Deformations due to principal stress rotation

Déformations dues à la rotation des contraintes principales

A.S.F.SAYÃO, Assistant Professor, Catholic University of Rio de Janeiro, Brazil (formerly Doctoral Student, University of British Columbia, Vancouver, Canada)

Y.P.VAID, Professor of Civil Engineering, University of British Columbia, Vancouver, Canada

SYNOPSIS A fundamental experimental study of the effects of principal stress rotations on the drained deformation response of a sand was carried out in a newly developed hollow cylinder torsional shear apparatus. Effect of both monotonic and cyclic rotations on sand over a range of relative densities and diverse initial stress conditions was investigated. It is shown that significant volumetric contractions and shear distorsions result from rotation of principal stresses,especially for looser specimens under higher principal stress ratios . Largest strains are generated in the first rotation cycle, with further cycles contributing to progressively smaller strains.

INTRODUCTION

In many geotechnical problems, soil loading is multiaxial and may also be accompanied by rotation of principal stress directions. In most models of soil behaviour, principal stress rotation effects are generally ignored. Recent experimental studies in the Hollow Cylinder Torsional (HCT) apparatus and the Directional Shear Cell (DSC) have, however, demonstrated that neglect of these effects could result in unconservative design (Symes et al, 1988; Wong and Arthur, 1986).

This paper presents results from a fundamental experimental study of the effects of principal stress rotation on the drained response of a sand. The study was carried out in a newly developed HCT apparatus at the University of British Columbia. A HCT specimen can be subjected to controlled changes in principal magnitudes (σ'_1, σ'_2, σ'_3) and directions (rotation α of σ'_1 to the vertical deposition direction). An alternative equivalent set of parameters: mean normal stress σ'_m, stress ratio $R = \sigma'_1/\sigma'_3$, intermediate stress parameter $b = (\sigma'_2-\sigma'_3)/(\sigma'_1-\sigma'_3)$ and rotation α, commonly used for HCT test interpretations, constitute the four stress parameters that influence sand behaviour.

THE HCT APPARATUS

The hollow cylinder test specimen is 15 cm external diameter, 10 cm internal diameter and 30 cm high. These dimensions were selected after giving appropriate considerations to keeping stress inhomogeneities within acceptable levels. Considerations in selecting adequate specimen geometry, together with specification of the 'no go' regions of the stress space, are discussed in detail by Hight et al. (1983)and Sayao and Vaid (1988).

The effect of radial shear due to end restraint was minimized by using polished anodized aluminum end platens and by the selection of a height to external diameter ratio of 2.0. The end platens were provided with thin radial ribs for transferring circumferential shear. Drainage from the specimen is provided by six small porous discs set 60º apart flush with each platen surface.

A double acting air piston is used to aplly the vertical load. Torque is applied by two pairs of identical air pistons together with a system of cables and pulleys. These allow application of monotonic or cyclic stress paths. Internal and external chamber pressures are independently applied by means of regulated air supply using air water interfaces separated from pressure chambers by diffusion spirals.

The apparatus and the instrumentation is described in detail in Vaid et al. (1988).

AVERAGE STRESSES AND STRAINS

The application of vertical force, circumferential torque, and external and internal confining pressures induces stresses σ_z, σ_r, σ_θ and $\tau_{z\theta}$ in an element in the wall of the HCT specimen. The associated four non-zero strain components are ε_z, ε_r, ε_θ and $\gamma_{z\theta}$.

Interpretation of results from HCT test is made by considering the entire specimen as a single element. Only σ_z is not dependent on the material constitutive law and is obtained by equilibrium considerations only. The remaining stress components correspond to the assumption of a linear elastic material. The values for σ_r, σ_θ and $\tau_{z\theta}$ are obtained by averaging over the volume of the specimen. Hight et al (1983) and others compute these stresses in a slightly different way. The differences arise partly on account of averaging across the wall instead of the volume of specimen, as well as assuming plastic constitutive law for evaluating $\tau_{z\theta}$. These differences, however, are minor and do not exceed 2% for the specimen dimensions and stress paths considered herein.

The radial stress σ_r in HCT test is normally the intermediate principal stress σ_2. Application of torque therefore causes rotation (α) of stresses σ_1 and σ_3 in the vertical plane perpendicular to the radial direction.

The strain components are calculated using considerations of compatibility of displacements together with the assumption of a linear variation of displacement across the specimen wall. These expressions are identical to those used by Hight et al (1983). Radial and circumferential strains are calculated from the measured changes in volume and height of the inner pressure chamber (Vaid et al., 1988).

EXPERIMENTATION

Ottawa sand, ASTM Designation C-109, was used in the tests. Ottawa sand is a uniform medium quartz sand with rounded particles (C_u = 1.9 and D_{50} = 0,4 mm). Reference minimum and maximum void ratios used are 0.50 and 0.82,respectively.

Specimens were prepared loose by water pluviation and then vibrated to the desired density . Saturation was ensured by using a back pressure of atleast 150 kPa and insisting on a B-value of at least 0.98. After forming, all specimens were first brought to a hydrostatic effective stress σ'_m = 50 kPa. The stress state (σ'_m, R, b) prior to the principal stress rotation phase was then achieved by a sequential application of R, b and σ'_m. Throughout this sequence α was zero.

Because of the variations in confining pressures appropriate membrane penetration corrections were applied to the measured volume changes according to the method suggested by Vaid and Negussey (1984). The resolution of the measuring system was better than 10^{-3}% for strains and 1 kPa for stresses.

RESULTS AND DISCUSSION

Test results are presented so as to illustrate the development of volumetric strain ε_{vol} and maximum shear strain γ_{max} (= $\varepsilon_1 = \varepsilon_3$) with principal stress rotation angle α. Typical response of a loose specimen (D_r = 20%) to cyclic changes in α is shown in Fig. 1. The specimen was under a stress state of σ'_m = 300 kPa, b = 0,5 and R = 2 that was held constant during rotation.

Principal stress rotation may be noted to induce contractive volume changes regardless of whether α is increased or decreased on either side of the vertical direction. ε_{vol} tends to be more significant for increasing phases of α than for the decreasing phases, with the largest contraction being associated with the first time increasing rotation phase. Contractive strains become progressively smaller with further rotation phases, regardless of direction. This behaviour is in accordance with the interpretations suggested by Symes et al.(1988). After the first forward rotation phase, the stress path moves beneath the bounding surface . As a consequence, irrecoverable strains become much less dominant, giving rise to smaller contractions with further rotations. This cumulative contaction would clearly imply

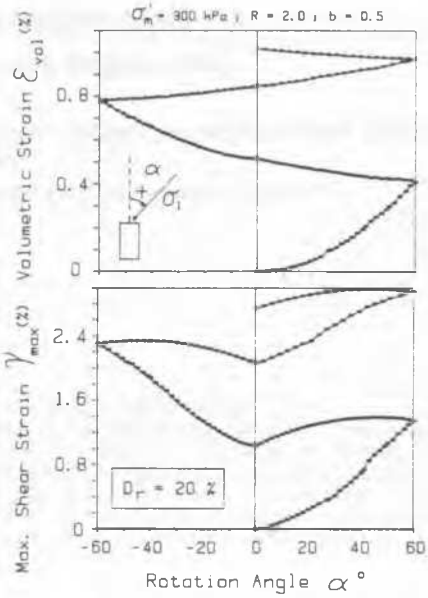

FIGURE 1 – Strain Development Due to Drained Cyclic Principal Stress Rotation

progressive pore pressure build-up under cyclic undrained conditions.

Like volumetric contactions, maximum shear strains accumulate progressively under cyclic changes in α. The strains increase, however,only on increasing rotation. Decrease in α towards the vertical direction tends to result in some recovery in the magnitude of shear strains . Residual shear strains nevertheless increase with each cycle (α = 0 → ±60 → 0) of principal stress rotations, though at a decreasing rate per cycle in the same direction.

Effect of Relative Density

Specimens for this test series were subjected to identical initial stress state (σ'_m = 300 kPa, b= 0.5 and R = 2) prior to initiating principal stress rotation. The results of first cycle rotation are illustrated in Fig. 2. Both ε_{vol} and γ_{max} induced due to rotation decrease progressively with increase in relative density. This would be expected because the degree of inherent anisotropy, which is primarily responsible for principal stress rotation effects, decreases as the relative density increases (Negussey and Vaid, 1986). Only sand with higher relative density (60%) responds with small dilation at initial stages of rotation. In all other cases, principal stress rotations induce progressive volume contractions, regardless of relative density and direction of rotation.

Similarly,induced shear strains decrease with increasing relative density under similar changes in α. Except for the dense sand (D_r=60%), γ_{max} continues increasing somewhat even after α starts decreasing from its maximum amplitude. Recovery in γ_{max} on decreasing α varies substantially with relative density.

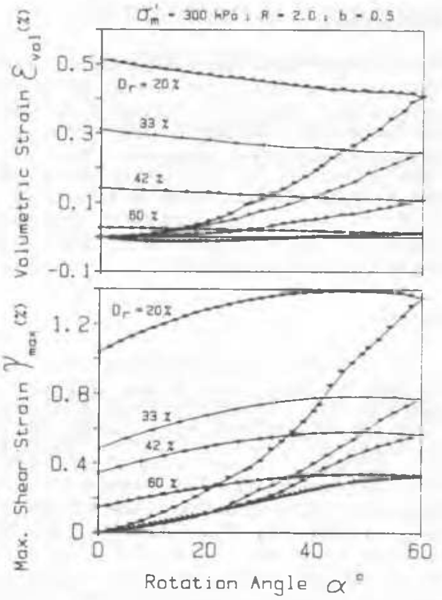

FIGURE 2 - Effect of Relative Density on Strain Development Due to Stress Rotation

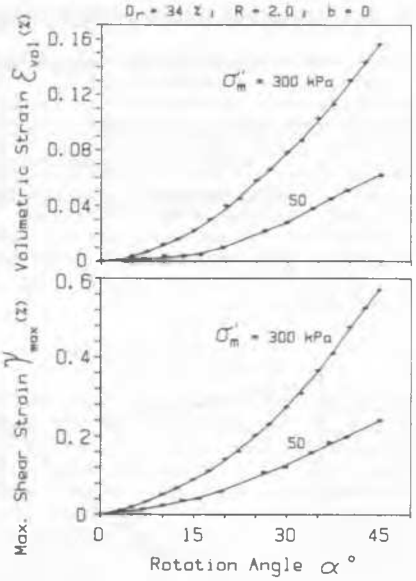

FIGURE 3 - Effect of Mean Effective Stress on Strain Development Due to Stress Rotation

Effect of Mean Effective Stress

Figure 3 shows the effects of forward principal stress rotation on two medium loose (D_r = 34%) specimens that have identical R = 2 and b = 0, but different σ_m' of 50 and 300 kPa. Much larger volumetric and shear strains are induced in sand at higher confining stress for a given rotation angle. The nature of the differences are qualitatively similar to those between loose and dense specimens at identical σ_m', illustrated in Fig. 2. The similar effects of decreasing relative density at constant confining stress, and increasing confining pressure at constant relative density, is a well recognized characteristic of granular materials.

Effect of Principal Stress Ratio

The three medium loose (D_r = 35%) specimens in this test series were subjected to identical σ_m' = 300 kPa and b = 0.5, but different values of R = 1.3, 2.0 or 3.0 prior to imposing principal stress rotations. Values of R greater than 3 were not used in this test series because of concern with excessive stress inhomogeneity across the specimen wall. The results of the first principal stress rotation cycles, illustrated in Fig. 4, show that for a given α both ε_{vol} and γ_{max} induced increase stress rotations have been found to preserve the nature of inherent depositional anisotropy (Rowe, 1971); Negussey and Vaid, 1986). Non-proportional loading to higher R levels, however, is likely to result in significant changes to this inherent anisotropy prior to imposing changes in α. This may account for part of the large difference between behaviour under principal stress rotations at low and high stress ratios.

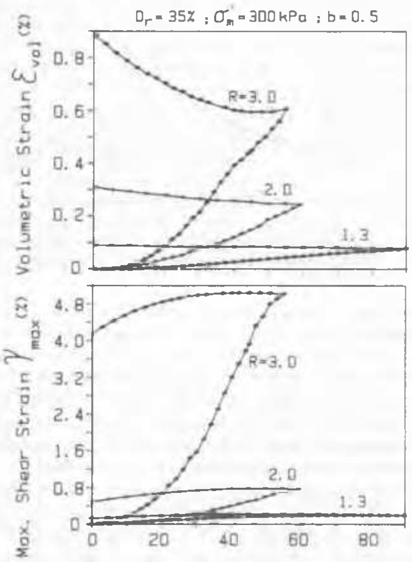

FIGURE 4 - Effect of Stress Ratio on Strain Development Due to Stress Rotation

Decrease in α, following its peak value in the forward direction, is associated with some recovery in γ_{max} similar to that observed in previous test series. The magnitude of recovery as a percentage of forward developed strains, however, decreases significantly with increase in R.

Effect of Intermediate Stress Parameter

In this test series, three medium loose specimen (D_r = 35%) were subjected to σ'_m = 300 kPa, R = 2 and different values of b = 0.0, 0.3 or 0.5, prior to imposing principal stress rotations. The range of b selected represents the majority of practical loading situations ranging from axisymmetric to plane strain. Volumetric and maximum shear strains induced during a rotation cycle are shown in Fig.5, and it may be noted that value of b does not appear influence the response of sand to principal stress rotations.

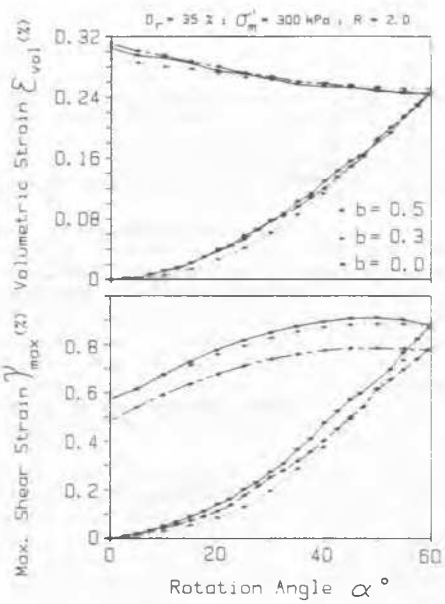

FIGURE 5 - Effect of Intermediate Stress Parameter on Strain Development Due to Stress Rotation

CONCLUSIONS

Continuous rotations of principal stress directions were applied to sand specimens under drained conditions. Progressive accumulation of both volumetric contraction and shear distortion was observed. At constant stress state (σ'_m, R, b), these deformations increase with decrease in relative density. Similarly, for a givem relative density, deformations increase with the level of σ'_m, and with the level of R.

The level of principal stress parameter b between 0.0 and 0.5 did not influence the strain response during principal stress rotations imposed on medium loose sand at fixed R and σ'_m.

Both volumetric contractions and shear strains were found to be more significant in the first rotation cycle. Progressively smaller cumulative straining was observed in subsequent cycles. Under drained loading conditions, this implied hardening effects of the previous rotation cycles. This type of behaviour, under undrained conditions, would lead to progressive softening due to accumulation of positive excess pore pressure.

ACKNOWLEDGMENTS

The work described in this paper has been supported by a grant from the Natural Science and Engineering Research Council of Canada. The National Research Council of Brazil (CNPq) provided financial support for A. Sayao's doctorate program at the University of British Columbia, while on leave from the Catholic University of Rio de Janeiro. Assistance of Kelly Lamb for preparation of the manuscript is gratefully acknowledged.

REFERENCES

Hight, D.W.; Gens, A. and Symes, M.J. (1983) - "The development of a new hollow cylinder apparatus for investigating the effects of principal stress rotation in soils". Geotechnique, Vol. 33, nº 4, 355-383.

Negussey, D. and Vaid, Y.P. (1986) " Sand deformation under proportional loading". Canadian Geotech. Journal, Vl. 23, nº 2, pp. 155-163.

Rowe, P.W. (1981) "Theoretical meaning and observed values of deformation parameters for soil". Proc. Roscoe Memorial Symposium , Cambridge, U.K. 143-194.

Sayao, A.S.F. and Vaid, Y.P. (1988) " A critical assessment of stress distribution in hollow cylinder test specimens". Soil Mechanics Series nº 123, Department of Civil Engineering, The University of British Columbia.

Symes, M.J.; Gens, A. and Hight, D.W. (1988) - "Drained principal stress rotation in saturated sand". Geotechnique, Vol.38, nº 1, 59-81.

Vaid, Y.P. and Negussey, D. (1984) "A critical assessment of membrane penetration in the triaxial test". Geotechnical Testing J., A. S.T.M., Vol.7, nº 2, 70-76.

Vaid, Y.P.; Sayao, A.S.F.; Enhuang, H. and Negussey, D. (1988) "The University of British Columbia Hollow Cylinder Torsional Apparatus". Soil Mechanics Series nº 118, Department of Civil Engineering, University of British Columbia.

Wong, R.K. and Arthur, J.R. (1986) " Sand sheared by stresses with cyclic variations in direction". Geotechnique, Vol.36, nº 2 , 215-226.