A unified theory of one-dimensional consolidation with creep

Une théorie unifiée de la consolidation unidimensionnelle et du fluage

G.IMAI, Professor of Civil Engineering, Yokohama National University, Yokohama, Japan

SYNOPSIS There are still unsettled problems regarding one-dimensional consolidation. Several important problems among them are examined in this paper. First of all, the rule of mass conservation is theoretically examined to get the correct equation. Experimental results are next presented to verify that creep acts during the whole process of consolidation, and to construct a set of constitutive equations involving the creep effect. In the last section, the effects of sample thickness as well as of secondary compression before loading on consolidation processes are examined on the basis of numerical calculations, of which techniques are also presented.

INTRODUCTION

Regarding one-dimensional consolidation several important problems are still unsettled. As a theoretical problem, for example, it has not been confirmed whether a mass conservation equation expressed by Eulerian coordinate system is equivalent or not to the one expressed by Lagrangian system. As a problem to be verified by experiments it is still a serious controversy whether creep acts or not during the primary consolidation stage; for instance, Mesri et al (1985) presented the data suggesting no creep action while Leroueil et al (1985) showed the contrary data. Furthermore, the effects of sample thickness on consolidation process are still not clear probably because the action of creep is still not understood.

In the following context these problems are examined to finally succeed in evaluating the effect of both creep and sample thickness on consolidation process.

FIELD EQUATIONS FOR CONSOLIDATION

Local mass conservation is correctly expressed by Eq.(1) for solid phase and by Eq.(2) for water phase when the Eulerian system (ξ,τ) is used.

$$\frac{\partial}{\partial\tau}\left[\frac{\rho_s}{1+e}\right]+\frac{\partial}{\partial\xi}\left[\frac{\rho_s}{1+e}\cdot v_s\right]=0 \tag{1}$$

$$\frac{\partial}{\partial\tau}\left[\frac{e\rho_w}{1+e}\right]+\frac{\partial}{\partial\xi}\left[\frac{e\rho_w}{1+e}\cdot v_w\right]=0 \tag{2}$$

Where, ξ = a location at which a skeleton point under consideration passes through at a time τ, e = void ratio in the neighborhood of the skeleton point, v_s = moving velocity of the point, v_w = velocity of water which passes through the point, and ρ_s, ρ_w = densities of solid and water (assumed to be constant, respectively).

Mass conservation of water is correctly considered when we observe the phenomena riding on the skeleton point and use the exit water velocity relative to the skeleton point, v.

$$v=\frac{e}{1+e}(v_w-v_s) \tag{3}$$

Substituting Eqs.(1) and (2) for (3), we get Eq. (4) which expresses the mass conservation of water.

$$\frac{\partial v}{\partial\xi}=-\frac{1}{1+e}\cdot\frac{\partial e}{\partial\tau}-\frac{v_s}{1+e}\cdot\frac{\partial e}{\partial\xi} \tag{4}$$

In the consolidation of a soil layer, the initial location of a skeleton point, a, can always be specified, but this is not the case for water particles. That is, the Lagrangian system (a,t) should be introduced for a skeleton point; where a = the initial location of the skeleton point considered in deriving Eq.(4) and t = time numerically equal to the value of τ. Because the skeleton point carries a relation $\xi=\xi(a,t)$, the next equations must be applied in the coordinate transformation.

$$\frac{\partial}{\partial\xi}=\frac{1+e_o}{1+e}\cdot\frac{\partial}{\partial a} \tag{5}$$

$$\frac{\partial}{\partial\tau}=\frac{\partial}{\partial t}-v_s\cdot\frac{1+e_o}{1+e}\cdot\frac{\partial}{\partial a} \tag{6}$$

These equations already contain the mass conservation of solid, $\partial a/\partial\xi=(1+e_o)/(1+e)$, where, e_o = the initial void ratio at location a. Applying Eqs. (5) and (6) to Eq. (4) the next equation expressing mass conservation is obtained.

$$-\frac{1}{1+e_o}\cdot\frac{\partial e}{\partial t}=\frac{\partial v}{\partial a} \tag{7}$$

This equation is the same with the one derived by Gibson et al (1967) which has been confirmed to be true.

Equations which express the balance of momentum for solid phase and water phase can be combined to yield the following equation;

$$v = \frac{k}{\gamma_w}\left[\frac{1+e_o}{1+e}\frac{\partial \sigma'}{\partial a} + \gamma'\right] \tag{8}$$

Where, γ_w and γ'=unit weight of water and submerged unit weight of soil, respectively, σ'=effective stress and k=coefficient of permeability.

When the reduced coordinate system, (z,t), is used, the rule of transformation from (a,t) to (z,t) is expressed by $(\partial/\partial a)=[1/(1+e_o)](\partial/\partial z)$. Applying this rule to Eqs.(7) and (8), a set of correct field equations for consolidation is obtained as follows;

$$-\frac{\partial e}{\partial t} = \frac{\partial v}{\partial z} \tag{9}$$

$$v = \frac{k}{\gamma_w}\left[\frac{1}{1+e}\frac{\partial \sigma'}{\partial z} + \gamma'\right] \tag{10}$$

CONSTITUTIVE EQUATION OF SOIL SKELETON

In order to find a unique e-σ'-t relationship which is applied to any location within a specimen, special experiments were carried out by use of the test apparatus shown in Fig.1. Seven thin specimens (ϕ=60mm, H=5mm) are interconnected in series to form a specimen of 35mm thickness. The measurement of displacement for every thin specimen and of pore water pressure at every boundary between adjoining thin specimens yields the distributions of e and σ' , respectively, and their change with time.

In order to make the whole specimen initially uniform and homogeneous, every thin specimen was artificially made in its own consolidation ring from a dilute slurry by use of the same procedures, then preconsolidated under the pressure of 0.4kgf/cm^2. The clay used is a fine fraction of Yokohama Bay mud. Its fraction under 0.02mm is 60% and the values of the L.L. and P.I. are 120% and 80, respectively.

In general, the following equation holds;

$$\frac{de}{dt} = \left[\frac{\partial e}{\partial \sigma'}\right]_t \frac{d\sigma'}{dt} + \left[\frac{\partial e}{\partial t}\right]_{\sigma'} \tag{11}$$

The quantity expressing creep is the second term on the right side. Its value can be obtained not by usual means but by the present test in which e-σ'-t relations at any location in a specimen can be obtained. An example showing the test results of e-logt relationships under constant σ' values is presented in Fig.2, which verifies without doubt that creep acts during the whole stage of consolidation.

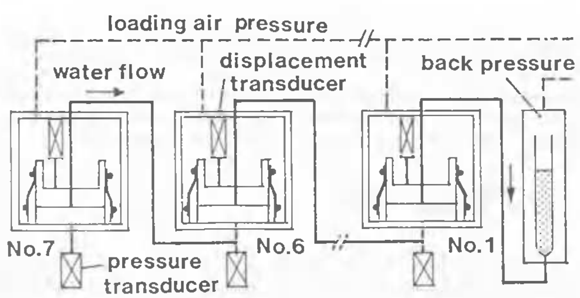

Fig.1 Scheme of interconnected consolidometer.

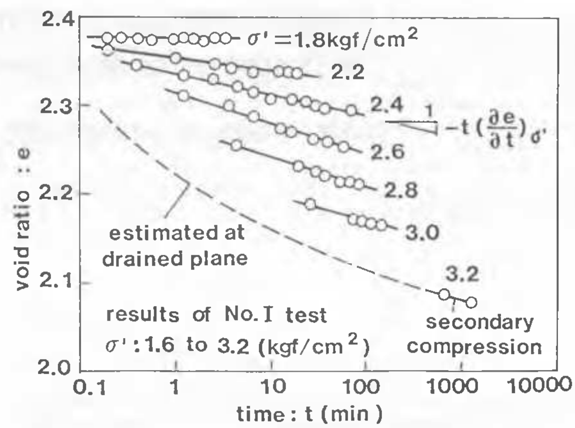

Fig.2. Creep action during the primary consolidation stage.

In order to examine whether a unique e-σ'-t relationship exists or not, the four different tests specified in Table I were carried out; the differences are in the initial states (e_o,σ'_o) and load increase ratio Δp/p. Every specimen underwent several stages of the 24hrs. step loading up to σ'_o over the preconsolidation stress of 0.4kgf/cm^2; that is, all the specimens were normally consolidated to the stress level of σ'_f.

Test results are summarized in Fig.3, where the quantity $\dot{e}$ (=de/dt in Eq.(11)) is used as the parameter which represents creep action. The results evidently show that a unique Δe-(σ'/σ'_o) relationship for a constant $\dot{e}$ value exists in spite of the differences of location in a specimen, initial state and load increase ratio. That is, an isotache type constitutive relationship $\dot{e}$=f(e,σ') holds for a soil skelton.

Any constant $\dot{e}$ line has a yield stress, σ'_y, over which the line is straight and parallel to the 24hrs. compression curve. Because yield occurrs at larger σ'_y values for larger $-\dot{e}$ values as shown in Fig.4, the yield must be caused by the rate effect (Leroueil et al, 1985).

An isotache-type constitutive relation forms a sheet of state surface in the $\dot{e}$-e-σ' space. A state point which represents a consolidation state at a location in a specimen must trace a

TABLE I The tests carried out

Case	H_o (mm)	e_o	σ'_o (kgf/cm^2)	σ'_f	Δp/p
I	35	2.4	1.6	3.2	1.0
II	35	2.7	0.8	2.4	2.0
III	35	2.7	0.8	3.2	3.0
IV	35	2.3	2.4	3.6	0.5

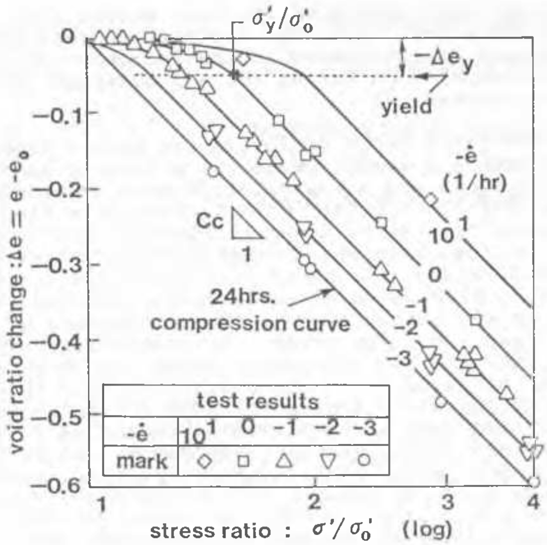

Fig.3 A unique relationship among $\dot{e}$, e and σ'.

state path on this surface as shown in Fig.5, which is constructed based on the results shown in Fig.3. According to this model, an instant increase in stress from σ'_o to σ'_f should occur at a location very near the drained plane with very little volume reduction, but thereafter gradual volume reduction due to creep follows. This conclusion is verified to be true by the broken line in Fig.2, where gradual decreases in volume at the drained plane are clearly recognized. That is, the idea of instantaneous compression first proposed by Bjerrum (1967) is not actual excepting the idea of delayed compression.

When the relationship shown in Fig.4 is approximated by $\log(\sigma'_y/\sigma'_o) = A\log(-\dot{e}) + B$, and the constant $\dot{e}$ lines for $\sigma' < \sigma'_y$ are approximated by parabolic curves which smoothly join the straight lines for $\sigma' > \sigma'_y$, the following constitutive equation (12) for $\sigma' < \sigma'_y$ and (13) for $\sigma' > \sigma'_y$ are obtained.

$$\log\frac{\sigma'}{\sigma'_o} = \frac{\Delta e}{\Delta e_y}(\Delta e - \Delta e_y)C_c - \frac{\Delta e}{\Delta e_y^2}(\Delta e - 2\Delta e_y)[A\log(-\dot{e}) + B] \qquad (12)$$

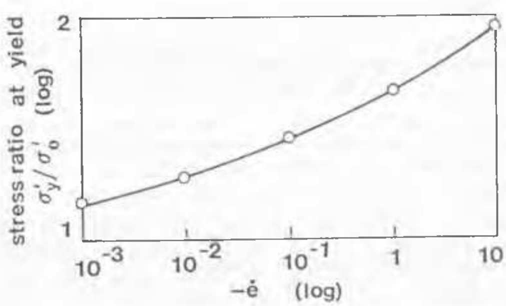

Fig.4 Dependency of yield stress on $\dot{e}$.

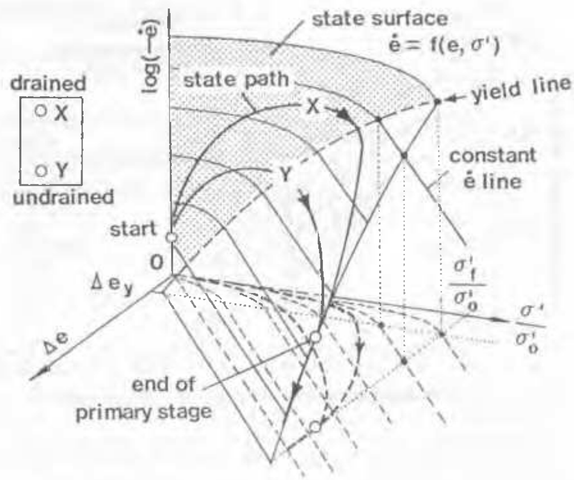

Fig.5 A state surface for one-dimensional consolidation.

$$\log\frac{\sigma'}{\sigma'_o} = -\frac{\Delta e - \Delta e_y}{C_c} + [A\log(-\dot{e}) + B] \qquad (13)$$

EFFECTS OF CREEP ON CONSOLIDATION

Ladd et al (1977) pointed out about Fig.6 that no creep acts during the primary stage in the case of curve A but it does so in the case of curve B. This idea, however, requires another important viewpoint that the secondary compression duration before loading may largely affect the consolidation process after the loading. In order to examine this problem, numerical calculations were carried out by use of Eqs.(9),(10),(12) and (13).

Calculation conditions are specified in Table II. In any case a clay layer is normally consolidated from 1.6 to 3.2 kgf/cm^2 under a single drainage condition. Cases 1 to 4 are calculations from the end of the primary stage (these cases are called EOP in the following context), and Cases 5 and 6 are calculations on clay layers which underwent secondary compression during a period equal to the time required for the layer in Case 4 to complete the primary stage (these cases are called ASC). The values of $\dot{e}$ at the initial states, $\dot{e}_o$, were calculated based on an equation $-\dot{e} = [\ln 10/(C_c A)]t$ which is derived from

TABLE II
Calculation conditions

Case No.	H	$-\dot{e}_o$ (1/hr)	Initial state
1	1cm	7×10^{-2}	EOP
2	10cm	7×10^{-4}	EOP
3	1m	7×10^{-6}	EOP
4	10m	7×10^{-8}	EOP
5	10cm	7×10^{-8}	ASC
6	1m	7×10^{-8}	ASC

EOP; End of primary stage
ASC; After secondary compression stage

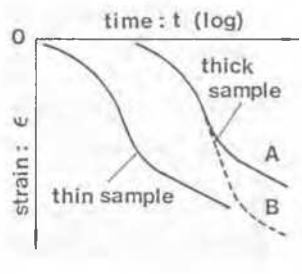

Fig.6 Two probable effects of sample thickness on consolidation.

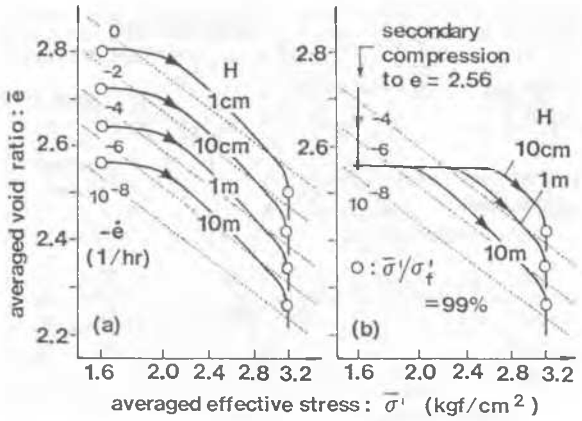

Fig.7 $\bar{e}$-log$\bar{\sigma}$' relationships for different H.

Eqs. (12) and (13); because the t value at the end of the primary stage for H=35mm is known from the Case I test, those t values for different thicknesses can be calculated by applying the H^2rule.

In order to solve Eqs. (9), (10), (12) and (13) a new technique was developed. If $\dot{e}(z,t)$,$\sigma'(z,t)$ and $e(z,t)$ are all known, $\dot{e}(z,t+\Delta t)$, $\sigma'(z,t+\Delta t)$ and $e(z,t+\Delta t)$ can all be determined as follows. First of all, $e(z,t+\Delta t)$ is determined by $e + \dot{e}\cdot\Delta t$, therefore $k(z,t+\Delta t)$ and $\gamma'(z,t+\Delta t)$ are also determined. When a value of $v(H-\Delta z/2,t+\Delta t)$ is assumed (H=sample thickness and Δz=distance between adjacent lattices), $\partial\sigma'/\partial z$ at $H-\Delta z/2$ is obtained by Eq.(10) and therefore $\sigma'(H-\Delta z,t+\Delta t)$ is obtained because σ' value at the drained plane, z=H, is always known. When substituting e and σ' values at $H-\Delta z$ for Eq.(12) or (13), $\dot{e}(H-\Delta z,t+\Delta t)$ is obtained. Therefore, $\partial v/\partial z$ at $H-\Delta z$ is obtained by Eq.(9) and then $v(H-3\Delta/2,t+\Delta t)$ is obtained. When these procedures are repeated, a value of $v(0,t+\Delta t)$ at the undrained plane is finally obtained by making an imaginary lattice at $z=-\Delta z$. If that value is not zero, another value $v(H-\Delta z/2,t+\Delta t)$ is reassumed to get the correct solution, $v(0,t+\Delta t)=0$.

Calculations were carried out by use of the following experimentally obtained data; C_c=1.0, Δe_y=-0.05, A=0.04, B=0.16 and logk=(e-13.9)/1.6.

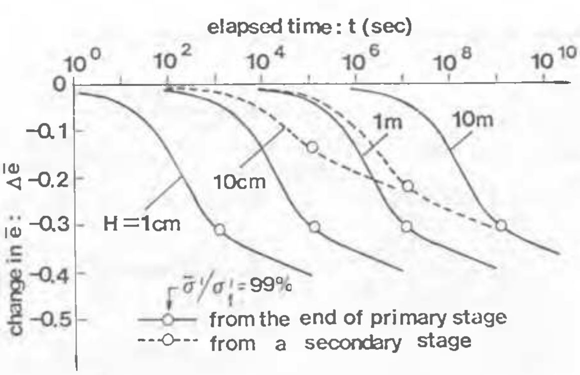

Fig.8 $\bar{e}$-logt relationships for different H.

Because of simplification, self weight of soil was neglected, γ'=0. The calculation results obtained are expressed in the following by use of averaged void ratios, $\bar{e}$, and averaged effective stresses, $\bar{\sigma}'$.

In the cases where consolidation starts from the EOP state, $\bar{e}$-log$\bar{\sigma}$' curves for different sample thicknesses are all similar figures as shown in Fig.7(a), therefore, $\bar{e}$-logt curves are also all similar figures as shown in Fig.8. On the contrary, in the cases where consolidation starts from the ASC state, the result corresponding to the type B curve is obtained as shown in Fig.8. The reason why both type A and B are obtained although creep action is taken into considerations for both EOP and ASC conditions comes from the fact that the values of $\dot{e}$ and e at the end of the primary stage are definite for each sample thickness; the fact is confirmed by comparing Figs. 7(a) and 7(b). Therefore, the amount of the primary compression for a sample largely depends on the initial state.

Regarding the time required to complete the primary stage, t(EOP), it should be noted that t(EOP) value is independent of the initial state, because t(EOP) is uniquely related to the $\dot{e}$ value at t(EOP) as precedingly stated.Therefore, the H^2 rule for t(EOP) is applied regardless of the difference of thickness and initial state.

CONCLUSIONS

The present studies can be concluded as follows:

(1) The correct mass conservation equation can be obtained even if the Eulerian coordinate system is used.
(2) Creep acts during the whole stage of consolidation.
(3) Constitutive relation is the isotache type.
(4) The amount of primary compression largely depends on a state before loading, but the time required to complete the primary consolidation is little influenced by the initial state.

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