ROLE OF NUMERICAL METHODS IN SOLVING PRACTICAL PROBLEMS

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SYNOPSIS: Problem solving in geotechnical engineering requires thorough site investigation, estimation of values for soil parameters, creative design, construction in which appropriate monitoring is done, and interpretation of that monitoring so as to confirm expectations of the design. Numerical methods can make a very significant contribution to the design analysis phase of the design process, they can also perform an invaluable function when it comes to interpreting measurements of displacements and pressures etc. This paper reviews the manner in which numerical methods fit into the wider spectrum of the solution of practical problems in geotechnical engineering. Progress in the development of numerical methods over the last quarter of a century or so has been nothing short of spectacular. It is suggested that possibly the greatest limitation to application of numerical methods in solving practical problems are the restrictions posed by difficulties in estimating values for soil property values.

INTRODUCTION

The topic for Discussion session 2.1, *Computer Applications in Geotechnical Engineering*, and this paper addressing the role of numerical methods in solving practical problems, could overlap with that of plenary session A, *Soil Properties*, in which Professor J. M. Duncan is giving a State-of-the-Art Report entitled: "*Role of advanced constitutive formulations in solving practical problems*". Herein it also seems necessary to mention constitutive modelling but it will not form a major feature of the material covered. Rather the emphasis will be focused on the *solving of practical problems* in geotechnical engineering and the place that computer methods play in this process. It might seem repetitive at a conference such as this to have two invited papers with the words *solving practical problems* in the title. However, this is not inappropriate as the objective of our sub-discipline of civil engineering is to solve practical problems. We know that there are a range of intellectually demanding and fascinating problems to challenge the geotechnical engineer, but even so our discipline exists to solve, subject to the constraint of minimising hazard and waste, and with due economy, those practical problems that arise in the development of the physical resources of the planet.

The question we have to address is the role of computer applications, and numerical methods in particular, in achieving our goals. For the purposes of this paper numerical methods are assumed to encompass all activities that use a computer for calculation, data processing, and presentation. This is clearly much broader than the traditional association of the term *numerical methods* with sophisticated computer codes based on discretisation procedures such as finite elements, boundary elements, discrete elements, and finite differences. In particular this definition includes the use of general purpose calculation facilities such as spreadsheets and similar software.

It is on interest to refer back to the Eleventh International Conference of the ISSMFE which took place in San Francisco in 1985. Ralph Peck delivered a most stimulating address to the closing session of the conference entitled "The last sixty years" (Peck, 1985). In this he emphasised the exceedingly rapid development of numerical techniques and suggested that a numerical calculation subculture had arisen. Since then this activity has become established as Computational Mechanics which embraces a much wider field than numerical applications in geotechnical engineering. It still remains true, as implied by Peck, that some of this work is not properly focused on applications. Nevertheless numerical methods have, and will continue to have, great potential for contributing to the problem solving process in geotechnical engineering.

Since the comments of Peck in 1985 a major development has been the amazingly rapid increase in the capabilities of the personal computer. These machines are now capable of performing in a modest time quite complex numerical calculations, calculations that a decade ago would have required a mainframe computer.

The development of computer techniques has been the most rapidly expanding aspect of our discipline for the last few decades. Because of this the literature on the subject is now vast and this paper must be limited in the space available to a survey of topics of which the author has some personal experience.

The main conclusion of the paper is that the application of numerical methods is one facet of the geotechnical problem solving process. Within this limitation though the range of applications is extensive. The current sophistication of numerical methods is such that our computational ability has outstripped our ability to determine numerical values for soil properties. At present this seems to be the main obstacle to the further application of numerical methods in solving practical problems. Nevertheless the rapid expansion of numerical capabilities over the last few decades has contributed to advances in many facets of geotechnical engineering.

PRACTICAL PROBLEMS IN GEOTECHNICAL ENGINEERING

There is a large range of practical problems in geotechnical engineering and it is not possible or wise to classify or list all of these, but it will helpful for our subsequent discussions to at least mention some of the more important categories.

Foundation Loading

This covers both near surface and deep foundations under static loads as well as dynamic forces from wind, wave, earthquake and vibrating machines. Estimates of the capacity of the foundation system as well as likely displacements are needed. These questions are traditional topics covered in any text book on foundation engineering. In addition environmental factors, such as fluctuating water table and seepage from below, shrinking and swelling, freezing and thawing, weathering and/or chemical changes in the soil and rock at the site need to be considered. When piles penetrate soft soil subsequent adjacent surface filling will cause consolidation of the soil surrounding the piles. This consolidation induces downdrag forces on the pile shaft which also need to be estimated.

Retaining Structures

Traditionally these are of the gravity type, those which derive support from embedment in the soil below, or those which have tied back support. Once again the standard design question involves the structural capacity of the wall, the factor of safety against sliding at the base, and prevention of a bearing failure beneath the wall which is manifested in a rotational collapse mechanism. There is also the question of movement and rotation of the wall under the expected loads. Additional concerns are the pressure distribution applied to the wall structure because of the method of backfill placement, freezing and thawing, drainage of the groundwater in the backfill, and earthquake loading. Many ingenious modern techniques of earth reinforcement and tieback/anchoring have been invented which give economical solutions to retaining problems but which raise questions of the long term durability and corrosion resistance of the system.

Excavations and underground services

Excavations are required for the short term in the construction of basements for multistorey buildings and for the construction of underground services. The main design requirements here are to maintain stability and limit deformations adjacent to the excavation so that nearby structures and underground services are not damaged, and to ensure that the safety of those required to work in the excavation is not at risk. In addition the effects of seepage towards the excavation need to be considered.

Amongst the most impressive and challenging of underground excavations made in soil are those for subways in major cities. These require lining and a major contribution to the loading on the lining is the state of stress in the ground before the excavation is made and also the position of the water table. In areas subject to seismic activity the effect of earthquakes on underground structures must be considered.

Underground lifelines and services are an essential part of the infrastructure in towns and cities. The performance of these under earthquake loading and the assessment of seismic robustness of these facilities is currently a vigorous area of research in many countries.

Slope Stability

In this case natural slopes have to be assessed and manmade slopes designed. Usually the factor of safety is the criterion for satisfactory performance and deformations are only of secondary importance. Groundwater is almost always the factor controlling stability. In recent years the stability of landfills having sophisticated liner systems has become important in many parts of the world.

In regions of seismic activity the question of earthquake effects on slope stability arises. Rather than focusing on factors of safety there is a consensus that permanent displacement of the slope may often be a more appropriate way to consider earthquake effects.

Groundwater sources and contamination

In recent years questions of groundwater supply and pollution have come to form an important interdisciplinary area in which geotechnical expertise contributes to the solving of problems. Extraction of groundwater has long been known to cause settlement which can be estimated using conventional theories of consolidation if the change in effective stress with time can be estimated. On the other hand contamination of groundwater by chemicals introduced because of waste disposal is a much more recent concern. As well as conventional seepage processes, diffusion of chemicals through soil water and the attachment (and release) of ions in the soil water to the surface of clay particles need to be considered. These estimations require information about non-classical soil properties and soil-water-contaminant interactions.

Rehabilitation and Repair

The final category of geotechnical "standard problem" mentioned here relates to the repair or strengthening of an existing foundation or the underpinning of an existing structure. In this case the capacity of the foundation is important, but likely to be more important are the movements experienced by the foundation during the repair.

SOLUTION OF GEOTECHNICAL PROBLEMS

To reach a satisfactory solution to a geotechnical problem information is needed under the following headings:

- Firstly we need to know something of the site and its geological environment as well as the processes leading to the formation of the soils present. We need information about the stratigraphy of the soil and rock and the variation in properties with depth and across the site. This understanding requires input from engineering geology, site investigation, field testing, and laboratory techniques. Eventually the information gathered will be synthesised into a geotechnical model of the site.

- In addition we require details of the loads that will be applied to or exerted by the facility to be constructed at the site and information about the required performance.

- Having an understanding of the site, the loads that will be applied, and the performance requirements of the proposed facility, values need to be estimated for the soil parameters which are of importance. This is done with traditional sampling and laboratory testing, correlations based on penetrometer results, in situ testing or combinations of these.

- The next step is to investigate possible solutions to the problem in hand and estimate the performance of the various options. This involves, among other things, calculations of load capacity and deformation under expected loads, and assessment of factors of safety or other measures of reliability, a process that will be referred to herein as *design analysis.* It is discussed below.

- A further stage in arriving at a solution is to consider methods of construction and such related processes as ground improvement.

- Constraints that need to be satisfied (usually finance and time) must be considered in the process of arriving at a solution. This often means that a solution must be reached without all the data that would regarded as necessary in an ideal world.

- The final step of the problem solving process will be the exercise of judgement.

What we are involved with here is a design process. Design in the widest sense embracing initial conception, investigation, idealisation of the soil profile, design analysis, detailed design, and development of a means of construction. This is a very broad activity requiring the synthesis of experience, insight, creativity and technical knowledge. The problem solving process is presented above as a sequence. In reality it is more likely to be cyclic with several passes through the steps outlined above (and perhaps others).

Design as envisaged here is an important area for the exercise of creativity in geotechnical engineering. As with other branches of engineering it is design which distinguishes engineering: the creation/manufacture/construct-ion of something which has never before existed; from science: the investigation of what already exists. This is not to deny that scientific investigation is a creative process or that scientific insight is not of great utility to the engineer, but it does emphasise the distinction between science and engineering. It also leads to the thought that the processes of developing and using numerical methods are at the scientific end of the spectrum of activities appropriate to geotechnical engineering.

DESIGN ANALYSIS

The major application of numerical methods in the above sequence is in the activity referred to as design analysis. By this is meant calculations that are done to provide understanding of the likely behaviour of the system under investigation. A secondary role for numerical methods is in facilitating the interpretation of data obtained from in situ tests done for the estimation of soil property values. Thus numerical methods make, at most, a partial contribution to the geotechnical problem solving process as they are essential at only one stage, design analysis, and may have a role in one of the others, the estimation of property values.

Sometimes solutions are reached not by use of analysis and numerical methods but by brilliant insight; a good example is the solving of the problems with the Cape Kennedy Causeway discussed by Peck (1969). In other cases the application of hard earned experience along with good construction practice is the mainstay of "design". Generally, however, some type of analysis and calculation sequence forms part of the geotechnical design process. The challenge of these calculations is the handle the complexity that is the norm rather than the exception in geotechnical engineering. The problems are inevitably difficult because we deal with a material the strength and stiffness of which may change during the life of the facility, the properties are difficult to determine because of sampling, size, and apparatus effects, and the material exhibits non-classical properties such as nonlinearity and dilatancy. In addition to difficulties with material properties we have complications as the soil has been formed by natural processes rather than placed in compliance with a specification. This will inevitably give properties which vary from point to point across a site, and there might also be layering or even more complex configurations of different materials. Coupled with this there are the boundary conditions which must be satisfied in the application of numerical methods.

To make progress with this analysis it is necessary to adopt a model, or a series of models, to apply to the process under investigation. The essence of modelling is to idealise the situation at hand and reduce the real complexity to an acceptable level. It is generally helpful to envisage this as taking several stages starting from very simple models, which usually have easily evaluated mathematical forms, and progressively engaging more sophisticated approaches, requiring the application of numerical methods, until sufficient understanding of the problem at hand is reached to enable a design to be completed with confidence. In this way the insights gained from simple models serve as stepping stones to the final solution and as points of reference in evaluating the results of sophisticated calculations. Linear elasticity and the Mohr-Coulomb strength model are examples of simple models. These are easily criticised but form a good starting place and in many cases the solutions associated with these models are expressed in closed form or in forms that are easily evaluated in specific cases. At the other end of the spectrum realistic models for soil stress-strain behaviour based on incremental work hardening/softening plasticity or other constitutive relations do not lead to solutions that can be expressed in simple forms so numerical analysis is required. For a given application knowledge of the solution to the related linear elastic problem and the related collapse situation gives bounds against which the output from sophisticated numerical modelling can be assessed.

There are two elements of sophistication or complexity in the modelling process. Firstly there is the basic mathematical complexity of the model itself. Secondly there is complexity associated with determining the input parameters for the model. Since modern computing facilities are so powerful a simple model which requires values for parameters which are very difficult to determine or whose physical significance is not clear is probably less useful than a complex model which requires values for parameters which are readily determined. Similarly a model requiring a small number of parameters is more likely to be useful than one requiring a large number of parameter values. Yet another view of this relates to seemingly small steps up the scale of sophistication. Isotropic linear elasticity is the simplest stress-strain model available and requires just two parameter values to describe a given soil. For a soil deposit formed by sedimentation an anisotropic elastic model, requiring five parameter values, is an obvious step along the path to more realistic modelling. However there is a very substantial jump in the difficulty of determining values for some of these five parameters, in this sense anisotropic elasticity is not a simple model.

EXAMPLES OF THE APPLICATION OF NUMERICAL METHODS

Herein a catalogue is made of the various ways in which numerical methods may contribute to design analysis both from the computational viewpoint and in the processing of field and laboratory data. The list makes no pretence to be exhaustive but rather reflects the experience of the author.

The listing is not in terms of distinct types of numerical analysis but rather aims to identify distinct activities to which numerical methods have been applied with success.

(a) Understanding and extending traditional design methods

Modern numerical techniques enable us to develop our understanding of traditional methods of design. For example finite element calculations have been useful in gaining appreciation, not provided by traditional methods of analysis, of the processes by which bearing capacity is mobilised or the mobilisation of active and passive earth pressure against retaining structures. In addition factors not considered in the classical analyses such as the state of in situ stress in the ground can be investigated by numerical methods.

Another example of this relates to consolidation. The classical calculations of Terzahgi are restricted to one dimensional drainage. When foundations are constructed at the surface of a uniform soil profile the consolidation mechanism is certainly not one dimensional drainage. In this case more complex analysis accounting for two and three dimensional effects shows that the consolidation time is very much faster, so fast in fact that moderate sized footings will often have achieved most of the consolidation settlement by the time construction is complete.

(b) Sensitivity analysis

Since soil properties may be difficult to determine and soil behaviour is complex sensitivity analysis provides insight into the relative importance of various parameters for a given problem. Even relatively simple hand calculations are tedious to repeat the many times required for sensitivity analyses. The availability of computer facilities encourages this activity which highlights those factors in a given problem that require the main effort.

(c) Effect of boundary conditions

Boundary conditions must be considered explicitly where a soil profile is underlain by rock and where there are inclined or even more irregular boundaries close to the foundation being designed. These situations are not well handled by analytical solutions but are readily addressed by numerical methods. Another boundary of importance is that between a structure and adjacent soil along which slip may occur. Once again this is handled well by available numerical methods.

An obvious situation in which the presence of boundaries is important is the estimation of settlement and other movements of a foundation. Boundaries and irregular geometry also affect bearing capacities and collapse loads. Finite element techniques have been applied with success to the calculation of upper and lower bound solutions for stability problems with boundary conditions not amenable to traditional techniques.

(d) Complex soil profiles and site complexity

It is more the rule than the exception that real soil profiles are complex, both in the vertical direction and laterally across a given site. Methods of coming to terms with this variability are not well developed. The recent rise in popularity of Geographical Information Systems provides a means of handling complex regional and site topographic data. In addition facilities of this type which handle subsurface data are now becoming available . These provide means of processing, assimilating and extrapolating borehole and other site investigation data. Colour graphics capabilities provide superb facilities for visualisation for which sophisticated numerical methods are vital as is high speed computer hardware. Cross sections in various directions can be drawn and three dimensional models of a particular site can be generated which are then available for examination from various viewpoints. Relations between proposed foundations and the soil layers present, the materials exposed during an excavation for the construction of a basement, or the materials exposed during the driving of a tunnel are examples of the types of visualisation possible with these facilities.

(e) Interpretation of data from monitoring of behaviour during and after construction

Under heading (c) above it was explained how numerical methods provide a means of handling complex boundary conditions. This becomes quite important when interpreting data obtained during monitoring of the behaviour of a foundation or excavation support system during construction, perhaps as part of the observational method (Peck, 1969). In such cases the site and boundary conditions are nearly always well beyond the range of existing closed form solutions so a numerical model provides the possibility of more realistic interpretation of observed behaviour. In this way a synthesis of information gained from numerical modelling with data obtained during monitoring leads to real understanding of site and foundation behaviour. Examples where this approach has yielded insight of practical value are: excavations in soft soils supported by sheet piling, embedded cantilever retaining walls in stiff clays, displacements during the formation of basement excavations, interpretation of piezometer readings in earth dams, and the rate of settlement of embankments constructed in stages over soft ground.

Numerical analysis could also be of assistance in planning the layout of instruments for monitoring and deciding on what positions would give the best information.

This process is clearly important for improving the understanding of the process being monitored but it also offers the possibility of extending general understanding of such construction processes and provides the empirical information for developing new design methods.

(f) Development of simple design methods

The traditional method of handling the output from a sophisticated piece of numerical analysis that has general application in design analysis is to plot the results as a chart using a suitable choice of dimensionless variables. This approach has the small disadvantage that the process of taking values off such a chart is tedious when the calculations need to be repeated several times as the design analysis is refined.

In recent years a useful procedure has appeared in which the results from a sophisticated numerical analysis are subjected to a nonlinear regression and simple formulae in terms of suitable dimensionless parameters are obtained. The interaction between two laterally loaded piles embedded in a homogeneous elastic soil is a very good example of this method. Randolf (1981) gives the following equations for the lateral interaction coefficients between a pair of piles of fixed head piles:

$$\alpha_{uF} = 0.3 \left[\frac{D}{s}\right] \left[2(1 + \nu) K\right]^{0.143} \left(1 + \cos^2 \xi\right)$$

where: D is the pile diameter
s is the pile spacing
ν is the Poisson's ratio for the soil
K is the ratio of the pile to soil Young's modulus
ξ is the angle between the direction of the applied load and the line joining the two piles
and α_{uF} is the interaction factor for a fixed head pile as defined by Poulos and Davis (1980).

For free head piles the interaction factors are obtained from α_{uF} as follows:

$$\alpha_{uH} = \frac{5\alpha_{uF}}{6}$$

$$\alpha_{uM} \approx \alpha_{\theta H} \approx \alpha_{uH}^2$$

$$\alpha_{\theta M} \approx \alpha_{uH}^3$$

where α_{uH}, α_{uM}, $\alpha_{\theta H}$, $\alpha_{\theta M}$ are the interaction factors defined by Poulos and Davis for free head piles.

Poulos and Davis (1980) present charts for interaction coefficients. No less than 17 diagrams are required to cover the range of possibilities for lateral interaction between two piles. Clearly the use of the above equations, inserted into a spreadsheet or other suitable software, provide a powerful method for performing design calculations.

The calculation of the behaviour of pile foundations, both single and groups, is now well served by formulae such as those given above. In a recent survey of pile foundation design analysis the author was able to present a set of formulae to cover many aspects of the prediction of pile foundation capacity and stiffness (Pender, 1993).

Gazetas (1991) gives formulae, derived following the process outlined above, for the static and dynamic stiffness and damping of shallow foundations of arbitrary shape and embedment. The basic numerical results were derived by boundary element calculations. Shape and embedment factors have been used for a long time with regard to bearing capacity estimates for shallow foundations. However the factors accounting for shape and embedment in the bearing capacity equations are arrived at by empirical means. The factors given by Gazetas for the effect of shape and embedment on shallow foundation stiffness, although arrived at by nonlinear regression, reflect the rigour of the boundary element calculations from which the regression data was derived. An example of the use of these equations in con

Previously one might have thought that there would always be a range of problems that would require mainframe hardware resources to obtain solutions. The advent of the above sets of equations means that with readily available software and modest computer resources it is possible to arrive at solutions of practical value and perform sensitivity analyses.

(g) Slope stability calculations

The calculation of slope stability is one of the oldest applications of numerical methods in geotechnical engineering. This is an ideal application for numerical methods as many calculations are needed to locate the critical failure surface geometry. In recent years a number of software packages have been developed for use on personal computers with very well developed user interfaces. This means that comprehensive stability analyses are possible with relative ease.

Along with these developments there has been something of a renaissance in methods for the assessment of slope stability in which general wedge methods and three dimensional analyses have been developed. In addition the application of minimisation techniques to the location of the critical failure surface geometry has been explored.

(h) Numerical analyses to back up simple models

Numerical methods provide a useful vehicle for investigating the validity of simple methods of calculation. One example relates to the dynamic behaviour of pile groups. Elastic analysis, Kaynia and Kausel (1982), using dynamic boundary element methods, have shown that pile group dynamic stiffness and damping are very complex functions of frequency as shown in Fig. 1 for the lateral stiffness (with no rotation) of a 2x2 fixed head pile group. Dobry and Gazetas (1988), exercising great physical insight, suggest that this behaviour can be modelled quite simply with the propagation of cylindrical waves. The results of their modelling are shown as discrete points in Fig. 1. It is seen that the approximate results are a very good match of those obtained with the boundary element calculations. This simplification opens the way for simple methods accurate enough for practical application to the estimation of dynamic pile group response. Dynamic pile group response is sufficiently complex that the Dobry an Gazetas simplified method would not be regarded as justified without the validation provided by appeal to the more rigorous boundary element analyses.

(i) Comparison of alternative models

An example of the use of alternative models relates to the lateral stiffness of long piles. The soil-pile interaction can be modelled using the elastic continuum approach or the Winkler discrete spring model. It is found that the two models give similar results. This opens the way to the use of the Winkler model for layered soil profiles and also for modelling nonlinear soil pile interaction. Examples of this are given by Pender (1993). It is of interest to note that the Winkler model has recently been used with good effect to provide a simplified method for the estimation of kinematic pile-soil interaction (Makris and Gazetas 1991, 1992 and Kavvadas and Gazetas 1993). This method gives results of sufficient accuracy for the solution of practical problems more easily than the boundary element calculations required for rigorous results.

(j) Insight into the effect of nonlinear behaviour

Many predictions in geotechnical have traditionally been based on the assumption that soil behaves elastically, even though it is well known that the stress-strain behaviour of geomaterials is nonlinear, exhibiting progressive decrease in shear stiffness and increasing dilatancy as failure is approached. Numerical methods provide a powerful means of investigating the significance of these phenomena. Early workers in the finite element field were quick to realise the potential of the method and since then it has been used extensively for the investigation of nonlinear behaviour. Finite difference and boundary element techniques are also used. In this section examples are given of some of these applications, not so much from the point of view of solving problems but rather as techniques that have helped develop background understanding which contributes to the exercise of judgement.

The first example concerns the stress distribution beneath a vertically loaded foundation. Finite element calculations have been done modelling the soil beneath the foundation with various nonlinear constitutive relations. It is found that the vertical stress distribution for the nonlinear soil model is very similar to that obtained for elastic behaviour. On the other hand it is clear that the horizontal stresses calculated with the nonlinear model are greater than those for the elastic case. The finding is found for the stress distribution both beneath the centre and edge the edge. It is concluded that nonlinear behaviour has a more significant effect on the horizontal than the vertical stress distribution. Another aspect of these results is the confirmation that deformations in a nonlinear material are localised closer to the load than they are for an elastic soil. The same conclusion is reached for the displacement of soil adjacent to a vertically loaded pile. This has important implications for pile groups as the vertical interaction factors between adjacent piles will be smaller in a nonlinear medium than in an elastic one.

Other examples of the development of insight from nonlinear calculations cover the spread of zones of failure beneath a foundation and adjacent to

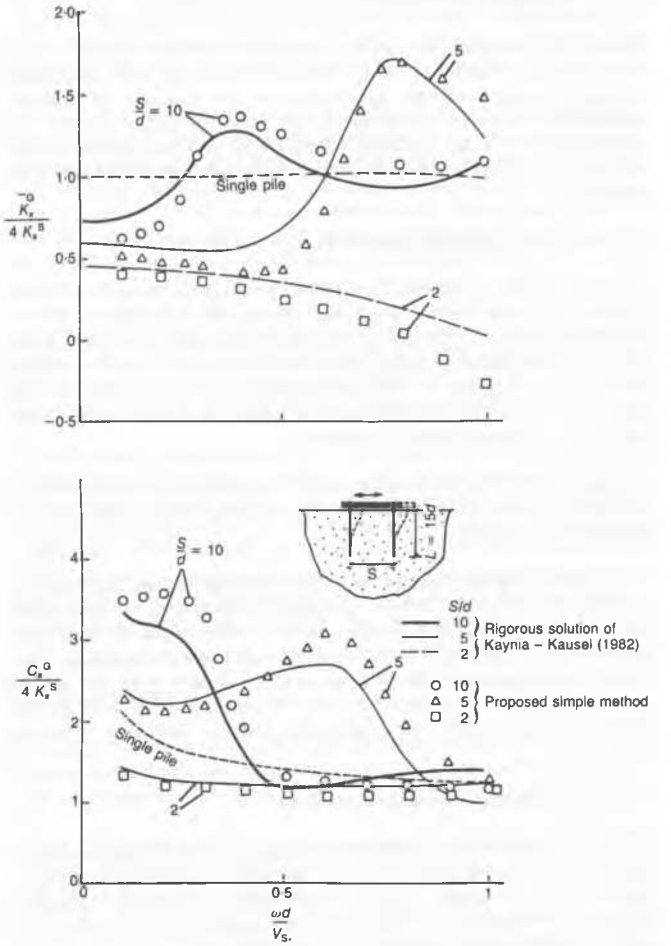

Fig. 1 Response of a 2x2 fixed head pile group to dynamic lateral loading. Full lines - boundary element calculations of Kaynia and Kausel (1982); symbols - Dobry and Gazetas (1988) cylindrical wave approximation.

retaining walls as loads increase. This links with the comments given in section (a) above.

The effect of nonlinear soil behaviour on the earthquake response of soil profiles has been a source of discussion between geophysicists and geotechnical engineers. The current status of these viewpoints is reviewed by Pender (1993a). It is clear that for cohesive soil profiles nonlinear effects are important. They are also important for saturated cohesionless soil profiles but in this case the added complication of the potential for liquefaction also arises. On the other hand profiles of very dense unsaturated cohesionless soils and stiff gravels usually respond elastically to earthquake excitation.

Important questions inevitably arise about the validity of the output from complex nonlinear computations. It is a simple matter to check that the software gives correct results to elastic problems but for the nonlinear case there are no benchmark solutions. Comparison of the output for the same problem obtained with different software packages can give some level of confidence. Use software to interpret results from field monitoring of real geotechnical situations, as mentioned under (e), is probably the most effective means of validation of such techniques.

(k) Situations too complex for traditional methods

Section (a) discusses the use of numerical methods to gain further understanding of standard design methods. However, the geotechnical setting of many important applications is too complex for standard approaches. Numerical investigation of these has helped greatly the development of design methods. Examples are propped retaining walls, incremental building of dams, tunnelling and the placement of tunnel lining systems.

(l) Estimation of property values from in situ testing

In situ testing is an important part of site investigation process and in the estimation of values for soil properties. The details of the tests are more complex than those for laboratory tests and the boundary conditions can be difficult to understand. Numerical analysis of the processes involved in these tests has helped greatly in the understanding of the measurements being made. This in turn provides information which can be used with greater confidence in solving practical problems.

In penetration testing large strain effects are involved, recent numerical analyses have been able to handle these, so giving further insight of and support to in situ testing.

In the above paragraph devices that penetrate the ground are mentioned. A relatively new and quite different technique for estimating the distribution elastic modulus with depth in a soil profile involves excitation of the soil deposit with a vibrator at the ground surface and systematic recording of the surface waves generated. The principal of this technique is not new but the practical application is dependent on efficient numerical methods for inversion of the recorded data to obtain the modulus distribution in the soil profile.

(m) Understanding of sampling processes

The process of recovering "undisturbed" specimens of soil is very complex and yet it is a central activity in much geotechnical investigation and in deriving input for design analysis. Once again sophisticated numerical analyses have been used to gain better understanding of the process of sampling and the state of the soil in the sampling tube. Like in situ testing the ability to handle large strains has been used to good effect in modelling the deformation of the soil adjacent to the sampler cutting shoe.

(n) Understanding of laboratory apparatus

Finite element analyses of the triaxial and simple shear tests has provided insight as to the stress distribution in specimens and the extent of nonuniformity. In this way it has been possible to investigate the effect of the actual nonuniform stress distributions and strains on the parameters measured in various tests.

(o) Control of laboratory testing processes

The combination of a personal computer with suitable data acquisition and instrument control facilities has revolutionised laboratory testing. Testing over complex stress paths is now routine so that soil parameters can be determined for the effective stress conditions expected to exist in the field.

The processing of data gathered during such test procedures can also be automated. These applications of numerical methods mean that laboratory testing is not now so demanding of technician time and therefore not so expensive in many parts of the world.

(p) Probabilistic methods

Probabilistic methods give helpful information about the meaning of factors of safety and emphasise the need to be conservative in variable soils or, more correctly, soils for which it is difficult to estimate representative values of strengths and stiffnesses. Probabilistic approaches are also the basis for reliability methods and they have an important role in the derivation of load factors, partial factors of safety, and performance factors in limit state design codes. A possible limitation of these methods is the need to know more than a single estimate for parameter values. In most cases it is difficult enough to come up with a single design value (and/or maximum and minimum range) for a soil property; the task of providing, in addition, information about the variance and, in some cases, the form of the distribution of property values for probabilistic calculations is usually formidable.

(q) Expert systems

The use of a computer to present expert systems information is included in this list of example applications as it is encompassed in the broad definition of numerical methods given in the introduction to the paper. In effect an expert system acts as a type of interactive manual assisting the user to locate appropriate responses to given queries.

SOURCES OF CONFUSION

It frequently occurs that the users of the results of numerical analyses are not those who performed the analysis in the first place. There is thus the possibility of confusion arising because of variability in matters of terminology. This is illustrated in this section with two examples in which terminology is confused, doubtless there are many others.

Modulus and Coefficient of Subgrade Reaction

The Winkler model is frequently used for foundation loading problems. It is well established that it gives good predictions of the lateral load stiffness of long piles. A parameter is needed to give the stiffness of the bed of Winkler springs. The basic relation is between applied pressure and spring displacement, a parameter is involved having units of force per unit volume (FL^{-3}) or, what is physically more meaningful, pressure per unit displacement. In the case of a long beam or laterally loaded pile the load per unit length is a more convenient parameter, this is the contact pressure multiplied by the beam width. The deflection of our beam or pile is then related to the local distributed load and the basic relation is between distributed load and displacement; a parameter is now involved which is the above parameter multiplied by the beam width and so having units of force per unit area (FL^{-2}). The names coefficient and modulus of subgrade reaction are applied to these two quantities.

The source of the confusion is that there seems to be no consensus as to which name is to be applied to the stiffness parameter having units FL^{-2} in the Winkler model. Various usages are given in Table I.

A simple way to resolve the problem would be to use the term modulus for the quantity having units of FL^{-2}, which is consistent with the common usage of the word, and coefficient for the term having units of FL^{-3}.

Table I: Terminology for subgrade stiffness in the Winkler model

author	term	units
Richart & Zia (1962)	coefficient	FL^{-3}
Scott (1981)	coefficient	FL^{-3}
Terzaghi (1955)	coefficient	FL^{-3}
Vesic (1961)	coefficient	FL^{-3}
Hetenyi (1946)	modulus	FL^{-3}
Bowles (1982)	modulus	FL^{-3}
Selvadurai (1986)	modulus	FL^{-3}
Den Hartog (1952)	modulus	FL^{-2}
Timoshenko (1956)	modulus	FL^{-2}
Matlock & Reese (1960)	modulus	FL^{-2}
Poulos & Davis (1980)	modulus	FL^{-2}
Zeevaert (1983)	modulus	FL^{-2}

Definition of Damping for Dynamic Foundation Behaviour

The dynamic behaviour of a foundation is conveniently handled by using the impedance which is a complex quantity having stiffness and damping terms. The impedance is expressed in a number of equivalent ways:

$$\mathfrak{C}_{\alpha\beta} = K_{\alpha\beta} + i\,\omega\,C_{\alpha\beta}$$

$$\mathfrak{C}_{\alpha\beta} = K_{\alpha\beta} + i\,a_o C'_{\alpha\beta}$$

$$\mathfrak{C}_{\alpha\beta} = K_{\alpha\beta} + i\,C''_{\alpha\beta}$$

where: $\mathfrak{C}_{\alpha\beta}$ is a component of the foundation impedance matrix
$K_{\alpha\beta}$ is a component of the foundation stiffness matrix
$C_{\alpha\beta}$ etc are components of the foundation damping matrix
$\alpha\beta$ refer to the moment and shear actions to which the impedance components apply, ie HH, HM, MH, and MM
ω is the excitation frequency is radians per second
a_o is a dimensionless frequency parameter depending on the dimensions of the foundation, the shear wave velocity of the soil, and the frequency of loading
and i $\sqrt{-1}$.

Once again the difficulty relates to units, in this case for the damping term. The equations above are equivalent in underlying concept. The damping term, the second in the above equation, must have units of stiffness but the equations give three different forms for the damping parameter (C), the possibility for confusion of the unwary is clear.

THE MAJOR DIFFICULTY IN SOLVING PRACTICAL PROBLEMS

As was mentioned at the beginning of this paper the development of numerical techniques in the last few decades has been quite amazing and so easily taken for granted. In little more than a decade since the introduction of the personal computer there has been an explosion of the capabilities of these machines and in those of work stations which have been developed subsequently. It is now feasible to do a sophisticated finite element and boundary element calculations on a desktop machine. From this perspective the comments made by Peck in 1985 about the numerical subculture may be even more valid now than then.

The difficulty of deciding on the appropriate input data for the numerical analyses remains the greatest limitation to the role of numerical methods in solving geotechnical problems. Denver and Ovesen (1994) emphasis that this is often the weak link in the geotechnical design process. The rapid growth in the availability of numerical analysis, both software and hardware, has far outstripped our ability to investigate a given site, describe the materials present there, and specify the appropriate properties.

THE FUTURE

The question to ponder at the end of this paper is the path to the future. In the last decade there has been exponential growth in computing power that can be purchased for a given sum. At present this growth shows no sign of slowing. The making of predictions may not be not wise in a time of such rapid changes.

Clearly computational facilities will continue to outstrip our ability to specify soil properties.

High speed graphics processing and 3D analysis, processes which currently push computer hardware to the limit and are only done on top-of-line machines, will become more common because of falling costs. With increasing sophistication verification of the output from such software will continue to be an important issue.

Will the growing sophistication of computer applications distance the users from the real world of geotechnical engineering? Basically ours is a "muddy boots" occupation which relies heavily on physical insight into the nature and properties of soil as an engineering material coupled with input from our senses of touch and sight. The place of computer applications and numerical methods is to assist this process not to dominate it.

What will the computer do to geotechnical judgement? Is there a risk of loosing the insight that leads to brilliant solutions to practical problems, such as that of Peck at Cape Kennedy, which make no use of numerical methods? The ability to come up with solutions like this says much for the creative ability of the unaided human mind. Earlier generations of geotechnical engineers had to reach solutions to practical problems without the benefit of the tools available today. Their extant works attest to their success. The current collective wisdom regarding the use of numerical methods is that they allow more refined and economical solutions as the problem in hand can be explored in greater depth and understood better. The dramatic fall in the cost of computer hardware means that these numerical explorations are available at modest expenditure. It is hard to envisage circumstances which will prevent these trends continuing.

The above two paragraphs give voice to frequently expressed concerns the validity of which cannot be denied. On the other hand the application sophisticated numerical methods has certainly enhanced our understanding of the complex behaviour of soil masses and the details of soil-structure interaction. Many of these advances have been made in such a way that the direct practical application may not be immediately apparent but, even so, understanding of the significance of various factors and the ability to perform sensitivity analyses are important developments.

CONCLUSIONS

This paper has attempted to show how numerical methods fit into the wider spectrum of the solution of practical problems in geotechnical engineering. It is concluded that they have a very important but not dominant role.

Numerical methods provide an opportunity to address complexity of soil profiles and boundary conditions as well as handling non-classical effects such as dilatancy, nonlinearity, coupling between soil skeleton and pore fluid etc. It seems likely that the best results will be achieved by using various levels of computational sophistication with the output from very simple calculations being assessed against those from complex approaches.

Problem solving in geotechnical engineering requires thorough investigation, creative design, construction in which appropriate monitoring is done and interpretation of that monitoring so as to confirm expectations of the design. Numerical methods can make a very significant contribution to the design analysis phase of the design process, they can also perform an invaluable function when it comes to interpreting measurements of displacements and pressures etc.

The advent of the personal computer coupled with general purpose computational software is replacing traditional design charts as a means of carrying out routine design calculations.

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