BEHAVIOUR OF LAYERED SYSTEMS UNDER REPETITIVE LOADING

COMPORTEMENT DES SYSTEMES MULTI-COUCHES SOUS CHANGEMENT REPETE

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SYNOPSIS: This paper presents ideas to stimulate discussion on the behaviour of soils and granular materials in road structures subjected to repeated traffic loading. A design philosophy is reviewed and stress-strain characteristics are discussed. The non-linear resilient properties of soils and granular materials are identified as a key feature in properly modelling layered systems under repeated loading. The "threshold stress" concept is shown to be useful for design purposes and this, together with other points, are identified for discussion.

INTRODUCTION

The layered systems considered in this paper are pavements for highways, though most of the material will have relevance for rail track and for other shallow load bearing stratified arrangements.

Pavement design is emerging from the empirically dominated approaches typified by the California Bearing Ratio (CBR) method (Porter, 1938) as a result of extensive research over the past thirty years. This work has led to an improved understanding of how soils and granular materials respond to repeated loading (Brown and Selig, 1991). It has also involved a range of studies into the full-scale performance of layered systems. An even greater effort has been put into research on bituminous materials.

The mechanistic approach to pavement design involves theoretical analysis to compute stresses and strains at critical locations as a result of applied transient wheel loading. The values of these parameters are then compared with permissible levels consistent with a defined level of performance for the pavement. This generally means prevention of serious cracking in the bituminous layers and of surface rutting in the wheel tracks. A number of organisations have developed design methods in the form of charts (Shell International, 1978) or computer programs (Brown and Brunton, 1984)) which adopt this philosophy. This field of activity has been covered by the series of International Conferences on Asphalt Pavements, the most recent of which was held in the U.K. during 1992 (ISAP, 1992).

Studies of soils and granular materials in the context of pavement design have lagged behind research on bituminous materials and have also been a minority interest within soil mechanics. The behaviour of soils under cyclic loading has been studied in the context of earthquake and off-shore structure foundation design problems but the detailed circumstances differ from those relevant to pavements.

PAVEMENT DESIGN PHILOSOPHY

Brown and Dawson (1992) outlined a two-stage approach to pavement design. The first stage (Fig. 1) involves construction traffic loading the pavement foundations, which consists of granular material placed over soil. The stress levels are high and the number of load repetitions are low. The design criterion is rutting. They suggested that the granular layer thickness should be selected to prevent significant development of plastic strains in the soil below. The problem of rutting as a consequence of plastic shear strains developing in the granular layer was identified but no detailed procedure suggested. This was because the computation of stresses near the surface of a directly loaded granular layer presents some difficulties. This matter is discussed later in this paper.

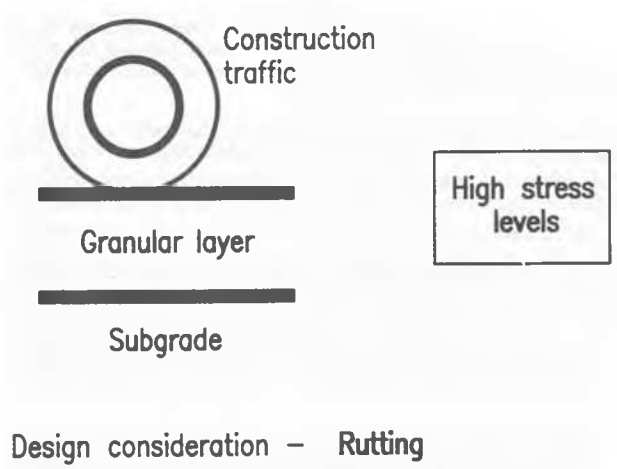

Fig. 1 Pavement foundation design.

Brown and Dawson proposed that the two layer pavement foundation, with layer stiffness E_g and E_s (Fig. 2), should be converted, for design purposes, into an equivalent single, semi-infinite layer having the same resilient characteristics as the composite and an equivalent stiffness E_f. The equivalence was based on equal resilient surface deflection. The second stage of design (Fig. 2) then involves determination of the thickness of the bituminous layer of stiffness (E_a) required over the foundation to give satisfactory long-term performance; typically over a 20 year period. This stage involves low stress levels in the foundation materials but very large numbers of repetitions. The design criteria are fatigue cracking in the asphalt and rutting.

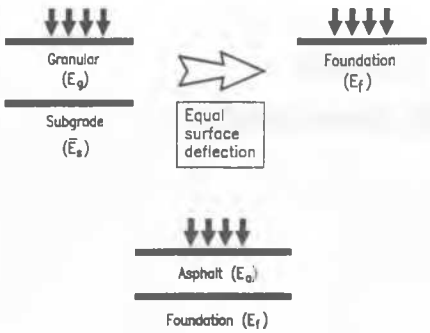

Fig. 2 Two stage pavement design.

Part of the Brown and Dawson philosophy envisages use of a dynamic plate loading test applied at the top of the foundation. This could be used to ensure that the assumed design value of resilient stiffness was being achieved and, if not, allow some adjustment to the quality of bituminous mixture to be placed above, recognising that it would be difficult to adjust thickness at that stage in the construction.

STRESS-STRAIN RELATIONSHIPS

In most soil mechanics problems involving analysis of the structure, attempts are made to characterise the soil as an elasto-plastic material. This recognises that both elastic and plastic strains are of importance but, usually, it is the latter which dominates. Since most problems involve monotonic loading, recoverable strain does not become a significant issue. For pavement design, it is usual to emphasise the resilient response, because, by definition, significant plastic strains are not permitted and the repeated loading nature of the problem emphasises recoverable strains. Fatigue cracking of bituminous materials is initiated when the resilient tensile strain is in the order of 100 to 300 microstrain, while serious rutting is associated with accumulated plastic strain of 1 or 2%. In soil mechanics terms, then, pavement engineering is a low strain problem.

Fig. 3 shows idealisations of shear stress/strain relationships for a soil or granular material under repeated loading at two peak deviator stresses well below failure in a triaxial test. Resilient (ε_r) and plastic (ε_p) shear strains are identified in Fig. 3(a) and the term resilient modulus (E_r) may be defined as: $E_r = q_r/\varepsilon_r$, based on the peak to peak values of deviator stress (q_r) and axial strain. The rate of increase in plastic strain with load repetitions decreases and may cease completely creating an "equilibrium" condition with a fixed hysteresis loop for successive load cycles if the deviator stress level is low enough. The maximum deviator stress associated with this situation is termed the "Threshold Stress" and is an important concept for design. Fig. 4 illustrates this point and also shows that at very high deviator stresses, the rate of plastic strain accumulation can accelerate leading to complete failure.

Fig. 3(b) shows the equilibrium situation at a higher deviator stress and comparison with Fig. 3(a) indicates that the resilient modulus decreases as the peak deviator stress increases. This is one manifestation of non-linear behaviour which is a matter of great importance in pavement engineering.

The term "resilient modulus" is used in preference to "elastic modulus" because of the non-linear hysteretic nature of stress-strain relationships for soils. Nonetheless, when carrying out structural analysis computations, values of resilient modulus are used for Young's modulus.

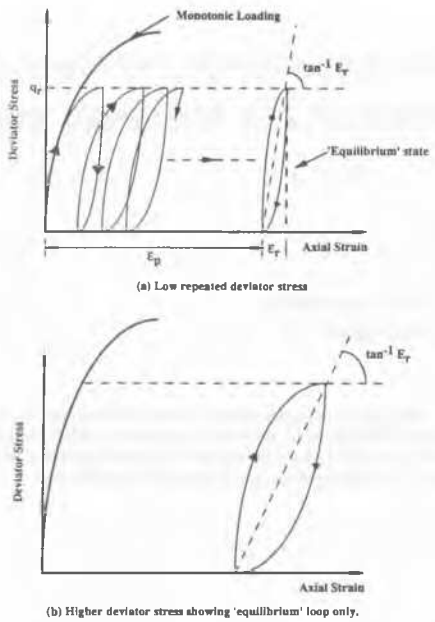

Fig. 3 Conceptual stress-strain relationships for soils and granular materials.

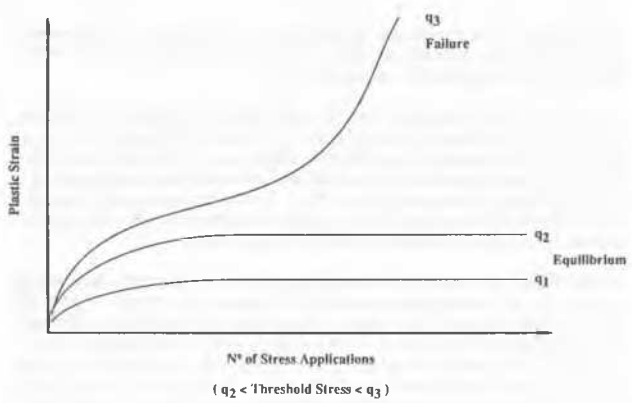

Fig. 4 Plastic strain accumulation under repeated loading.

USE OF GEOSYNTHETICS

Geosynthetics are often used in pavement construction. When the role is that of a filter or separator, there is no direct influence on analysis. The indirect influence is in the prevention of deterioration to soil or aggregate properties. When the role is one of reinforcement, then the presence of the geosynthetic must be taken into account. A reinforcing action can only be mobilised at relatively high strains. Consequently, geosynthetic reinforcement is principally of interest for haul roads consisting of granular material placed over soft ground. Pavement foundations may be regarded as high quality haul roads since the allowable rut depth is lower (40 mm) than for a haul road (75 mm).

RESILIENT BEHAVIOUR OF SOILS AND GRANULAR MATERIALS

For pavement design purposes, the non-linear resilient response of soils and granular materials has generally been expressed as a stress-dependent resilient modulus. This contrasts with the approach used in earthquake engineering where a strain dependent shear modulus has been adopted (Hardin and Drnevich, 1972).

In a pavement, the system is generally "stress controlled" rather than "strain controlled" as a consequence of loading by vehicle wheels. In earthquakes, the soil deposits above rock level are in a strain controlled condition caused by imposed deformations from the rock below.

Any fundamentally based stress-strain relationship should be able to cater for both situations. However, laboratory testing programmes have tended to try and reproduce, usually in a triaxial cell, the conditions appropriate to the problem under investigation.

For pavement engineering, Fig. 5 shows the stress regime considered appropriate for an overconsolidated undisturbed clay. The stress space used in this figure is deviator stress (q) against mean normal effective stress (p') being the average of the three principal effective stresses. For a saturated soil, the transient stress resulting from a passing wheel load (Fig. 6) will result in no change of effective stress (assuming isotropy) in what is an undrained situation (Graham and Houlsby, 1983). The peak stress level is likely to be within the notional yield surface created by preconsolidation and essentially resilient response can be expected in a well designed situation.

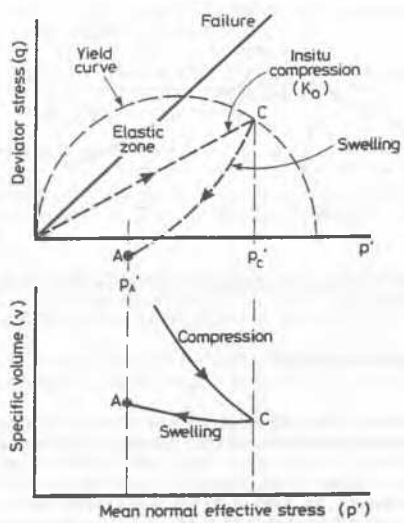

Fig. 5 Stress history for an element of subgrade soil.

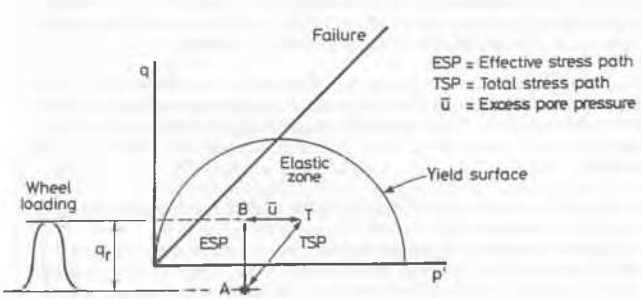

Fig. 6 Stress path caused by transient wheel load.

Laboratory studies reported by Brown et al (1987) indicated a stress-dependent resilient shear modulus given by:

$$G_r = A\, q_r \left[\frac{p_o'}{q_r}\right]^{l} \qquad (1)$$

where p_o' is the mean normal effective stress at the end of consolidation (p_A' in Fig. 5) and q_r is the deviator stress caused by wheel loading (Fig. 6). The constants A and l characterise the particular soil. For a saturated soil $E_r = 3G_r$.

The stress history for soil which is disturbed and recompacted during construction is less clear. However, the high stresses imposed by compaction plant are likely to create effective over-consolidation for soils which are close to saturation.

Brown et al (1990) demonstrated that equation (1), modified to replace p_o by suction, can be used to characterise partially saturated soils with degrees of saturation in excess of 85%. For a typical silty clay ($w_L = 37\%$, $w_P = 18\%$), the constants in equation (1) have values in the order of: A = 300 (undisturbed), 120 (recompacted) and l = 1.5 (undisturbed), 2.1 (recompacted). In more arid climates, other work (Richards and Gordon, 1972, Fredland et al, 1977) has clearly demonstrated the important influence of soil suction on resilient characteristics.

A great deal of research has been conducted on granular materials to improve understanding of the mechanical properties relevant to pavement design. The materials involved are crushed rock or gravel in a dry or partially saturated state. Consequently, under applied stress, both shear and volumetric strains will develop. The non-linear resilient response has been characterised by stress dependent shear and bulk moduli and various models have been published (Pappin and Brown, 1980, Boyce, 1980). That by Mayhew (1983) is convenient for the purposes of illustrating basic principles:

$$G_r = \frac{q_r \times 10^6}{3\left(\frac{p_r'}{B}\right)^{m} \eta_r} \qquad (2)$$

$$K_r = \frac{p_r' \times 10^6}{\left(\frac{p_r'}{C}\right)^{n} \left(1 - D\,\eta_r^2\right)} \qquad (3)$$

where q_r, p_r' are the changes in deviator and mean normal effective stress respectively as a consequence of transient loading and η_r is the corresponding change in stress ratio (q_r/p_r').
B, C, D, m and n are constants for a particular material. For a crushed carboniferous limestone, the following values have been reported (Mayhew, 1983): $B = 1.59 \times 10^{-5}$, m = 0.37, $C = 3.05 \times 10^{-7}$, D = 0.124, n = 0.35.

Equations (2) and (3) demonstrate that resilient shear modulus depends principally on the change in stress ratio under transient loading, noting that m < 1. Bulk modulus is, essentially, seen to be influenced principally by the change in mean normal effective stress. It should be noted, however, that resilient shear modulus is affected by normal stress and bulk modulus by stress ratio. The strain contours published by Pappin and Brown (1980) demonstrated these interactions.

The stress conditions in a pavement under a moving wheel load are quite complex and not exactly reproduced in the repeated load triaxial test, the experimental arrangement most commonly used. This is illustrated by Chan and Brown (1994). Work by Chan (1990) with a Hollow Cylinder Apparatus has demonstrated that resilient behaviour is unaffected by the rotation of principal planes which occur as a wheel traverses a point in the pavement. The same cannot, however, be said about plastic strains.

PLASTIC STRAIN AND THE THRESHOLD STRESS CONCEPT

The problem of plastic strain accumulation in soils below pavements can be minimised by ensuring that the threshold deviator stress at the soil surface is not exceeded. Brown and Dawson (1992) suggested that the threshold stress may be taken as twice the soil suction for design purposes but further data are required to refine this criterion. Consequently, in studying the response of soils in layered systems which are properly designed, the subgrade may be assumed to behave as a non-linear resilient material.

Brown and Selig (1991) have suggested that the threshold stress for granular materials should be expressed as the peak stress ratio (q/p') relative to that at failure. They proposed 70% of the failure value as the maximum stress ratio to minimise plastic strain development.

Chan and Brown (1994) have reported data on plastic strain accumulation in repeated load Hollow Cylinder tests. They demonstrated that the rotation of principal planes (or reversal of shear stress direction), does have a significant influence. Consequently, repeated load triaxial tests are likely to underestimate plastic strains, particularly those associated with volume change.

It is possible to use the threshold stress concept to ensure plastic strains in the granular layer are negligible for completed pavements. However, the high stresses imposed by construction traffic operating directly on this layer mean that plastic strains must be considered for the first stage of design.

Large amounts of data have been accumulated for resilient behaviour of granular materials but studies of plastic strain have been more limited. Consequently, further research is needed to properly understand plastic strain accumulated in granular materials.

STRUCTURAL ANALYSIS OF LAYERED SYSTEM

Closed loop solutions to determine stresses, strains and deflections in linear elastic layered systems subjected to circular uniformly distributed surface loads have been widely available for the past 25 years. Shell's BISAR (Whiteoak, 1990) and ELSYM 5 (Ahlborn, 1972) are typical of such programs and are available for PC's.

The application of such analysis to materials with non-linear resilient properties has involved an approximate iterative procedure in which the non-linear layers are sub-divided into artificial layers. A "seed" value of Young's modulus is used for each layer, and stresses are computed, usually at the centre of the layer on the axis of symetry of the wheel load. The values of Young's modulus are then "updated" by using the relationship between this parameter, taken as resilient modulus, and the stress conditions (e.g. Equations 1 to 3). Computations proceed until a convergence to a satisfactory result is achieved.

In reality, stress conditions due to imposed wheel loading vary horizontally as well as vertically. The procedure summarised above does not accommodate the horizontal variation. This may not create serious errors when computing effects on the centre line of the load. However, for off-axis positions, the horizontal variation may be important.

An example of this is the calculation of surface deflection "bowls" used in pavement evaluation. This involves an iterative procedure to match a theoretical deflection profile with that measured in situ using devices such as the Falling Weight Deflectometer (Brown et al, 1987). Such procedures allow the effective resilient modulus of each layer in a pavement to be determined.

Brunton and de Almeida (1992) proposed an approximate method to accommodate horizontal variations in Young's modulus. In calculating the surface deflection at a particular radial distance from the centre of the load, they considered the stress conditions in the soil and granular material at that same radius to determine the appropriate Young's modulus.

The non-linear resilient characteristics of soils and granular materials can be more effectively accommodated in pavement analysis by use of the finite element method. Several programs, such as SENOL (Brown and Pappin, 1981) and ILLIPAVE (Raad and Figueroa, 1980) have been specifically developed for this purpose. A recent program with some improved facilities is known as FENLAP and has been described by Brunton and de Almeida (1992). This program allows the user to select an appropriate non-linear resilient model from several which are available.

An important element of layered systems analysis, which is incorporated in the finite element solutions but not specifically in the layered system programs, is the need to compute self weight stresses *a priori*. This only applies to non-linear analysis. It raises the question of the approximate value to be assumed for K_0, the coefficient of earth pressure at rest. While this can be deduced from undisturbed soils using expressions such as that proposed by Mayne and Kulhawy (1982), no such procedures are available for compacted granular materials. It is usual to assume $K_0 = 1$ but "locked-in" horizontal stresses are caused by compaction and these have never been satisfactorily quantified.

Programs such as FENLAP are not able to converge on a solution when the load is applied directly to the granular layer. This is caused by the discontinuities associated with the edge of the wheel load and the high shear stresses in the neighbouring elements. Inclusion of a thin bituminous surfacing overcomes this problem. However, for pavement foundation design this is a false situation. It is desirable that plastic strains should be computed so that rut depths can be determined. This requires, either an appropriate elasto-plastic model for the granular layer, which has yet to be adequately defined from laboratory testing, or an alternative approximate technique. One such suggestion involves a mechanism of wedges in the granular layer as illustrated in Fig. 7 (Thom et al, 1993). The rut depth may be deduced from the proportion of shear strength which is mobilised for the system.

Milligan et al (1989) presented a method for the static analysis of haul roads which can accommodate reinforcement at the granular material/subgrade interface when present. Unlike previous methods which consider the reinforcement as a tension membrane effect (Giroud and Noiray, 1981), Milligan et al consider the horizontal shear stress imposed on the subgrade surface and its effect on the bearing capacity.

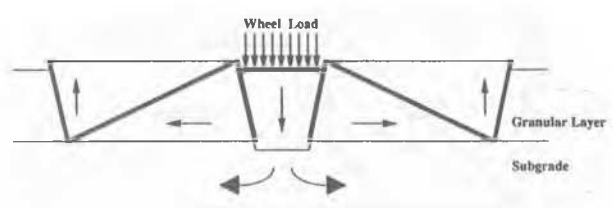

Fig. 7 Proposed wedge mechanism for pavement foundation analysis.

EXPERIMENTAL RESULTS

Early experiments with stress and strain transducers installed in pavement sections to obtain measurements in various directions (Brown and Bush, 1972) clearly revealed the non-linear behaviour of soils and granular materials. The measurements were combined by superposition to compute stress and strain invariants, the relationships between which were markedly non-linear. This predated the careful laboratory experiments with triaxial tests incorporating "on-specimen" strain measurements (Boyce and Brown, 1976, Brown et al, 1980) which confirmed the non-linear characteristics.

Recent data shown in Table 1 obtained from the Nottingham Pavement Test Facility show values of stresses and strains generated by moving wheels at particular locations and how these compare with computed values obtained by the FENLAP program (de Almeida, 1993) using non-linear stress-strain relationships. The comparisons are shown to be reasonable.

Brown et al (1986) have previously demonstrated the need to properly allow for non-linearity when modelling the pavement subgrade to compute resilient surface deflections in "back-analysis" computations. Use of a linear elastic subgrade led to poor predictions of critical stresses and strains in the pavement.

Recognising the need to improve on the CBR method for characterising soils and granular materials for pavement design, recent work at the University of Nottingham has been concentrating on the development of simplified repeated load test methods (Dawson et al, 1993). These allow credible values of resilient modulus, threshold stress and relative resistance to plastic strain to be determined using simplified versions of the repeated load triaxial test. Related work by Rogers and Brown (1993) at Loughborough University has concentrated on development of a suitable dynamic field test device for use on pavement foundations.

Table 1. Comparison of Measured Parameters and Values computed using FENLAP

Parameter	Location	Measured	Computed
Tensile strain (με)	Bottom of asphalt	-600	-666
Vertical stress (kPa)	Top half of granular	180	123
Vertical stress (kPa)	Centre of granular	110	67
Vertical strain (με)	Top of subgrade	1100	1110

(57mm asphalt, 225 mm crushed limestone, silty clay subgrade)

POINTS FOR DISCUSSION

The above review of relevant material properties and methods of analysis for layered systems forms a framework for discussion at the Conference.

Particular attention should be given to:

(a) Resilient stress-strain relationships.

(b) The threshold stress concept.

(c) Accumulation of plastic strain in granular materials.

(d) Analysis of pavement foundations, particularly with respect to plastic strains in granular layers.

(e) Simplified test methods to determine material properties.

(f) Field evaluation of pavements.

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