PULLOUT RESISTANCE FORMULA OF SHORT VERTICAL SPHERICAL ANCHORS IN SOIL

CAPACITE PORTANT DES TIRANTS GEOTECHNIQUE COURT VERTICAL DANS LE SOL AVEC LA ZONE D'ANCRAGE SPHERIQUE

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SYNOPSIS: One of the geotechnical anchor divisions is based on the character of the anchor embedment (anchor tail). According to this division distinction is made among anchors with: a) point, b) line, c) plane and d) volume embedment. Because of the simplicity of installation, the line embedment anchors are in wide use at the present time, but a significant increase of bearing capacity is achieved, if a volume embedment anchor is applied. A special case of the volume embedment anchor is a spferical tail anchor. In this paper an interaction mechanism anchor - soil is described for the case of short vertical spherical anchors installed in a c-ϕ soil. Commencement is made with the eqiulibrium state theory of Vesić (1965) for the case of expanding spherical cavity located at the shallow depth under the soil surface. Vesić (1971) applied this theory to determine the ultimate bearing capacity of short vertical anchors with plane embedment (circular and square anchor plate and strip anchor plate). The bearing capacity formulas Vesić derived separately for cohesionless ($c=0, \phi\neq0$) and for cohesive ($c\neq0, \phi=0$) soils. It is shown in the paper how the above stated work of Vesić has been adapted to the spherical tail anchors. The ultimate bearing capacity equation, which is given for such anchors, is derived for a c-ϕ soil. Finally, an actual example is added, showing an extremely good conformity of the computed with the in situ recorded ultimate bearing capacity of the short vertical spherical anchor.

INTRODUCTION

A geotechnical anchor simply may be taken to consist of the following 3 main parts (Fig.1.):
① anchor head, ② free length, ③ embedment (anchor tail).

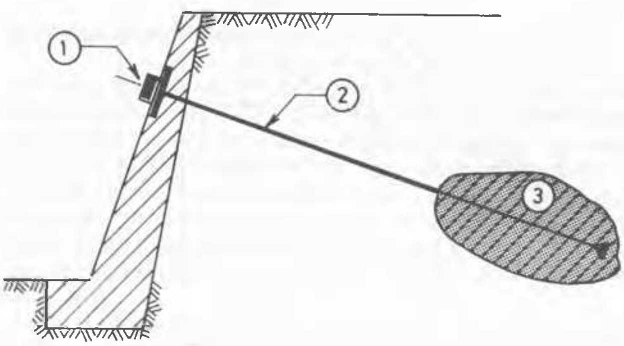

Fig. 1. Main parts of geotechnical anchor

Among various possibilities of classifying the geotechnical anchors, there exists also a classification according to the manner in wich a force is transmitted from embedment into soil. As this transmission primarily depends on the shape of the embedment, four embedment types may generally be distinguished (Muhovec,1983): point, line, plane and volume embedments.
Spherical anchor embedments represent a special case of the volume embedments. In this paper, a gradual presentation of a model of computation will be given, which at the end leads to the ultimate bearing capacity formula of a short vertical geotechnical anchor installed in c-ϕ soil. The proceeding is based on the works of Vesić (1965,1971) representing a modification and a synthesis of the fundamental Vesić's expression for the bearing capacity of the anchors in the soil. Comparison between anchor bearing capacity obtained by the theoretical expression and in-situ testing results is given, too.

RETROSPECTIVE VIEW OF THE VESIĆ'S THEORY

The problem of the ultimate bearing capacity of short vertical anchors installed in a c-ϕ soil, Vesić approached in particular manner (Vesić, 1965).

General Model

Vesić (1965) studied the effect of an explosive point charge expanding a spherical cavity close to the soil surface (Fig.2.).

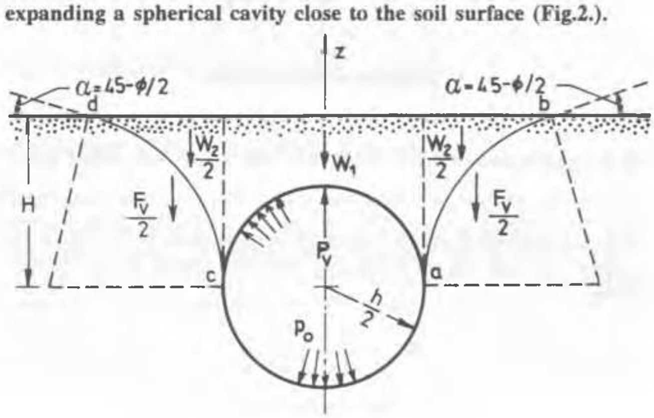

Fig. 2. Vesić's equilibrium state theory of a spherical cavity expansion in soil

Referring to Fig.2. it can be seen that, if the distance H is small enough, there will be an ultimate pressure p_o (as a result of gases generated by explosion) that will overcome the shearing strength of the soil along the critical slip surfaces ab and cd. At that time the diameter of the spherical cavity is equal to h. The geometry of the slip annuloid is indicated in the Fig.2.
Using the fundamental condition of the static equilibrium $\Sigma F_z = 0$, there will be:

$$P_v - (W_1 + W_2) - F_v = 0 \qquad (1)$$

where, F_v denotes a vertical component of internal force developed through the annuloid slip surface. Vesić developed expressions for each individual force and substituted them into equation (1). As a result, he obtained an expression for the unknown ultimate pressure p_o:

$$p_o = c\bar{F}_c + \gamma H \bar{F}_q \qquad (2)$$

where:

$$\bar{F}_q = 1 - \frac{2}{3}\left[\frac{(h/2)}{H}\right] + A_1\left[\frac{H}{(h/2)}\right] + A_2\left[\frac{H}{(h/2)}\right]^2 \qquad (3)$$

$$\bar{F}_c = A_3\left[\frac{H}{(h/2)}\right] + A_4\left[\frac{H}{(h/2)}\right]^2 \qquad (4)$$

A_1, A_2, A_3, A_4 are functions of soil internal friction angle ϕ.

Anchors in Cohesionless Soil

The above theoretical concept Vesić(1971) applied to determine ultimate bearing capacities of short vertical anchors with circular anchor plate (Fig.3.).

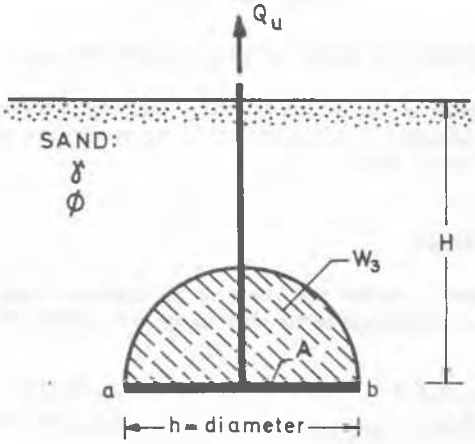

Fig. 3. Application of theory to anchor with a circular anchor plate

If the hemispherical cavity above the anchor plate is imagined to be filled up with soil material (γ), then the weight W_3 of the fill will be:

$$W_3 = \frac{2}{3}\pi\left(\frac{h}{2}\right)^3\gamma \qquad (5)$$

The weight W_3 produces an average pressure on the circular anchor plate of a surface area $A = \pi(h/2)^2$:

$$p_1 = \frac{W_3}{A} = \frac{1}{3}\gamma h \qquad (6)$$

If a cohesionless soil ($c=0$, $\phi \neq 0$) is assumed, the earlier given expression (2) reduces to:

$$p_o = \gamma H \bar{F}_q \qquad (7)$$

Summation of the pressures p_o and p_1 gives an ultimate pressure q_u, which will be effective on the upper surface of the circular anchor plate at the time the anchor is pulled out of soil. Therefore:

$$q_u = \frac{Q_u}{A} = p_o + p_1 = \gamma H\bar{F}_q + \frac{1}{3}\gamma h = \gamma H\left[\bar{F}_q + \frac{2}{3}\,\frac{(h/2)}{H}\right] \qquad (8)$$

Substituting expression (3) into (8) gives:

$$q_u = \gamma H\left\{1 + A_1\left[\frac{H}{(h/2)}\right] + A_2\left[\frac{H}{(h/2)}\right]^2\right\} = \gamma H F_q \qquad (9)$$

Finally, expression for ultimate force causing the onset of anchor extraction or soil failure, is given by:

$$Q_u = A q_u = \frac{h^2\pi}{4}\gamma H F_q \qquad (10)$$

The factor F_q is called a breakout factor,

$$F_q = 1 + A_1\left[\frac{H}{(h/2)}\right] + A_2\left[\frac{H}{(h/2)}\right]^2 \qquad (11)$$

and it represents a function depending on geometric anchor design relations ($H/(h/2)$) and on soil internal friction angle ϕ (through A_1 and A_2).

In Fig.4. the breakout factor F_q is plotted as a function of the ratio H/h and of soil internal friction angle ϕ.

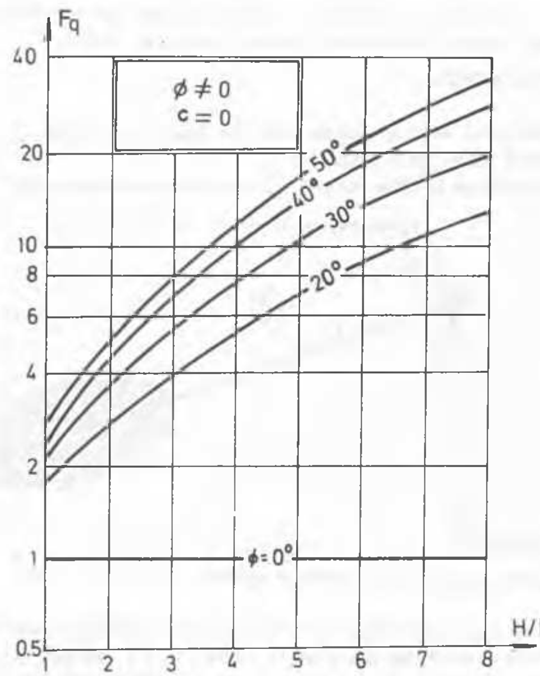

Fig. 4. Plot of Vesić's(1971) breakout factor F_q for anchors with a circular anchor plate

Anchors in Cohesive Soil

The approach is made on the assumption that in a completely saturated cohesive clayey soil, conditions prevail under which only undrained shearing strength is mobilized, that is, $\phi=0$, $c=c_u$.

Expression for the ultimate bearing capacity for an anchor with circular embedment ($A = \pi(h^2/4)$) in a cohesive soil is derived in analogy to the case of cohesionless soil:

$$Q_u = A(\gamma H + cF_c) \qquad (12)$$

where F_c = breakout factor with respect to the soil cohesion (Fig.5.).

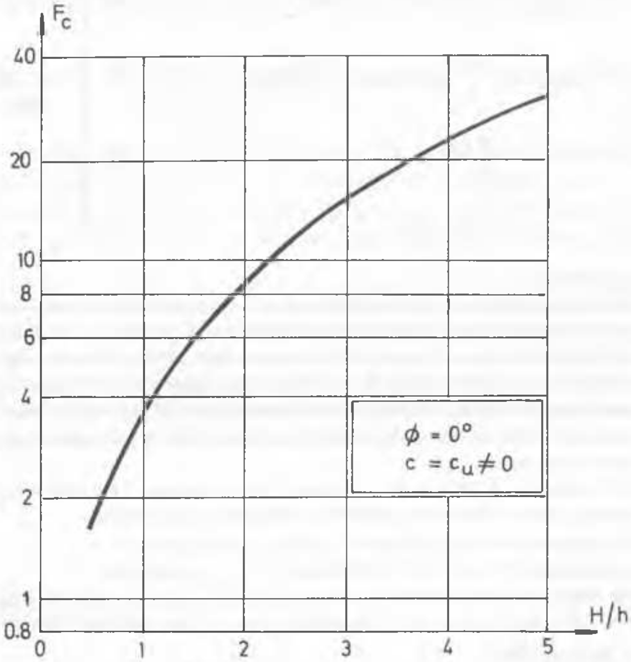

Fig. 5. Plot of Vesić's(1971) breakout factor F_c for anchors with shallow circular shaped embedment

Application Range of Vesić's Theory

Comparing some recent model test results concerning the anchor bearing capacity, Das (1990) found out that Vesić's theory in all probability gives a closer estimate for shallow anchors embedded in softer clay. Das reports further, that the breakout factor F_c gradually increases with embedment ratio H/h but only to a certain maximum value, remaining constant thereafter irrespective of further increase of the anchor depth H (Fig.6.).

Accordingly, the maximum value of F_c (max $F_c = F_c^*$) is reached at $H/h = (H/h)_{cr}$.

Anchors located at $H/h > (H/h)_{cr}$ are referred to as long (or deep) anchors. For these anchors, at ultimate load, local shear failures in soil located around the anchor take place. These failures continuously progress in the direction in which the anchor is extracted.

On the other hand, all anchors located at $H/h < (H/h)_{cr}$ are considered as shallow anchors.

In consequence of the experimental results of Meyerhof (1973), it appears that for anchors with circular and square embedment the following holds:

$$\left(\frac{H}{h}\right)_{cr} = 7{,}5 \qquad (13a)$$

and for the case of strip shaped embedments:

$$\left(\frac{H}{h}\right)_{cr} = 13{,}5 \qquad (13b)$$

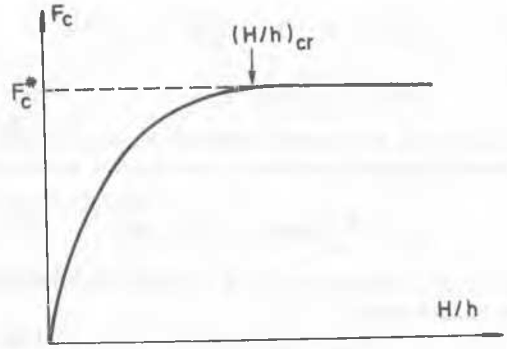

Fig. 6. Nature of variation of factor F_c with the embedment ratio (H/h)

BEARING CAPACITY OF SHORT VERTICAL SPHERICAL ANCHORS

The consideration of the vertical anchors installed closly under the soil surface is continued. Instead of a circular anchor plate, we now consider an anchor with spherical embedment (Fig.7.).

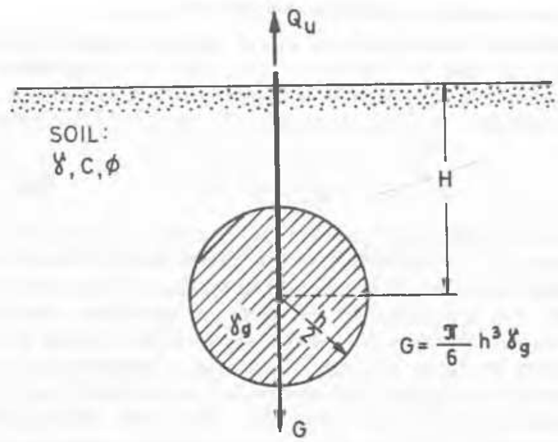

Fig. 7. Short anchor with spherical embedment

Modification of Bearing Capacity Equations

Expression (10) and (12) which are valid for bearing capacity of anchors with circular embedment, will now be modified so that they may be used for anchors with spherical embedments.

Briefly, the difference consists in a favourable effect the weight difference ($G-W_3$) will have on the bearing capacity of spherical anchors:

$$\Delta Q_u = G - W_3 \qquad (14)$$

This weight difference is directly added to the Vesić's bearing capacity expression (now indicated by $\bar{Q}_u$). G is a weight of the spherical embedment (of an average unit weight γ_g) and W_3, as earlier, weight of soil in an imagined hemisphere above the circular embedment. Therefore, the bearing capacity for a cohesionless soil ($c=0$, $\phi \neq 0$) will be:

$$Q_{us} = \bar{Q}_{us} + \Delta Q_u \qquad (15)$$

According to equation (10):

$$\bar{Q}_{us} = A\gamma H F_q$$

and:

$$\Delta Q_u = G - W_3 = \frac{\pi}{6}h^3\gamma_g - \frac{\pi}{12}h^3\gamma$$

that is

$$\Delta Q_u = \frac{h^3\pi}{12}(2\gamma_g - \gamma) \qquad (16)$$

Finally, expression for modified ultimate bearing capacity of a short vertical spherical anchor in a cohesionless soil takes the form:

$$Q_{u\phi} = \frac{h^2\pi}{4}\left[\gamma H F_q + \frac{h}{3}(2\gamma_g - \gamma)\right] \qquad (17)$$

For the case of a cohesive soil ($\phi=0$, $c=c_u\neq0$), the modification procedure is analogous:

$$Q_{uc} = \bar{Q}_{uc} + \Delta Q \qquad (18)$$

Substituting and rearranging gives:

$$Q_{uc} = \frac{h^2\pi}{4}\left[\gamma H + cF_c + \frac{h}{3}(2\gamma_g - \gamma)\right] \qquad (19)$$

It is important to notice that breakout factors F_q and F_c, entering the equations (17) and (19), are identical to those in the original bearing capacity equations (10) and (12) which means that their values may again be determined by the aid of the plots in Fig.4. and Fig.5.

Bearing Capacity of Spherical Anchors in a c-ϕ Soil

If spherical anchors embeded in soil having an angle of internal friction ϕ and a cohesion c (c-ϕ soil) are considered, an approximative expression for bearing capacity of such anchors can be established by using equations (17) and (19) in the following way:

$$Q_u = Q_{u\phi} + Q_{uc} - G - W_s \qquad (20)$$

On the right side of the equation (20) four terms appear. The first of them ($Q_{u\phi}$) depends on internal friction angle (ϕ) but it also includes the weight of the spherical embedment (G) and the weight of the soil contained in the body of revolution above the embedment ($W_\phi=W_1+W_2$), what is expressed by equation (1) and indicated by Fig.2. The second term (Q_{uc}) depends on the soil cohesion (c) but again, it includes the both weights, that of the spherical anchor tail (G) and that of the soil column above embedment ($W_c=W_1$). As it is seen, the weights of embedment and of soil overburden are included twice in the two first terms.
The excessive weight is eliminated by the remaining two terms on the right side of equation (20).
It should be emphasized that in his original work (Vesić, 1971) the author does not obtain the same ballast annuloid above anchor embedment, if transition is made from a cohesionless model to the special case c=0, ϕ=0 in comparison with that annuloid which results, if transition is made from a cohesive model to the special case c=0, ϕ=0. Shortly, in the first case the weight of the ballast annuloid is $W_\phi = W_1 + W_2$ and in the second one, $W_c = W_1$ (Fig.2.).
When combining expressions (17) and (19) for the purpose of setting a single but approximative bearing capacity equation for anchors in a c-ϕ soil, a weight $W_c = W_1$ is set for the fourth term on the right side of equation (20) and denoted as W_s. In this way a logical conception is completed, according to which a basic term in the bearing capacity equation is $Q_{u\phi}$ and then, because of existence of soil cohesion, the term Q_{uc} is added to that basic term. This undoubtedly must increase the total bearing capacity Q_u, irrespective of the imperatively required subtraction of the two last terms (G and W_s) which had to be substituted into equation (20) just because of the fact the term Q_{uc} is added. Namely, if the term $W_\phi = W_1 + W_3$ would be subtracted, it might often be the case that the addend Q_{uc} is smaller than the subtrahend W_ϕ, and this would be a paradox, because in cases like this the existence of soil cohesion would decrease the total anchor bearing capacity Q_u.
Considering the given argumentation, the four terms in the bearing capacity equation (20) are:

$$\left.\begin{aligned}
Q_{u\phi} &= \frac{h^2\pi}{4}\left[\gamma H F_q + \frac{h}{3}(2\gamma_g - \gamma)\right] && (a)\\
Q_{uc} &= \frac{h^2\pi}{4}\left[\gamma H + cF_c + \frac{h}{3}(2\gamma_g - \gamma)\right] && (b)\\
G &= \frac{\pi}{6}h^3\gamma_g && (c)\\
W_s &= \frac{\pi}{4}h^2\gamma\left(H - \frac{h}{3}\right) && (d)
\end{aligned}\right\} \qquad (21)$$

EXAMPLE

Within the scope of the scientific research project being realized for several years at the Geotechnical Engineering Faculty in Varaždin, of the University of Zagreb (Croatia), a total of 27 short vertical anchors have been installed and tested (Krajcer 1993). Fifteen of them being anchors with spherical embedment (using a controlled explosive effect in the embedment zone induced by a low-shattering power explosive).
For anchors of the group indicated G2 (4 pieces), the following average data values are available (lab and in-situ data):
Soil properties: $c=10$ kN/m² $\phi=25°$ $\gamma=19$ kN/m³
In-situ data: H=1,2 m h=500 mm $Q_{u\ in\ situ}=40{,}5$ kN
For filling the embedments a low consistency concrete MB 30 with γ = 24 kN/m³ was used. A practical value of the bearing capacity is to be verified.
For H/h = 1,2/0,5 = 2,4 → F_q = 3,9 F_c = 11 (plots in Fig.4. and Fig.5.) From equations (21):

$Q_{u\phi}$=18,40 kN Q_{uc}=27,02 kN G=1,57 kN W_s=3,85 kN

So: Q_u = 40,00 kN = $Q_{u\ cal}$

CONCLUSION

With regard to the fact that in this case the ratio $Q_{u\ in\ situ}/Q_{u\ cal}$ has a value close to one (1,01), a better confirmation of the validity of the presented model of computation is not needed. Within the framework of the mentioned scientific research, good results were obtained also for the remaining anchors.
This model, however, should be subjected to a further examination in practice and the actual bounds of application thus established for the suggested solutions.

REFERENCES

Das,B.M.(1990),"Earth Anchors",Elsevier,Amsterdam-Oxford-New York-Tokyo.

Krajcer,M., Muhovec,I. & Pranjić,J.(1993),"Ground Anchor Bearing Capacity Improvement by Blasting in Bore-Hole",paper submitted to III Intl. Conf. on Case Histories in Geotechnical Engrg. St. Louis, USA.

Meyerhof,G.G.(1973), "Uplift Resistance of Inclined Anchors and Piles", Proc., VIII Intl. Conf. Soil Mech. Found. Engrg., Moscow, USSR.

Muhovec,I.(1983), "Geotechnical Anchors", paper in Croatian, comprised in a book: "Mehanika stijena, temeljenje, podzemni radovi", DGIT-Zagreb, DMSPR SR Hrvatske. Zagreb, Croatia.

Vesić,A.S.(1965), "Cratering by Explosives as an Earth Pressure Problem", Proc., VI Intl. Conf. Soil Mech. Found. Engrg., Montreal, Canada.

Vesić,A.S.(1971), "Breakout Resistance of Objects Embedded in Ocean Bottom", J. Soil Mech. Found. Div., ASCE, 97 (9).