This paper was downloaded from the Online Library of the International Society for Soil Mechanics and Geotechnical Engineering (ISSMGE). The library is available here:

https://www.issmge.org/publications/online-library

This is an open-access database that archives thousands of papers published under the Auspices of the ISSMGE and maintained by the Innovation and Development Committee of ISSMGE.
Limit pressure of pressuremeter tests
La pressione limite des essais pressiométriques

V.N. GHIONNA, Associate Professor of Geotechnical Engineering, Università degli Studi di Pavia, Pavia, Italy
M. JAMIOLKOWSKI, Professor of Geotechnical Engineering, Politecnico di Torino, Turin, Italy
R. LANCELOTTA, Associate Professor of Geotechnical Engineering, Politecnico di Torino, Turin, Italy
M. MANASSERO, Ph.D., Senior Engineer, Ingegneria Geotecnica, Turin, Italy

SYNOPSIS: On the basis of the flow rule proposed by Rowe (1968) and a numerical solution for the interpretation of the Self-Boring Pressuremeter Tests (SBPT), a procedure to assess the limit pressure $p_{\text{lim}}$ of cylindrical cavity in pure frictional material is presented. The results obtained using the proposed approach are validated using the results of the SBPT's.

1 INTRODUCTION

A large number of SBPT's have been performed in the calibration chamber (CC) under controlled boundary conditions (see Table 1). Using these results, a procedure to evaluate the ultimate stress of a cylindrical cavity is proposed. The method assumes that the Rowe's (1962) stress dilatancy theory applies and that the sand is failing at the condition of constant volume when the critical state is reached at the cavity wall. The instant at which this condition is reached is evaluated using a previously developed numerical method suggested by Manassero (1988). The analysis of this latter aspect enables a solution of general validity, to be obtained. The solution highlights how a reliable estimate of $p_{\text{lim}}$ from SBPT can be made only provided that the expansion strains are large enough to reach the condition of critical state at the cavity wall ($\epsilon_{\text{CV}} \geq 15\%$).

Simplified procedures, previously reported in the literature, can, in this context, lead to a significant overestimate of $p_{\text{lim}}$.

2 PROPOSED METHOD

Referring to a cylindrical co-ordinate system, the basic stress and strain definitions are:

- $\sigma'_r$, $\sigma'_\theta$: respectively radial and hoop principal effective stresses;
- $p':$ radial effective stress at the cavity wall;
- $\epsilon'_r = -d\epsilon_r/d\tau$: radial principal strain;
- $\epsilon'_\theta = -\epsilon_r/\tau$: hoop principal strain;
- $\epsilon$: radial displacement;
- $r = r_0 + \epsilon$: current radial distance;
- $R = r_0 + \epsilon R$: current radius of expanding cavity;
- $\epsilon = -\epsilon R/R$: hoop principal strain at cavity wall;
- $R = r_0 + \epsilon R$: current radius of expanding cavity;
- $p'_r$: volumetric strain in plane strain conditions;
- $\gamma = \epsilon'_r - \epsilon'_\theta$: shear strain.

The general equations of equilibrium and strain compatibility around the cylindrical cavity are:

\[
\begin{align*}
\frac{d\sigma'_r}{dr} &= \frac{\sigma'_r - \sigma'_\theta}{r} \quad \ldots \ (1) \\
\frac{d\sigma'_\theta}{dr} &= \epsilon'_r - \epsilon'_\theta \quad \ldots \ (2)
\end{align*}
\]

The same equations (1) and (2) in terms of $r/d\tau$ become:

\[
\frac{\sigma'_r - \sigma'_\theta}{d\sigma'_r} = \epsilon'_r - \epsilon'_\theta \quad \ldots \ (3)
\]

When the critical state condition ($d\epsilon_{\text{V}}=0$) is reached:

\[
\epsilon'_r = \epsilon'_\theta + C_1 \quad \ldots \ (4)
\]

and upon integrating:

\[
\epsilon'_r = \epsilon' + C_1 \quad \ldots \ (5)
\]

and moreover:

\[
\frac{\sigma'_r}{\sigma'_r - \sigma'_\theta} = K'_{\text{CV}} = \frac{1 - \sin\phi_{\text{CV}}}{1 - \sin\phi_{\text{CV}}} \quad \ldots \ (6)
\]

where:

- $\phi_{\text{CV}}$: constant volume friction angle.

Combining eqs. (4), (5) and (6) with eq. (3) and rearranging it follows:

\[
\frac{d\sigma'_r}{\sigma'_r} = \frac{K'_{\text{CV}} - 1}{2} \left( \frac{d\epsilon}{C_1 + 2 - \epsilon'_\theta} \right) \quad \ldots \ (7)
\]

Solving eq. (7) one can write:

\[
\ln \frac{2}{1 - K'_{\text{CV}}} = \ln \left( \frac{C_1 + 2 - \epsilon'_\theta}{C_2} \right) \quad \ldots \ (8)
\]

where from eq. (5):

\[
C_1 = \epsilon'_r + \epsilon'_\theta = \epsilon_{\text{CV}} = \text{constant}
\]
Table 1. Result of proposed method from SBPT's in calibration chamber.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>D_R</th>
<th>OCR</th>
<th>(\sigma'_h)</th>
<th>(\sigma'_v)</th>
<th>(K_o)</th>
<th>(q_{c})</th>
<th>(\epsilon_{cv})</th>
<th>(\epsilon_{cv}^*)</th>
<th>(\gamma_{cv})</th>
<th>(P'_{cv})</th>
<th>(P'_{lim})</th>
</tr>
</thead>
<tbody>
<tr>
<td>208</td>
<td>43.2</td>
<td>1.00</td>
<td>112.8</td>
<td>45.13</td>
<td>0.400</td>
<td>5491</td>
<td>-8.93*</td>
<td>-1.92*</td>
<td>15.93*</td>
<td>312*</td>
<td>770</td>
</tr>
<tr>
<td>209</td>
<td>49.1</td>
<td>1.00</td>
<td>116.7</td>
<td>51.99</td>
<td>0.441</td>
<td>6840</td>
<td>-8.93</td>
<td>-1.88</td>
<td>15.97</td>
<td>409</td>
<td>1008</td>
</tr>
<tr>
<td>210</td>
<td>53.3</td>
<td>1.00</td>
<td>512.1</td>
<td>244.27</td>
<td>0.479</td>
<td>17582</td>
<td>-7.93*</td>
<td>-1.65*</td>
<td>14.22*</td>
<td>1403.1*</td>
<td>3680</td>
</tr>
<tr>
<td>211</td>
<td>64.6</td>
<td>2.86</td>
<td>110.9</td>
<td>82.40</td>
<td>0.747</td>
<td>25385</td>
<td>-8.93*</td>
<td>-3.65*</td>
<td>14.20*</td>
<td>1516.9*</td>
<td>3891</td>
</tr>
<tr>
<td>212</td>
<td>47.5</td>
<td>2.78</td>
<td>112.8</td>
<td>83.39</td>
<td>0.740</td>
<td>7883</td>
<td>-8.93*</td>
<td>-0.50*</td>
<td>17.41*</td>
<td>655.5*</td>
<td>1573</td>
</tr>
<tr>
<td>213</td>
<td>42.4</td>
<td>1.00</td>
<td>113.8</td>
<td>53.96</td>
<td>0.476</td>
<td>5788</td>
<td>-8.93*</td>
<td>-2.54*</td>
<td>15.97*</td>
<td>455.6*</td>
<td>1140</td>
</tr>
<tr>
<td>214</td>
<td>92.3</td>
<td>1.00</td>
<td>514.6</td>
<td>225.63</td>
<td>0.439</td>
<td>47457</td>
<td>-8.93*</td>
<td>-2.78*</td>
<td>15.07*</td>
<td>2096.4*</td>
<td>5272</td>
</tr>
<tr>
<td>215</td>
<td>45.4</td>
<td>7.57</td>
<td>510.8</td>
<td>60.8</td>
<td>0.927</td>
<td>11472</td>
<td>-8.93*</td>
<td>-1.17</td>
<td>19.02*</td>
<td>640.5*</td>
<td>1452</td>
</tr>
<tr>
<td>216</td>
<td>42.4</td>
<td>1.00</td>
<td>113.8</td>
<td>53.96</td>
<td>0.476</td>
<td>5788</td>
<td>-8.93*</td>
<td>-2.54*</td>
<td>15.97*</td>
<td>455.6*</td>
<td>1140</td>
</tr>
<tr>
<td>217</td>
<td>64.6</td>
<td>2.86</td>
<td>110.9</td>
<td>82.40</td>
<td>0.747</td>
<td>25385</td>
<td>-8.93*</td>
<td>-3.65*</td>
<td>14.20*</td>
<td>1516.9*</td>
<td>3891</td>
</tr>
<tr>
<td>218</td>
<td>65.4</td>
<td>7.66</td>
<td>59.8</td>
<td>56.90</td>
<td>0.980</td>
<td>10077</td>
<td>-9.92</td>
<td>0.64</td>
<td>20.47</td>
<td>640.5*</td>
<td>1452</td>
</tr>
<tr>
<td>219</td>
<td>65.9</td>
<td>5.46</td>
<td>112.9</td>
<td>101.04</td>
<td>0.902</td>
<td>13815</td>
<td>-8.48</td>
<td>-0.92</td>
<td>16.03</td>
<td>958.0</td>
<td>2365</td>
</tr>
</tbody>
</table>

\(\sigma'_{ho}\) : vertical and horizontal effective stress at midheight of the specimen just before expansion;

\(\sigma'_{vo}\) : overconsolidation ratio

\(\epsilon_{cv}\) : attained total volumetric strain when constant volume behaviour of soil around the expanded cavity is reached.

Rewriting eq.(8) at the cavity wall in terms of measured parameters \(p'\) and \(\epsilon\) one gets:

\[
\ln \frac{2}{1 - \frac{K_o}{q_c}} \ln p' = \ln \left[ \frac{\epsilon_{cv} - \epsilon}{\frac{\epsilon_{cv}}{2} - \epsilon} \right] + C_2 \quad \ldots (9)
\]

To find the value of the constant \(C_2\) one can refer to the method of interpretation of SBPT's in sands proposed by Manassero (1988). This method incorporates the non linear nature of the stress-strain behaviour of sand, and assumes that Rowe's stress-dilatancy concept applies. Therefore:

\[
\frac{\sigma'_{ho}}{K_o} = \frac{1}{\sigma'_{vo}} \frac{d\epsilon}{d\gamma} \quad \ldots (10)
\]

Combining eq.(10) with eq.(3) the following final solution is obtained (Manassero, 1988):
\[ \frac{d\sigma'^I}{d\epsilon^I} = - \frac{\sigma'^I}{\epsilon^I} \left( 1 - K^I_{\epsilon} \frac{d\tau^I}{d\epsilon^I} \right) \]

The above equation has been integrated using the finite differences technique under the assumption that:
- At the start of a SBPT \( \epsilon^I=\epsilon^F=0 \) and \( \sigma^I=\sigma^F=\sigma^F_{ho} \) (\( \sigma^F_{ho} \) initial in situ horizontal stress).
- The relation between \( \sigma'^I \) and \( \epsilon^I \) can be derived directly from the SBPT expansion curve as a function of measured \( \rho' \) and \( \epsilon \) (Manassero, 1988).

The above mentioned procedure allows the complete stress-strain and effective stress path of a sand to be computed from the results of a drained SBPT expansion test knowing \( \rho'^I_{CV} \).

Figure 1 shows an example of the stress-strain, volumetric vs. shear strain relationships, and effective stress paths computed on the basis of a SBPT performed in CC in dense Ticino sand (Bellotti et al., 1988).

Returning to the assessment of \( C_2 \), by substituting \( \epsilon=\epsilon^CV \) and \( \rho'=\rho^CV \) into eq. (9) one can find:

\[ C_2 = \ln \left\{ \frac{p'^CV}{\gamma^CV} \left[ \frac{2}{1 - K^CV} \right] \right\} \]

being:
\( \psi^CV = \epsilon^CV - \epsilon^CV \): cavity shear strain at the point where the sand reach the critical state \( (\Delta_{\epsilon^F}=0) \) named constant volume point in Figure 1.

where:
- \( \epsilon^CV \): hoop cavity strain
- \( \epsilon^CV \): radial cavity strain
- \( p'^CV \): cavity effective stress

Now considering that when \( \epsilon^R = -\epsilon^F \):

\[ \epsilon = \frac{\epsilon^R}{\epsilon^F_{ho} + \epsilon^R} = -1 \quad \text{and} \quad \rho' = \rho'^{lim} \]

one obtains the equation for the effective limit cavity stress:

\[ \rho'^{lim} = \rho'^CV \left[ \frac{\epsilon^CV + 2}{\epsilon^CV} \right] \frac{(1-K^CV)}{2} \]

An illustration of the proposed procedure to assess \( \rho'^{lim} \) from SBPT is shown in Figure 2.

Table 1 gives the results obtained after applying the above outlined procedure for the evaluation of \( \rho'^{lim} \) to the results of 45 SBPT's performed in the CC in Ticino sand (Bellotti et al., 1988). With the values of \( \epsilon^CV \) reported in the Table 1 one can infer that the use of eq. (13) under the assumption that \( \epsilon^CV=0 \) leads to an underestimate of \( \rho'^{lim} \) which does not exceed 8%.

Therefore, according to eq. (9) \( \rho'^{lim} \) can be evaluated following a simplified procedure consisting of plotting the experimental \( \rho' \) vs. \( \epsilon \) data on a double logarithmic scale and then constructing a line with a slope of 0.5 \( (1-K^CV) \)

from the last point until the intersection with the point corresponding to \( \epsilon=100\% \). An example of application of this simplified procedure to an expansion curve obtained from CC tests in medium dense Ticino sand is shown in Figure 3. It is important to stress that this simplified approach assumes implicitly that the pressuremeter membrane has been expanded enough to reach the critical state condition at the cavity wall.
3 REMARKS

On the basis of the previous statements and with reference to the data given in Table 1, the following comments can be made:

a. The analysis of the available SBPT's following the complete procedure outlined above have indicated that in about 50% of the tests the maximum attained cavity strain (10%) was not sufficient to reach the critical state in the sand at the cavity wall. However, the ε_{v} observed at the end of these tests was already very small. This highlights the importance of using pressuremeter devices which allow higher expansion strains than that of the Caskometer probe used in this research if one wants to evaluate p_{lim} in a reliable manner.

b. In the tests where the critical state condition at the cavity wall has not been reached the assessment of p_{lim} from the slope of the apparently straight line in the log p' vs. log ε plot corresponding to terminal part of expansion tests lead to an overestimate of the ultimate cavity stress.

c. The fact that during an expansion test the maximum ε measured was not sufficient to induce critical state conditions at the cavity wall can be perceived from the fact that the slope of the terminal part of log p' vs. log ε plot is greater than 0.5 (1-K_{SV}).

Similar solutions have been obtained in the past by Ladanyi (1963) and Hughes et al. (1977). Ladanyi (1963) assumed the relationship between volumetric and shear strain idealized in Figure 4. Failure was assumed to occur at point B and the volumetric strain ε_{v} is obtained by trial and error, until a straight line is found when plotting log p' vs log (ΔV/V + ε_{v}). Hughes et al. (1977) assumed the stress–dilatancy theory is valid and the failure occurs under constant ratio of principal stresses. Their eq.(18) is almost identical to the eq.(9) proposed in this note.

REFERENCES


