Determination of soil stiffness parameters in stress path probing tests

Détermination des paramètres de dureté des sols dans des essais à chemin de contraintes contrôlés

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SYNOPSIS: Special stress path probing tests are convenient for determining anisotropic elastic stiffness parameters for soils. These tests are relatively simple to perform using a hydraulic stress path triaxial cell. Anisotropic elastic stiffness parameters for undisturbed samples of London Clay were determined from stress path probing tests and used to predict the behaviour in further probing tests. The results demonstrate that the stress-strain behaviour of London Clay, for continuous loading paths, is in good agreement with a simple anisotropic elastic model.

1 INTRODUCTION

The need to predict ground movements under working loads has prompted experimental and theoretical research directed towards examining the stress-strain behaviour of soils at relatively small strains and at states of stress relatively far from failure. The practical requirement is now for routine laboratory tests which measure small strain soil stiffness parameters suitable for relatively simple constitutive equations.

For heavily overconsolidated soils for which the state remains within the yielding condition the stress-strain characteristics are usually taken to be elastic but anisotropic (e.g. Atkinson, 1975). For isotropic elastic soils only two elastic parameters are required which may be obtained from a single drained triaxial compression test. For anisotropic elastic soils however five independent elastic parameters are required and these cannot be obtained from the results of a single triaxial compression test. One parameter, the cross shear modulus, can best be measured in special simple shear tests but the other four can be measured in drained triaxial tests with suitable stress paths. These tests are relatively easily performed in a simple hydraulic stress path triaxial cell and, measurements of stiffness which are satisfactory for many practical purposes can be made using conventional instrumentation.

2 BASIC STRESS-STRAIN RELATIONSHIPS FOR OVERCONSOLIDATED SOIL

A general constitutive relationship for material behaviour may be written as

$$\{\delta\epsilon\} = [C]\{\delta\sigma'\} \qquad (1)$$

where [C] is a compliance matrix which contains functions of a number of material parameters. Equation 1 is written in terms of small increments of effective stress and of strain and, if the terms in [C] are allowed to change with loading, it describes non-linear behaviour.

Equation 1 may be written in terms of invariants of stress and strain. For loading with axial symmetry convenient invariants are $q' = (\sigma_a' - \sigma_r')$, $p' = 1/3(\sigma_a' + 2\sigma_r')$, $\epsilon_s = 2/3(\epsilon_a - \epsilon_r)$ and $\epsilon_v = (\epsilon_a + 2\epsilon_r)$. The constitutive equations then become

$$\delta\epsilon_s = (1/3G')\delta q' + J_1\delta p' \qquad (2)$$

$$\delta\epsilon_v = J_2\delta q' + (1/K')\delta p' \qquad (3)$$

where G' is a shear modulus, K' is a bulk modulus and J_1 and J_2 couple together shear and volumetric effects. For perfectly elasto-plastic materials $J_1 = J_2$ and for isotropic and elastic materials $J_1 = J_2 = 0$ in which case shear and volumetric effects are decoupled. It should be noted that the moduli in equations 2 and 3 are not necessarily elastic parameters and may not be constants.

For soils, values for the moduli in equations 2 and 3 depend on a number of factors, the most important being the current state, the stress history and the current loading path. For the present we will consider only overconsolidated samples and relatively small strains so that the states will remain inside the state boundary surface and the strains will remain elastic.

For isotropic compression and swelling $\delta q' = 0$ and equation 3 becomes

$$\delta\epsilon_v = (1/K')\delta p' \qquad (4)$$

but we also have

$$\delta v = -\kappa\,\delta p'/p' \qquad (5)$$

where κ is the gradient of the isotropic unloading-reloading line (Schofield and Wroth, 1968). Dividing equation 5 by the specific volume v and noting that $\delta\epsilon_v = -\delta v/v$ we have

$$\delta\epsilon_v = (1/vp')\kappa\,\delta p' \qquad (6)$$

If it is assumed that the other moduli in equations 2 and 3 depend on the current state in a similar manner the constitutive equations may be written

$$\begin{Bmatrix} \delta\epsilon_s \\ \delta\epsilon_v \end{Bmatrix} = \frac{1}{vp'} \begin{bmatrix} \gamma & j_1 \\ j_2 & \kappa \end{bmatrix} \begin{Bmatrix} \delta q' \\ \delta p' \end{Bmatrix} \qquad (7)$$

It may be noted that vp' is a constant factor in many of the current soil models including Cam Clay (Schofield and Wroth, 1968). The parameters in equation 7 may be regarded as basic soil parameters and the current state is taken into account by vp' so that the conventional elastic stiffnesses in equations 2 and 3 depend on the current state.

Conventional oedometer compression and swelling tests and conventional drained and undrained triaxial compression tests are inconvenient for examining the form of equation 7 and for determining design values for the parameters for a particular soil. The principal problem is that both $\delta q'$ and $\delta p'$ vary in both sets of tests. A more convenient set of tests to determine values for the soil parameters is to perform stress path probes with $\delta q' = 0$ (i.e. isotropic loading and unloading) and with $\delta p' = 0$ (i.e. constant mean effective stress) respectively. From equation 7, with $\delta q' = 0$ we have

$$\delta p'/\delta\epsilon_v = vp'/\kappa \qquad (8)$$

$$\delta\epsilon_s/\delta\epsilon_v = j_1/\kappa \qquad (9)$$

and with $\delta p' = 0$ we have

$$\delta q'/\delta\epsilon_s = vp'/\gamma \qquad (10)$$

$$\delta\epsilon_v/\delta\epsilon_s = j_2/\gamma \qquad (11)$$

For an ideal elastic material $j_1 = j_2$ and, recalling that for an isotropic and elastic material $j_1 = j_2 = 0$, the values of j_1/κ and j_2/γ are measures of the degree of elastic anisotropy. The basic stiffness parameters γ, κ, j_1 and j_2 may be obtained from tests with $\delta q' = 0$ and with $\delta p' = 0$ making use of equations 8 to 11. These may be checked independently by carrying out tests with stress paths at ± 45° (i.e. with $\delta q'/\delta p' = \pm 1$).

3 LABORATORY TESTS

Laboratory tests were carried out on samples of blue London Clay using a hydraulic stress path triaxial apparatus. The samples were all relatively heavily overconsolidated (with values of OCR estimated to be approximately 10) and so the states remained inside the state boundary surface throughout the tests. They were subjected to the relatively short ($\delta q' = \delta p' = \pm 80$ kPa) loading paths shown in Fig. 1. Measurements of strain were made using conventional instruments taking particular care to avoid errors. All tests were drained with constant back pressure u_o = 200 kPa and the loadings were applied slowly

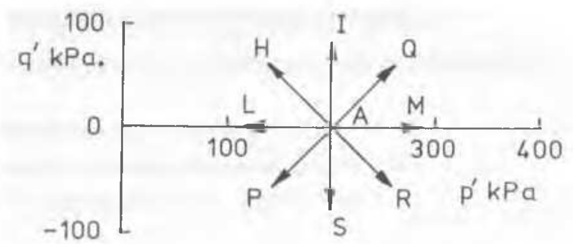

Fig. 1 Test Paths

at a rate of 1 kPa per hour. To avoid small strain effects due to changes of direction of the loading path the penultimate loading paths were always in the same direction as the current loading path; this is illustrated in Fig. 2 which shows the current loading path AQ from Fig. 1 and the previous loading path XA in the same direction as AQ.

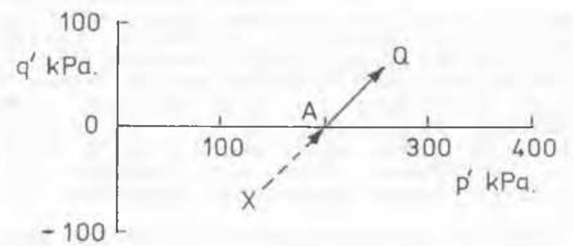

Fig. 2 Previous and Current Paths

The tests were carried out in a microcomputer controlled hydraulic stress path triaxial cell (Atkinson, Evans and Scott, 1985). The cell is similar to the conventional stress path triaxial cell (Bishop and Wesley, 1976) but enlarged to take sample sizes up to 150mm diameter. For the present tests the samples were 100mm diameter and 200mm long.

Samples of blue London Clay were obtained from the Building Research Establishment test bed site at Brent Cross in North London (Powell and Uglow, 1986). The samples were taken in 100mm diameter thin-walled tubes slowly pushed into the bottom of bore holes at depths of 8.1 to 8.5 m. The basic classification properties of the samples were LL = 71 and PL = 26 and the natural water content was 27 %. In the laboratory the samples were extruded from the tubes, cut to length and mounted in the triaxial cell in the usual way. The samples were firstly compressed isotropically and undrained to p = 300 kPa. The back pressure was then set to the measured pore pressure, the drainage valve opened and the samples allowed to come to equilibrium; this resulted in only very small strains. The drainage leads were flushed through with de-aired water and, if necessary, the process of compression and flushing repeated until the measured value of B exceeded 0.98. The samples were then consolidated isotropically to initial states p' = 200 kPa with a back pressure u = 200 kPa; the initial specific volume was v = 1.809.

4 DISCUSSION OF LABORATORY TEST RESULTS

Figs. 3 and 4 show the results of two constant q' and two constant p' tests respectively. The most striking feature of these data are that both the stress-strain characteristics and the shear strain-volumetric strain characteristics are approximately linear and the gradients are approximately the same for loading and for unloading. This is largely due to the immediate past loading path being in the same direction as the probing path; for stress path rotations the stress and strain characteristics for the probing path become significantly non-linear.

Since the stress-strain and shear strain-volumetric strain characteristics are linear the gradients of the lines in Figs. 3 and 4 may be given by single values which are both tangents and secants. Values are summarised in Table 1 where the stiffnesses have been normalised by dividing by vp' = 1.809 x 200 = 362 kPa. Taking mean values and substituting into equations 8 to 11 the elastic parameters are κ = 0.036, γ = 0.020, j_1 = 0.007, j_2 = 0.008.

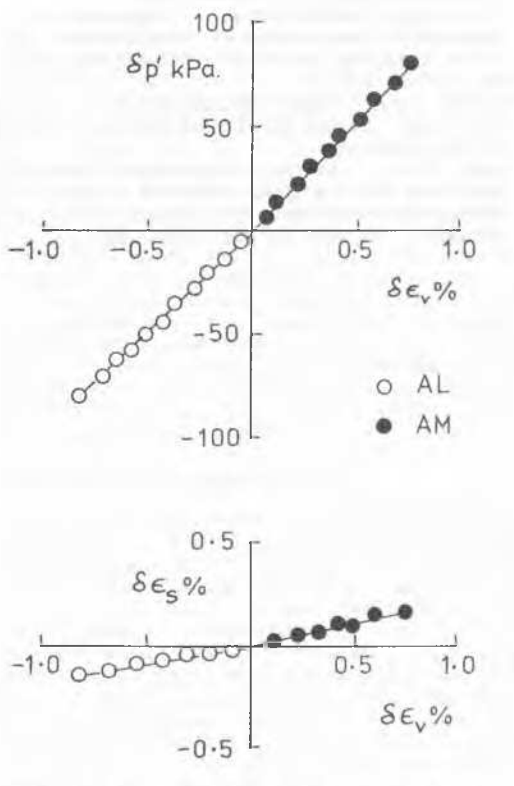

Fig. 3 Results of Constant q' Tests

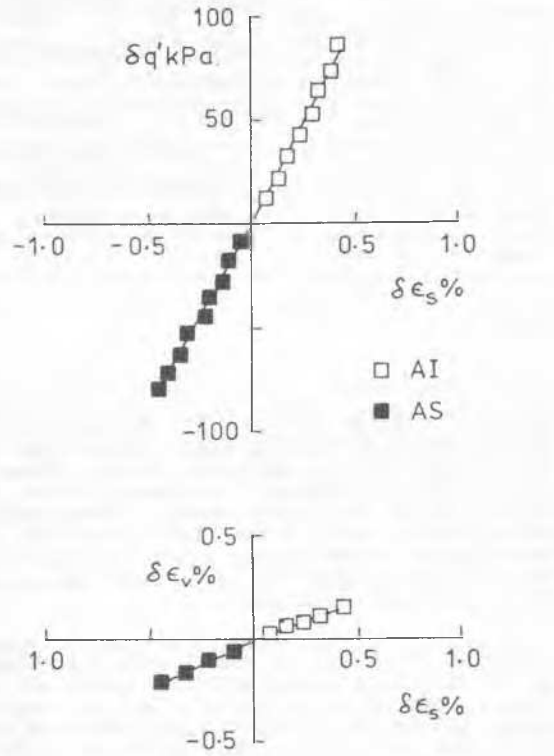

Fig. 4 Results of Constant p' Tests

Probe	Path	$\frac{1}{vp'}\frac{\delta q'}{\delta \epsilon_s}$	$\frac{1}{vp'}\frac{\delta p'}{\delta \epsilon_v}$	$\frac{\delta \epsilon_v}{\delta \epsilon_s}$	$\frac{\delta \epsilon_s}{\delta \epsilon_v}$
AL	$\delta q'$ = 0	-	27	-	0.18
AM	$\delta q'$ = 0	-	29	-	0.20
AI	$\delta p'$ = 0	53	-	0.36	-
AS	$\delta p'$ = 0	49	-	0.44	-

Table 1.

The value of κ is close to values found from other isotropic and one dimensional compression tests on undisturbed samples of blue London Clay from the Brent Cross site. The values of j_1 and j_2 are approximately equal indicating that the stress-strain behaviour was essentially of an elastic nature. Values of j_1 and j_2 are not, however, zero indicating that the samples were anisotropic.

In all tests with stress paths at ± 45° the stress-strain and shear strain-volumetric strain characteristics were approximately linear and the gradients are summarised in Table 2. Also shown in Table 2 are the corresponding gradients calculated from

Probe	Path $\frac{\delta q'}{\delta p}$	$\frac{1}{vp'}\frac{\delta q'}{\delta \epsilon_s}$ Expt	Calc	$\frac{1}{vp'}\frac{\delta p'}{\delta \epsilon_v}$ Expt	Calc	$\frac{\delta \epsilon_s}{\delta \epsilon_v}$ Expt	Calc
AH	− 1	79	77	34	36	−0.47	−0.46
AR	− 1	93		43		−0.46	
AQ	+ 1	37	37	25	23	0.70	0.61
AP	+ 1	36		21		0.58	

Table 2.

equation 7 with $\delta q'/\delta p' = \pm 1$ and with values for the basic parameters found from tests with $\delta q' = 0$ and $\delta p' = 0$ and given above. There is good agreement between the experimental and calculated values in all cases. These results demonstrate that both the form of the constitutive relationship given in equation 7 and the values for the basic soil parameters given above are acceptable.

Comparing equations 2 and 3 with equation 7 the conventional shear and bulk moduli G' and K' are given by $G' = vp'/3\gamma$ and $K' = vp'/\kappa$. Taking values of γ and κ given above we have G' = 6 MPa and K' = 10 MPa, which may also be found directly from Figs. 3 and 4. These values are smaller than those back calculated from field observations of structures on the London Clay (e.g. St. John, 1975). The present values were obtained from tests in which the previous stress path was in the same direction as the current stress path and so the stiffnesses do not correspond to very small strains. For cases where there is rotation of the stress path the very small strain stiffnesses measured in probing tests have been found to be substantially greater and of the same order of magnitude as field values.

5 CONCLUSIONS

Anisotropic elastic parameters for soil may be determined simply from two special stress path probing tests with $\delta q' = 0$ and $\delta p' = 0$ respectively. These probing tests may be carried out conveniently in a hydraulic stress path triaxial cell.

Anisotropic elastic parameters obtained from these tests on undisturbed samples of blue London Clay were used to calculate the behaviour in other stress path probing tests and the results agreed well with experimental observations. The values for the anisotropic elastic parameters correspond to loading and unloading without rotation of the stress path and the values will be different when the paths rotate.

These results demonstrate that the stress-strain behaviour of the samples which are typical of heavily overconsolidated clays, was anisotropic and elastic. The constant q' and constant p' tests have advantages over conventional triaxial compression tests for examining the stress-strain behaviour of anisotropic soils and for measuring stiffness parameters.

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