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## INSTABILITY ANALYSIS FOR TAILINGS SLOPES ANALYSE DE L'INSTABILITE DES BARRAGES DE REMBLAI

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**SYNOPSIS:** Experimental and theoretical studies have shown that cohesionless soils can become unstable inside the failure surface. Instability or the inability to sustain a given load has been observed in compressing soils of low permeability under undrained conditions. Such conditions may be encountered in loose fine sands and silts. The analysis of a tailings slope, which should remain stable according to conventional limit equilibrium stability procedures, is presented to show that it could become unstable due to small disturbances and proceed to fail catastrophically.

### INTRODUCTION

Stability postulates by Drucker (1951) and by Hill (1958) imply that granular materials exhibiting nonassociated flow may be unstable when exposed to certain stress paths *inside* the failure surface. Series of conventional triaxial tests on fully saturated and partly saturated specimens have been performed under drained and undrained conditions to study the regions of stable and unstable behavior (Lade et al., 1987, 1988; Lade and Pradel, 1990; Lade, 1992). The results of these experiments show that the stability postulates by Drucker and by Hill fail to capture the conditions for instability of granular materials.

For specimens that compress and have degrees of saturation higher than critical, undrained conditions lead to effective stress paths with decreasing shear stresses, and run-away instability is observed provided the yield surface opens up in the outward direction of the hydrostatic axis. Thus, instability occurs *inside* the failure surface. Instability is not synonymous with failure, although both may lead to catastrophic events. The location and determination of the instability line is discussed in detail.

If instability is defined as a condition for which the current, applied shear stress cannot be sustained for perturbations in the stress state, then compressive as well as dilative materials may be considered to be unstable, whether drained or undrained, in the region where the yield surface opens up in the outward direction of the hydrostatic axis (Lade, 1992). In this region, plastic strains

can be produced under decreasing stresses. For undrained conditions and compressive material, the instability is self-sustaining and unconditional, i.e. it is not dependent on conditions outside the soil element. For drained conditions, the instability is conditional, i.e. the decrease in load carrying capability depends on the reduction in effective confining pressure. This reduction may occur as a decrease in total confining pressure or as an increase in pore pressure.

The analysis of a tailings slope, which should remain stable according to conventional limit equilibrium stability, is presented to show that it could become unstable due to small disturbances and proceed to fail catastrophically. The conditions of instability, both unconditional and conditional, are recognizable in different zones within the potentially unstable tailings slope. The proposed analysis procedure is not based on limit equilibrium procedures, which involves simultaneous shear failure along a slip surface. Rather, the procedure involves determination of the state of stress in the slope and evaluation of stresses relative to the instability line. Instability occurs as a progressive event which requires initiation at a point or in a zone within the slope. Thus, a slip surface is not recognizable from the analysis procedure.

### DETERMINATION OF INSTABILITY LINE

Instability and failure are two different behavior aspects of soils that exhibit nonassociated flow (Lade, 1989). Although both may lead to cata-

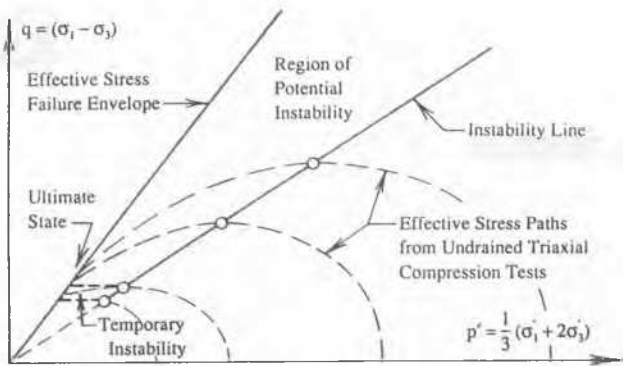


Fig. 1. Location of instability line in  $p'$ - $q$  diagram.

strophic events, they are not synonymous.

It is the fact that loading of a compressible soil (resulting in large plastic strains) can occur under decreasing stresses that leads to unstable behavior under undrained conditions. Loose, fine sands and silts have relatively low permeabilities, and small disturbances in load or even small amounts of volumetric creep may produce undrained conditions in such soils, and instability of the soil mass follows. As long as the soil remains drained, it will remain stable in the region of potential instability.

When the condition of instability is reached, the soil may not be able to sustain the current stress state. This stress state corresponds to the top of the undrained effective stress path, corresponding to  $(\sigma_1 - \sigma_3)_{max}$ , which occurs slightly after but very close to the top of the teardrop shaped yield surface. Fig. 1 shows a schematic  $p'$ - $q$  diagram in which the line connecting the tops of a series of effective stress paths from undrained tests on loose soil provides the lower limit of the region of potential instability. In the region above this instability line the soil can deform plastically under decreasing stresses. Experiments show that this line is straight (Lade, 1992). Since it traces the top points of the yield surfaces which evolve from the origin of the stress diagram,

the instability line also intersects the stress origin.

A region of temporary instability is located in the upper part of the dilating zone, as shown in Fig. 1. It is a region where instability may initially occur, but conditions allow the soil to dilate after the initial instability, thus causing the soil to become stable again. The approximate lower limit of the temporary instability region may be obtained from the intersection of the instability line and the total strength envelope, as shown schematically in Fig. 2. For very loose soils the total strength envelope intersects the stress origin, and the region of potential instability reaches down to the origin of the stress diagram. Previous studies of sand instability have been presented by Sladen et al. (1985) and Vaid and Chern (1985).

### INSTABILITY OF TAILINGS SLOPE

An example slope is analyzed to demonstrate that a tailings slope or a spoil heap consisting of loose granular materials with permeabilities near those of fine sands and silts may become unstable under essentially static loading conditions. Such slopes are often not engineered, but simply created by dumping material in a loose state.

The slope shown in Fig. 3 has an inclination of horizontal: vertical = 2:1 (inclination angle = 26.6°), a height of  $H = 45.7$  m, and effective strength parameters of  $c' = 0$  and  $\phi' = 30^\circ$ . The soil in the slope is assumed to have a void ratio of 0.89 and a dry density of  $13.75 \text{ kN/m}^3$ . It is also assumed to be nearly saturated, say due to rain water infiltration, with a degree of saturation of 97%, and the total density is therefore  $\gamma_t = 18.2 \text{ kN/m}^3$ . Only vertical water flow inside the slope is assumed and the pore water pressure is therefore dissipated as the water infiltrates the slope.

Conventional slope stability methods indicate that the most critical failure surface for a cohesionless slope is parallel to the slope surface. Thus, an infinite slope stability analysis

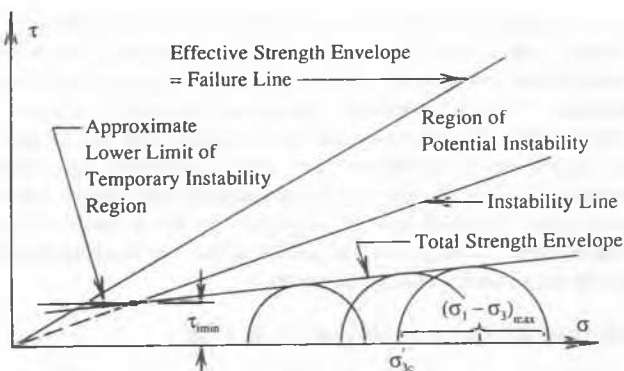


Fig. 2. Determination of approximate lower limit of temporary instability region for cohesionless soil.

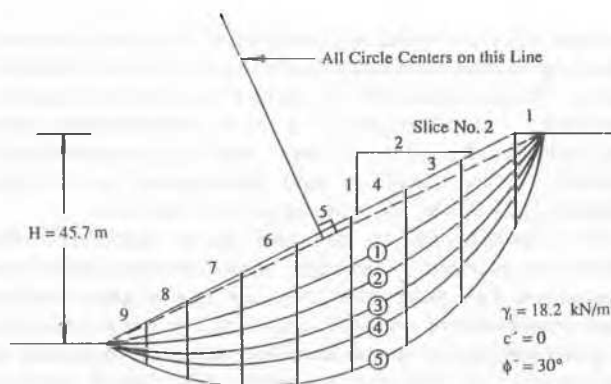


Fig. 3. Example slope with circular slip surfaces for analyses of static state of stress.

of a slope with vertical seepage produces

$$F = \frac{\tan \phi'}{\tan \alpha} = \frac{\tan 30^\circ}{\tan 26.6^\circ} = 1.15 \quad (1)$$

indicating that the slope should be stable.

To determine whether the slope can become unstable, it is necessary to compare the states of stress in the slope with those required to produce unstable behavior. Conventional slope stability analyses methods may be employed to evaluate the state of stress to an approximate degree. This approach has been used to determine the consolidation stress states in slopes (e.g. Lowe and Karafiath, 1960).

Spencer's slope stability method (Spencer, 1967; Pyke, 1984) was employed to determine the approximate effective stress state along five circular slip surfaces as seen in Fig. 3. Since none of these circles are critical, they all produce factors of safety above unity, and the stress states can be calculated along each slip surface. Because the calculation procedure requires the factor of safety to be constant along a given circle, the computed stress states are located on a straight line given by

$$\tau = \frac{c' + \sigma' \cdot \tan \phi'}{F} = c'_d + \sigma' \cdot \tan \phi_d \quad (2)$$

For a cohesionless slope  $c' = c'_d = 0$ , and the straight line goes through the origin of the  $\tau$ - $\sigma'$  diagram. Fig. 4 indicates the stress states obtained from the five circles shown in Fig. 3. Some of the stress states reach up into the region of potential instability, whose lower limits are described by  $\phi'_i = 19^\circ$  and  $\tau_{\min}/p_a = 0.50$ . Thus, the slope is potentially unstable according to the concepts presented here.

To obtain the region within the slope in which the stresses reach values that are in the area of potential instability in Fig. 4, the stress states (calculated at the middle of each slice) have been plotted in Fig. 5(a). This diagram shows values of  $\tau/p_a$  versus the horizontal distance from the toe.

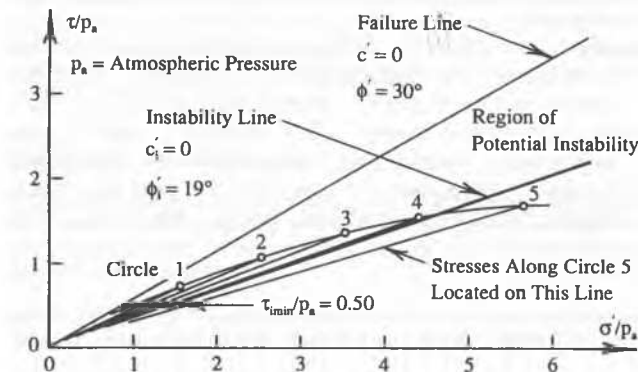


Fig. 4. Stress states along circular slip surfaces reach into region of potential instability.

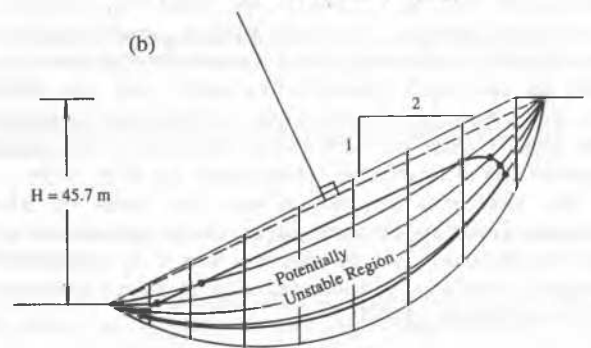
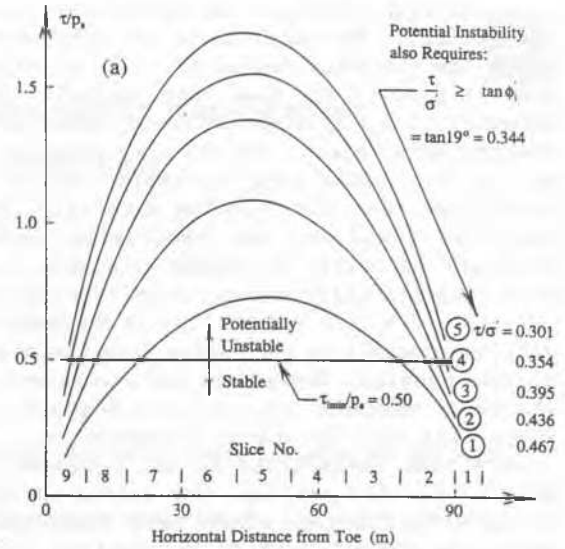


Fig. 5. (a) Detailed analysis of stress state relative to instability line, and (b) determination of region of potential instability in slope.

The stress states for which

$$\tau/p_a \geq \tau_{\min}/p_a \quad \text{and} \quad \tau/\sigma' \geq \tan \phi'_i \quad (3a,b)$$

are within the region of potential instability. The range of stresses for which these inequalities are fulfilled is then transferred to the respective circles in Fig. 5(b), and a region of potential instability may be identified within the slope. The reliability of this method for determination of the stress state in a slope was studied by Lade (1992).

According to this analysis procedure, whose results are shown in Fig. 5(b), a region is present in which instability may be induced under essentially static loading conditions, e.g. due to increasing stresses caused by the weight of additional rain water. Once a local zone of instability has been created, the resulting pore pressure buildup will propagate and enlarge the unstable region in the slope. The instability initiated in the potentially unstable region is self-sustaining, i.e. the material is not dependent on any further outside perturbations, and it

consequently exhibits unconditional, run-away instability. The material in the sloping surface above the unstable region is in a dilating mode, and initially it does not exhibit unstable behavior, i.e. it forms a rigid crust on top of the unstable region. As the pore pressure builds up in the underlying unstable region, water penetrates into the dilating material, increases the pore pressure, and eventually causes the dilating material to become unstable as well. Thus, the initially stable crust is conditionally unstable, i.e. its instability is dependent on the continued supply of pore water from the underlying unstable region. Therefore, a progressively larger volume of unstable soil will be engaged, and the slope will fail by static liquefaction.

Note that instability is not produced along a particular slip surface, but rather in a volume of soil within the slope, and classical slope stability methods cannot be used to produce a factor of safety. A slope with a large region of potentially unstable soil is perfectly stable as long as the soil remains drained. But the moment the soil becomes undrained, it becomes unstable, and the volume of unstable material will spread beyond the boundaries indicated on Fig. 5(b).

The analysis procedure and the mode of slope failure considered here is in close agreement with the mechanics, the geometry, and the sequence of events in static liquefaction of a slope presented by Casagrande (1975).

## CONCLUSIONS

Soils exhibit nonassociated flow and they may therefore become unstable *inside* the effective stress failure surface. Experiments have shown that soils are stable as long as they remain under drained conditions (Lade and Pradel, 1990). Instability may be obtained under undrained conditions in the region where the yield surface opens up in the outward direction of the hydrostatic axis. Initiation of instability also requires that the soil tends to compress during undrained shear. Thus, loading (i.e. hardening inside the failure surface resulting in large plastic strains) can occur under decreasing stresses, and this leads to unstable behavior under undrained conditions. The location of the lower boundary for instability, herein called the instability line, is identified. Together with the effective stress failure line it defines the region of potential instability. The instability of a slope representing a tailings dam or a spoil heap is analyzed. Conventional stability analysis indicates the slope to be stable. The analysis procedure presented here clearly show the potential for instability, as actually observed in many tailings dams or spoil heaps of granular materials with properties of loose, fine sands and silts.

## Acknowledgment

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