VALIDATION OF FINITE ELEMENT SOLUTIONS

VALIDATION DE SOLUTIONS A ELEMENTS FINIS

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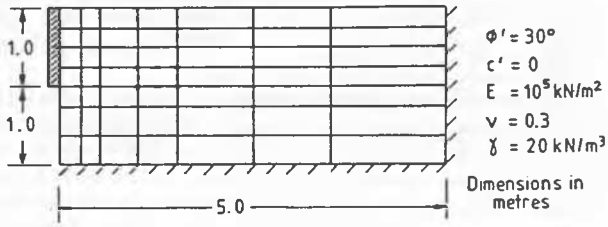
INTRODUCTION

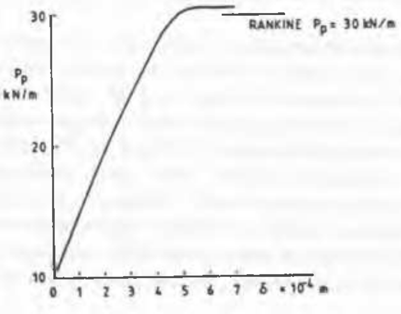
In a brief presentation such as this in a relatively complicated area of analysis, attention is concentrated on the two areas possibly of greatest general interest, namely, robust algorithms and the nature of possible 'benchmark' problems.

'SIMPLE' ELASTOPLASTIC PROBLEMS IN GEOTECHNICS

In standard geotechnical engineering practice, deformation analyses (serviceability limit state) and collapse analyses (ultimate limit state) are still usually calculated separately in independent computations. Finite element analyses have for a long time in principle given analysts the power to handle the ultimate state as a culmination of serviceability states but robust algorithms have perhaps been lacking. At any rate there are some 'simple' problems which any geotechnical analyses program must be able to cope with before confidence, can be gained to proceed further. These 'simple' problems concern soil where no excess porepressures are generated during the deformation process, often referred to as 'drained' problems and soil (particularly "clay" in the normally consolidated state) which remains saturated and without drainage of water during the deformation process, often referred to as 'undrained' problems. There are excess porewater pressures in such undrained clays, which are actually not easily retrievable from the simple analyses (see below), but since the 'undrained' strength is a constant, a Tresca analysis delivers the load-deformation behaviour to collapse.

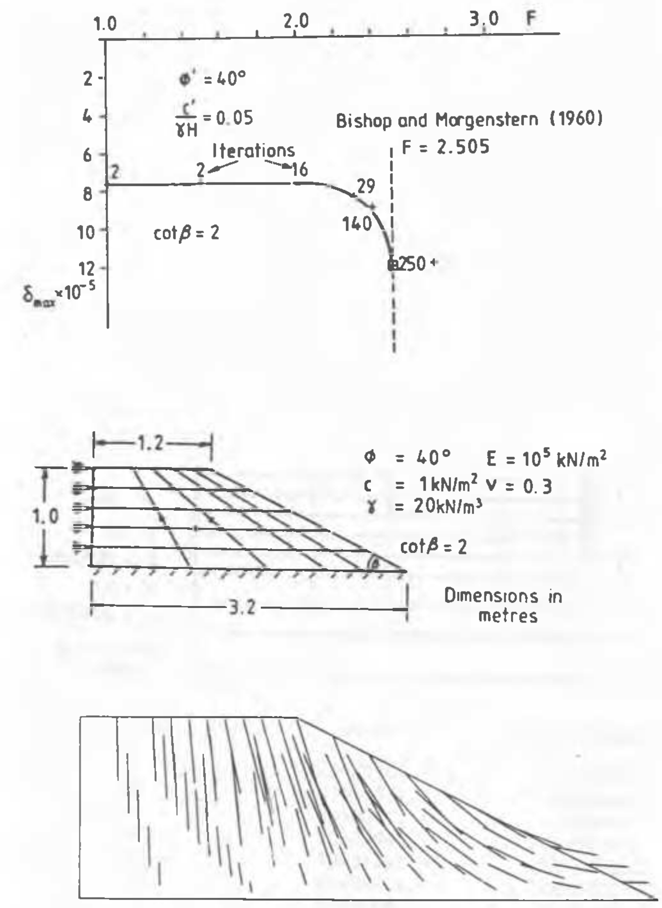


Displacement	Force	Iterations
.2000E-04	-.1097E+02	2
.8000E-04	-.1385E+02	12
.1600E-03	-.1750E+02	3
.2400E-03	-.2106E+02	10
.3200E-03	-.2441E+02	16
.4000E-03	-.2770E+02	21
.4800E-03	-.2989E+02	9
.5400E-03	-.3061E+02	38
.6200E-03	-.3065E+02	12
.6600E-03	-.3066E+02	3
.7000E-03	-.3068E+02	3



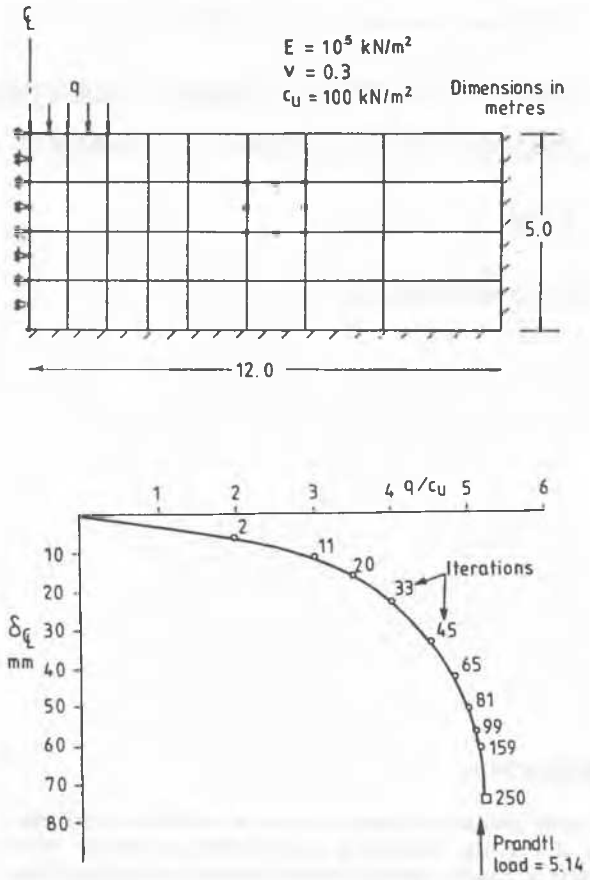
Example 1 The 'Rankine' Wall : Drained Sand

For the mesh shown, and a 'sand' with the given elastoplastic properties, the wall on the left is displaced incrementally towards the sand. The horizontal displacements at the wall are fixed, but the vertical displacements are free, simulating a "smooth" wall. For this case, the Rankine passive pressure is 30 kN/m while the analysis yields a clear ultimate pressure of about 30.7 kN/m. This could of course be improved by mesh refinement but the analysis clearly "works".



Example 2 The 'Taylor' Slope : Drained Sand

For the mesh shown, and a 'sand' with the given elastoplastic properties, the slope is loaded by gravity and the strength properties in terms of c' and tan ϕ' reduced by the same progressively increasing factor until collapse occurs. Taylor's value of the critical factor is 2.505 and the FE calculation could not be continued beyond that point indicating collapse. Therefore the program "works". However, a cursory inspection of displacement vectors at failure seems to reveal no circular 'slip path' mechanism such as would be computed by a classical limit geotechnical analysis. Better visualisation is needed.



Example 3 The 'Prandtl' Footing : Undrained Clay

For the mesh shown, and a 'clay' with the given elastoplastic properties, the footing is loaded with progressively smaller load increments to collapse. This clearly occurs very close to the Prandtl load of 5.14 Cu and so the program "works".

The above examples could be used as benchmark problems for "simple" analysis programs.

REMARKS ON PROGRAM DESIGN

The above three examples are taken from Smith and Griffiths (1982). In that text the FORTRAN 77 coding is reproduced in full and can be checked if necessary. A modular style is adopted, using portable subroutines, so that any subroutine module can easily be replaced by another. It may be of interest that this programming strategy is further facilitated by Fortran 90 (Smith, 1993).

WHEN IS "SIMPLE" TOO SIMPLE?

Soils are frictional materials whose behaviour is dominated by effective stress. Therefore, if excess porepressures exist, their calculation is imperative for most problems, and rather complicated field problems are routinely solved by specialists (eg Hicks and Smith, 1988).

As a benchmark for more complicated analyses in terms of effective stresses, we may return to Example 3 above, and recompute it.

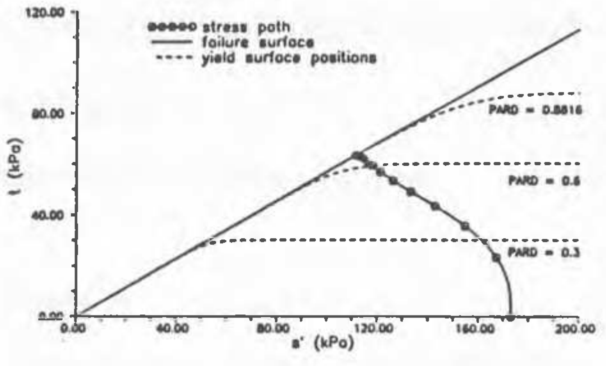


Fig. Example 4 a

The 'Prandtl' Footing : Undrained Clay (Effective Stress Analysis)

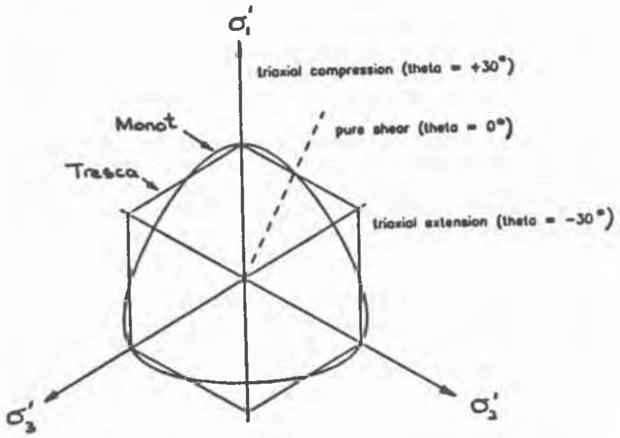


Fig. Example 4b

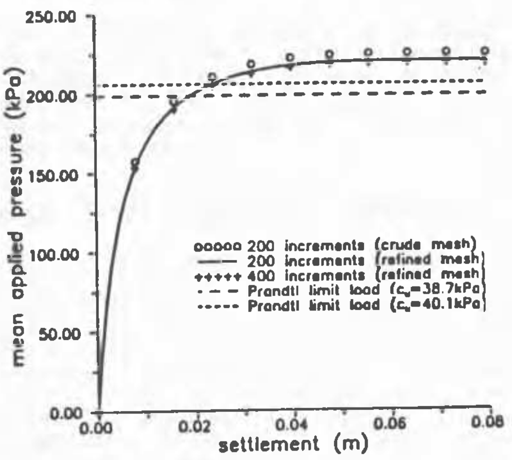
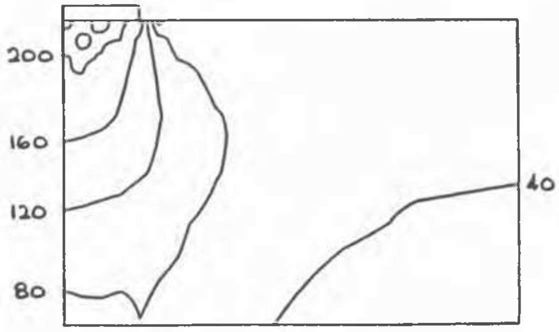


Fig. Example 4c



(f) excess pore pressure contours (contour interval = 40 kPa, minimum contour = 40 kPa, maximum contour = 200 kPa)

Fig. Example 4d

The effective stress path, computed for a conventional ($\sigma_2 = \sigma_3$ = constant) undrained triaxial test on a clay using the constitutive model MONOT is shown in Fig. Example 4a.

For an isotropic starting stress state $\sigma_1^1 = \sigma_2^1 = \sigma_3^1 = 100$ kPa, the equivalent c_u is 38.7 kPa in triaxial compression. However, most sophisticated models such as MONOT depart significantly from Tresca (or Mohr-Coulomb) approximations in the π-plane, see Fig. Example 4b.

Therefore, although collapse is unambiguously reached in the computations, see Fig. Example 4c, the ultimate load can exceed the Tresca value by about 10% due to the directions of the stress paths (Lode angles) in the π-plane. Particular care is needed as some codes use a completely incorrect circular failure envelope in the π-plane.

The extra bonus of the effective stress analysis is that correct excess porewater pressures are now computed as shown in Fig. Example 4c.

REFERENCES

Hicks, M.A. and Smith, I.M. (1987). Class A Prediction of Arctic Caisson Performance Geotechnique, 38(4) : 589-612.

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