particles of sand.

The application of the saw-blade dilatancy model to the test data for the loose saturated sand proves particularly interesting. In this test, the attainment of an ultimate condition of  $\phi'_{mh}=43^\circ$  coincided with a significant and steady rate of dilation, with  $v=7^\circ$ . The saw-blade dilatancy model suggests that the sand was shearing ultimately on a plane inclined at  $7^\circ$  to the horizontal, with  $\phi'_{mv} = 43^\circ - 7^\circ = 36^\circ$ . The steadily increasing vertical displacement of the loading platen at a constant shearing load was not indicative of dilation, but rather of a mass movement of the sand above the failure zone at an angle of  $7^\circ$  to the horizontal. This analysis suggests that, during shearing, a plane of relative weakness was encountered at  $7^\circ$  to the horizontal and that further shearing movement was concentrated on that plane.

Thus, it may be concluded that a critical state of shearing at constant shearing load and no volume change was achieved in all four tests. The initial points of confusion in the conventional plots are resolved and the 'rogue' result for the loose saturated sand makes sense in the end. The respective contributions of intergranular friction and of particle interlocking to the shear strength of sand are effectively separated through the systematic application of a simple model to the test data.

### CONCLUSION

Carefully structured laboratory work can make a significant contribution to the learning process, where it is used to clarify fundamental ideas. The concept of shearing embodied in the saw-blade dilatancy model is grasped readily by the student. Its application to the analysis of the direct shear test on sand provides the student with clear evidence of the pursuit of the critical state by the sand under shear, regardless of initial density. Of particular importance is the ability of the analysis to deal adequately with experimental data, which might otherwise be a source of confusion.

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