Shear strength of sand from CPT

La résistance au cisaillement des sables à partir du CPT

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SYNOPSIS: A brief review of the theoretical and empirical solutions used to predict shear resistance of clean sands from cone resistance (q_c) is reported. The reliability of several approaches is evaluated comparing friction angles $\phi'(q_c)$ predicted from CPT's performed in a calibration chamber with ϕ'(TX) measured in triaxial compression tests on two pluvially deposited sands (TS and HS).

1 INTRODUCTION

A brief review of the interpretation of static cone penetration test (CPT) results to assess shear strength of clean sands is presented. This presentation involves the analysis of theoretical and empirical solutions used to predict the angle of friction ϕ' of clean sands from the cone resistance q_c measured in a fully drained penetration test. The reliability of these solutions is evaluated comparing values of ϕ' inferred from q_c measured during calibration chamber (CC) tests with ϕ'(TX) deduced from triaxial compression tests. Both measurements were performed on pluvially deposited predominantly silica Ticino and Hokksund sands. The main characteristics of these sands can be found in the paper presented to this conference by Baldi et al., 1989.

2 SHEAR STRENGTH OF COHESIONLESS SOILS

The most important aspect of the shear strength behaviour of cohesionless soils is their non-linear failure envelope (De Beer, 1965; Vesic & Clough, 1968). Because of this aspect, the angle of friction ϕ' of a given sand is not uniquely defined but depends on the density and magnitude of the effective normal stress on the failure plane at failure σ'_{ff}. Therefore, any value of ϕ' inferred from penetration test results corresponds to a secant friction angle ϕ'_s linked to a specific level of σ'_{ff}. The description of the non-linear strength envelope was given by Baligh (1975) using the following expression:

$$\tan\phi'_s = \tan\phi'_o + \tan\alpha \left[\frac{1}{2.3} - \log\left(\frac{\sigma'_{ff}}{p_a}\right)\right] \quad \ldots(1)$$

where:

ϕ'_s = peak secant friction angle at σ'_{ff}

σ'_{ff} = effective normal stress on the failure plane at failure

p_a = reference stress, assumed equal to 98.1 kPa

ϕ'_o = secant angle of friction at σ'_{ff}=267 kPa

α = angle which describes the curvature of the failure evelope.

Baldi et al. (1986) have shown that α increases with increasing relative density D_R. As a first approximation, for silica sands the value of α can be evaluated sands using the following expression:

$$\alpha = \left[\frac{D_R - 0.2}{0.8}\right] \cdot 10°; \qquad \text{for } \alpha \geq 0°, \qquad ..(2)$$

where D_R is expressed as a fraction of one. The shear strength of cohesionless soils is related to the rate of dilation at failure which in turn depends on the soil density, level of mean effective stress and soil mineralogy. These factors are reflected in Rowe's (1962) stress–dilatancy theory which has recently received a simple but conceptually sound formulation by Bolton (1984; 1986).

Bolton (1986) has shown that the peak secant friction angle (ϕ'_s) of many sands from triaxial tests can be estimated from the empirical expression;

$$\phi'_s = \phi'_{cv} + 3\ I_D \qquad \ldots(3)$$

where ϕ'_{cv} is the friction angle at constant volume and I_D is the relative dilatancy index given by:

$$I_D = D_R\ (Q - \ln p'_f) - 1 \qquad \ldots(4)$$

being p'_f the mean effective stress at failure and Q an empirical constant depending predominantly on the mineralogy of the sand. Bolton (1986) suggested the following general values for Q:

Grain Type	Quartz	Feldspar	Limestone	Anthracite	Chalk
Q	10	10	8	7	5.5

For most silica sands a value of Q=10 was suggested. Figure 1 presents the generalized variation of ϕ'_s–ϕ'_{cv} for silica sand proposed by Bolton (1986) and reflects in a clear manner the non-linear nature of the strength envelope of

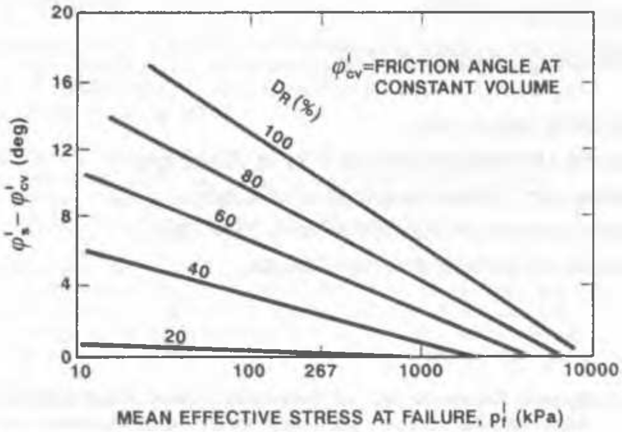


Figure 1. Generalized variation of triaxial peak ϕ'_s with mean effective stress.
After Bolton, 1986.

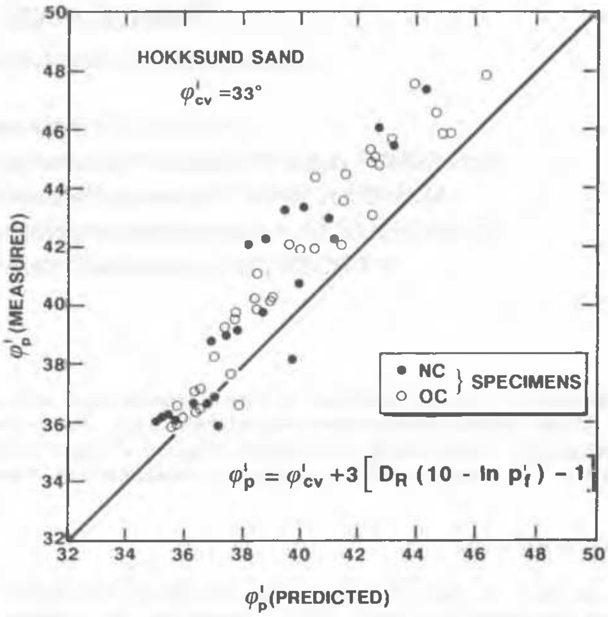


Figure 2. Validation of Bolton's (1984) dilatancy theory.

cohesionless soils. Figure 2 presents the results of triaxial tests on Hokksund sand used to evaluate Bolton's stress-dilatancy theory. Although Hokksund sand is a predominantly silica sand, Bolton's formulation underpredicts ϕ'_s by about 2° to 3°. Hence, in this case, since all variables (ϕ'_{cv}, p'_f, D_R) are known, the lack of agreement between ϕ'_s(measured) and ϕ'_s(predicted) must be linked with the assumed value of Q=10.

An alternate manner to account for the influence of density and stress level on ϕ' is to refer to the state parameter (ψ) (Been & Jefferies, 1985). This parameter is defined as the difference between the current void ratio (e) of the sand and its void ratio at steady state (e_{ss}), for the same mean effective stress p'. The state parameter ψ consistently combines the influence that e (or D_R) and p' have on the behaviour of cohesionless soils at failure or close to it. Therefore, ψ generally correlates well with ϕ'_s (see Figures 3 and 4) and with the maximum rate of dilation at failure.

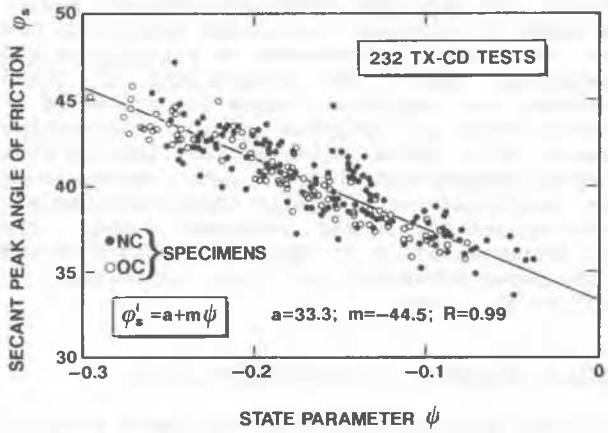


Figure 3. Friction angle of Ticino sand vs. state parameter.

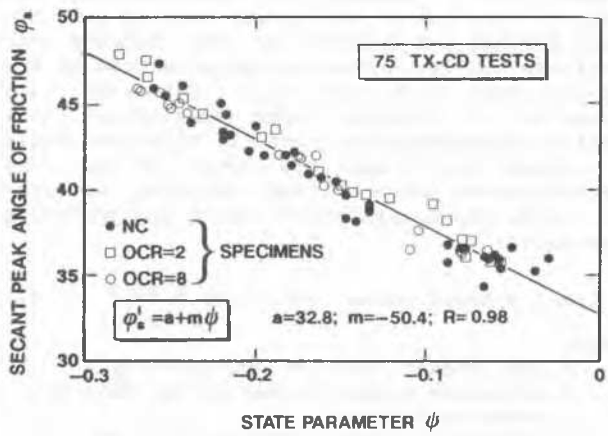


Figure 4. Friction angle of Hokksund sand vs. state parameter.

3 INTERPRETATION OF CPT FOR ϕ'_s

The existing methods for estimating ϕ'_s from CPT can be grouped as follows:

- bearing capacity theories based on rigid-plastic soil models;
- bearing capacity theories involving the theories of expanding cavities in elastic-perfectly plastic soil;
- empirical correlations.

Among the solutions assuming the rigid-plastic soil models, the most frequently used is the one proposed by Durgunoglu & Mitchell (1973) (D+M), see Figure 5.

Although the D+M method cannot account for soil compressibility, CC test results on predominantly silica sands have shown that this approach provides reliable predictions of ϕ'_s (Robertson & Campanella, 1983; Baldi et al., 1986; Mitchell & Keaveny, 1986). However, as shown by Mitchell and Keaveny (1986), the D+M

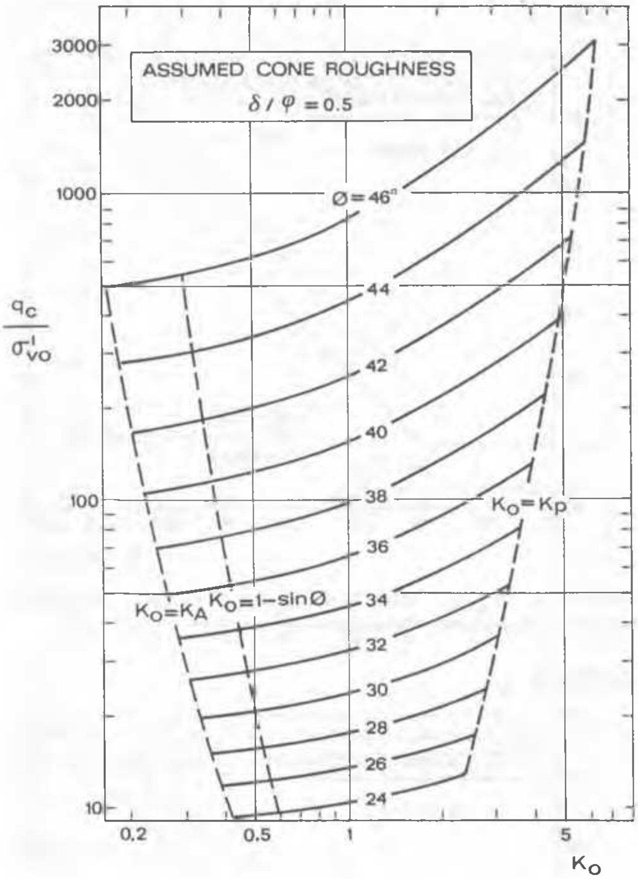


Figure 5. Chart for predicting peak friction angle (ϕ') from CPT for uncemented, unaged, silica sands using Durgunoglu & Mitchell bearing capacity theory (Adapted from Marchetti, 1985).

method as well as others based on classical bearing capacity theories:

- requires the knowledge of σ'_{ff} around the cone penetrometer because of non-linearity of the strength envelope and
- tends to conservatively underestimate ϕ'_s, a trend which increases with increasing sand compressibility.

As to the former aspect, the problem is far from being solved satisfactorily. In first approximation, the value of σ'_{ff} can be estimated following the suggestions of Meyerhof (1957), De Beer (1965) and Schmertmann (1982).

To account for soil compressibility when interpreting CPT results for ϕ'_s, it is necessary to resort to bearing capacity theories based on cavity expansion concepts (Vésic, 1972; Baligh, 1976). As shown by Mitchell & Keaveny (1986), the Vésic (1972; 1977) expansion theory provides a good prediction of $\phi'_s=f(q_c)$ for most sands studied including highly compressible sands.

Unfortunately, the cavity expansion analyses require considerable input data regarding soil stiffness in the elastic region and volumetric strain in the plastic region, rendering its use in practice difficult. The cavity expansion approach also requires an estimate of the average σ'_{ff} around the penetrating cone. In his recent work, Delladonna (1988) has produced a chart giving the ratio of σ'_{ff}/q_c for cohesionless soils referring to the failure mechanisms under the cone tip postulated by Meyerhof (1963) and Vésic (1977). His analysis indicated $\sigma'_{ff}/q_c \simeq 0.1$ for $\phi'_s=\phi'_{cv}$ to $\simeq 0.04$ at $\phi'_s=45°$.

During the past decade several empirical correlations have been developed to relate q_c to ϕ'_s. Among the early works one of the most notable is by Schmertmann (1978) who correlated q_c to ϕ'_s using D_R and grain size distribution.

However, even in silica soils, this approach has the problem that a non-unique correlation exists between q_c and D_R, and it also suffers from the lack of definition of σ'_{ff} to which the obtained ϕ'_s should be referred.

In the early 1980's, some authors proposed empirical correlations based on the results of large CC studies (Lunne & Christoffersen, 1984; Robertson & Campanella, 1983) relating ϕ'_s to q_c/σ'_{vo}. These correlations proved to be quite effective in uncemented, unaged silica sands that are approximately NC, where $K_o \simeq 0.5$.
In fact, the CC studies have shown that the penetration resistance is almost completely controlled by the initial effective horizontal stress. Therefore any empirical correlation between ϕ'_s and q_c should more rationally include σ'_{ho} or K_o instead of σ'_{vo}.

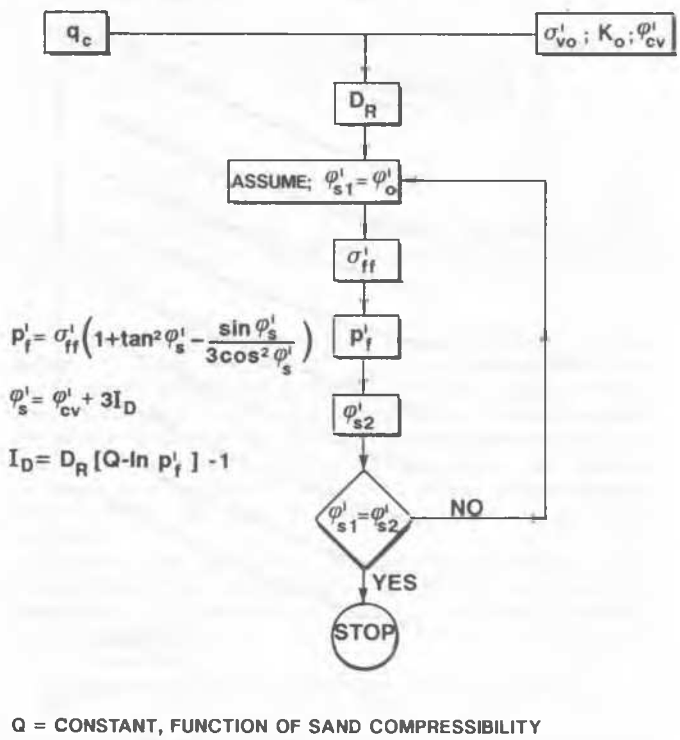


Figure 6. $\phi'_{peak}=f(q_c)$ of sand from Bolton's stress-dilatancy theory (Bolton, 1986).

Recently, Jamiolkowski et al. (1988) combined CC test results with Bolton's (1984; 1986) stress dilatancy theory and proposed a method which allows one to evaluate $\phi_s'=f(q_c)$ and incorporates K_o.

The proposed procedure consists of the steps shown in Figure 6. Figure 7 shows the results for silica sand expressed in terms of ϕ_o', defined as the secant angle of friction at $\sigma_{ff}'= 267$ kPa. For values of $\sigma_{ff}' \neq 267$ kPa, ϕ_s' should be evaluated using formula (1) with values of α selected as a function of D_R. Figures 8 through 11 show comparisons between ϕ_s' predicted from q_c and ϕ_s' measured from triaxial compression tests. The method leads to an underprediction of ϕ_s' by about 1° to 2° which is of the same order of magnitude as that shown in Figure 2 representing the validation of Bolton's theory (1984; 1986) in Hokksund sand.

The empirical approach suggested by Been et al. (1986) and modified by the writers is based on the state parameter ψ. A summary of this approach is shown in Figure 12.

The state parameter approach incorporates the mean effective stress (p') and hence it requires the independent measurement (or estimate) of σ_{ho}' or K_o.

For cone penetration in sands the penetration resistance (q_c) is generally significantly larger than the total mean stress (p); hence there is little error in assuming:

$$\frac{q_c-p}{p'} \simeq \frac{q_c}{p'} = \frac{3\,q_c}{\sigma_{vo}'\,(1 + 2K_o)} \qquad \text{.. (5)}$$

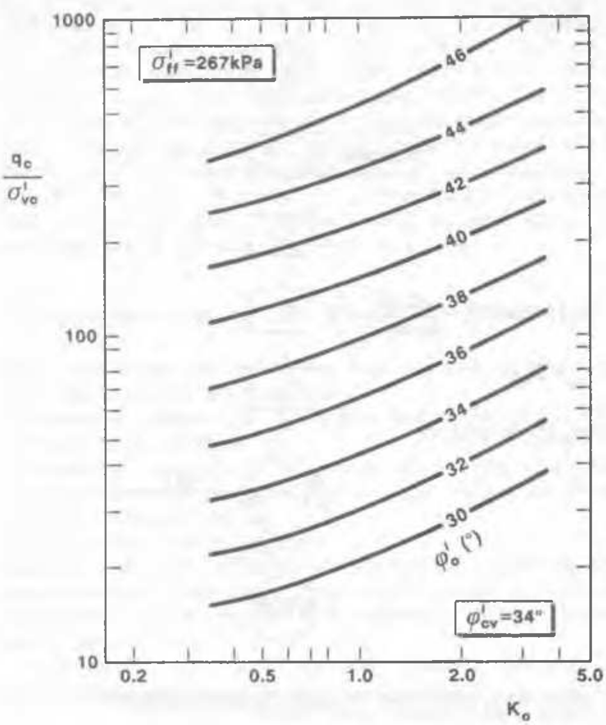


Figure 7. Friction angle ϕ_s' of silica sand using Bolton (1986) stress-dilatancy theory. Adapted from Jamiolkowski et al., 1988.

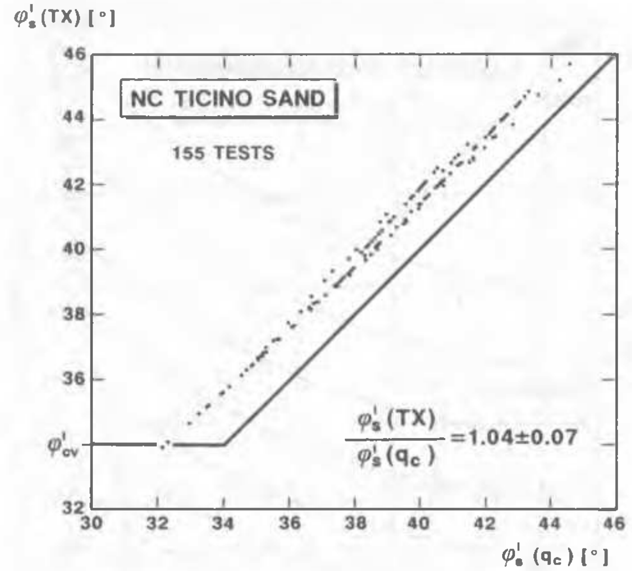


Figure 8. $\phi_s(q_c)$ for Ticino sand using Bolton's stress-dilatancy theory.

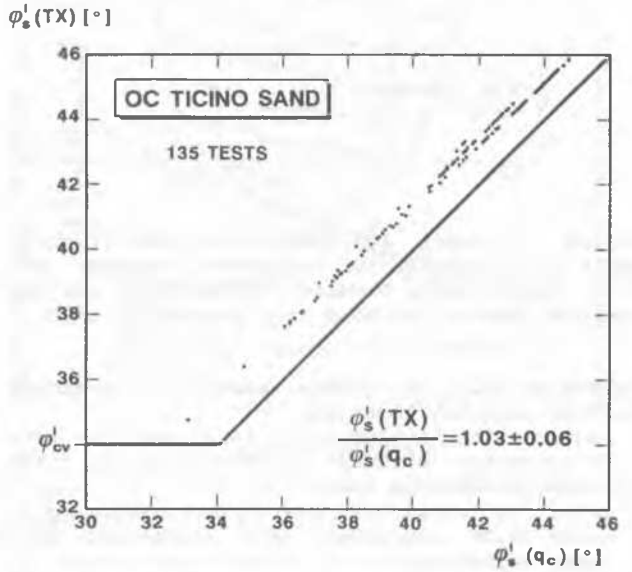


Figure 9. $\phi_s(q_c)$ for Ticino sand using Bolton's stress-dilatancy theory.

Figure 12 includes two axes for q_c/σ_{vo}', one is approximately representative of a normally consolidated (K_o=0.5) and the other of a quite highly overconsolidated (K_o=1) sand. The major contribution with the state parameter approach shown in Figure 12 appears to be the incorporation of sand compressibility in the form of the slope of the steady state line, λ_{ss}. The value of λ_{ss} appears to be an indirect measure of the compressibility of a sand. A potential problem with the state parameter approach may result from the need to obtain a series of samples to determine λ_{ss}. This could complicate the interpretation if the grain size distribution

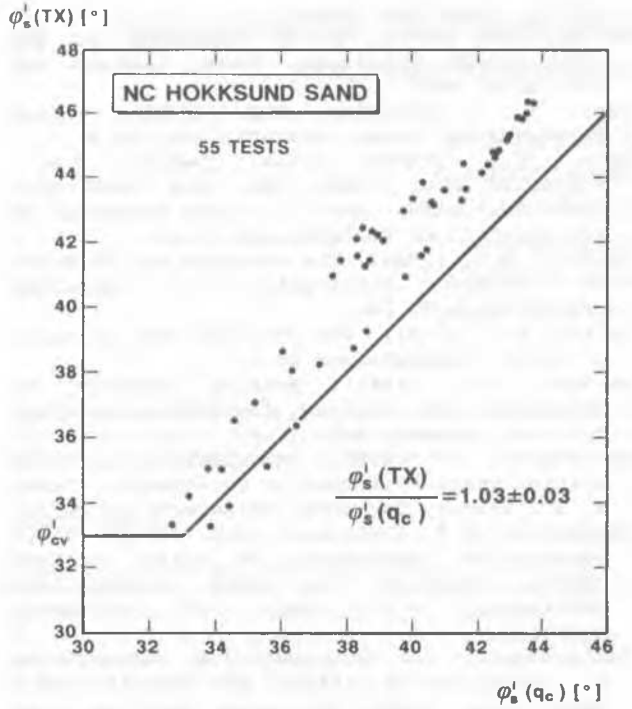


Figure 10. $\phi'_S(q_C)$ for Hokksund sand using Bolton's stress dilatancy theory.

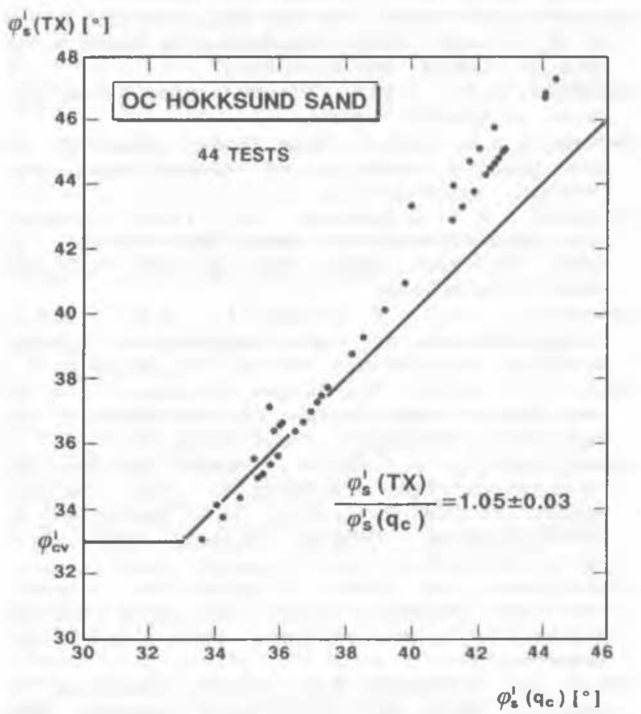
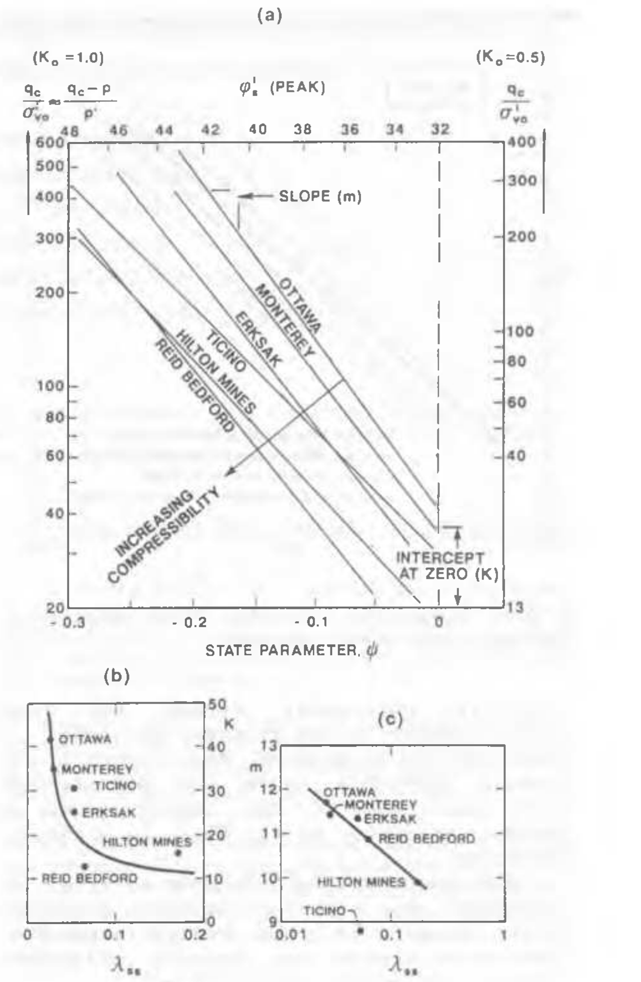


Figure 11. $\phi'_S(q_C)$ for Hokksund sand using Bolton's stress dilatancy theory.



$$\frac{q_c - p}{p'} = K \exp(-m\psi)$$

Figure 12. Evaluation of state parameter and friction angle from CPT (Adapted from Been et al., 1986).

and mineralogy of a deposit varies rapidly with depth. The value of λ_{ss} is sensitive to variations in fines content for the same sand. The influence of non-linearity in the strength envelope also needs to be incorporated in an explicit manner. The ϕ'_S values shown in Figure 12 were determined from triaxial compression tests with the same consolidation p' used to define ψ.

Figure 13 compares the state parameter with the methods by Durgunoglu & Mitchell (1973), Robertson & Campanella (1983), and Jamiolkowski et al. (1988).

4 SUMMARY

The existing experience concerning the interpretation of ϕ'_S from CPT shows:

- Any reliable approach to the problem should take into account the soil compressibility,

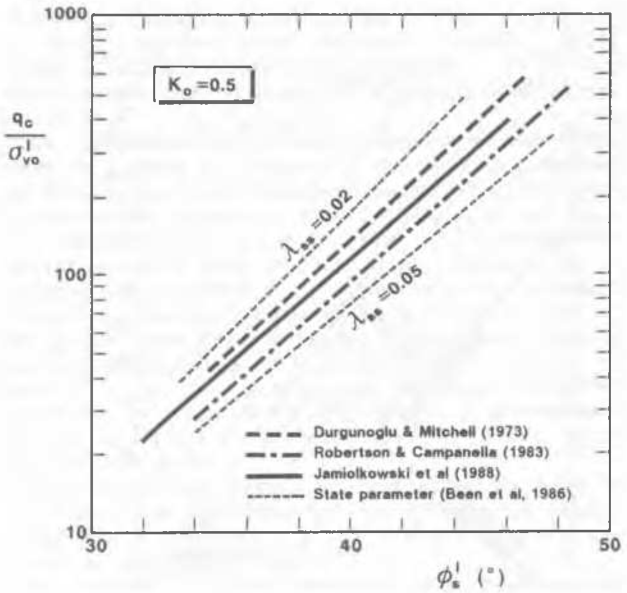


Figure 13. Comparison between state parameter approach and other major methods.

the in situ horizontal stress and the non-linear nature of the strength envelope.

- In predominantly uncemented non-crushable or moderately crushable sands the D+M method based on rigid plastic soil model gives a reasonable estimate of ϕ_s which is slightly conservative.
- In a more compressible crushable material it is necessary to use the bearing capacity theories based on the cavity expansion concepts which account for elastic stiffness and plastic volumetric strain of soil.
- As an alternative it is possible to use empirical approaches originating either from Bolton's (1984) stress-dilatancy theory or from the state parameter concept.
- In all cases the use of $\phi'_s(q_c)$ in design requires the capability to calculate or to estimate σ'_{ff}, in order to account for the non-linearity of the strength envelope.
- Finally one has to realize that q_c is a complex function of a large number of soil parameters and therefore all methods used to estimate ϕ'_s from q_c are approximate in nature.

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