Limit pressure of pressuremeter tests
La pression limite des essais pressiométriques

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SYNOPSIS: On the basis of the flow rule proposed by Rowe (1968) and a numerical solution for the interpretation of the Self-Boring Pressuremeter Tests (SBPT), a procedure to assess the limit pressure p_{lim} of cylindrical cavity in pure frictional material is presented. The results obtained using the proposed approach are validated using the results of the SBPT's.

1 INTRODUCTION

A large number of SBPT's have been performed in the calibration chamber (CC) under controlled boundary conditions (see Table 1). Using these results, a procedure to evaluate the ultimate stress of a cylindrical cavity is proposed. The method assumes that the Rowe's (1962) stress dilatancy theory applies and that the sand is failing at the condition of constant volume when the critical state is reached at the cavity wall. The instant at which this condition is reached is evaluated using a previously developed numerical method suggested by Manassero (1988). The analysis of this latter aspect enables a solution of general validity, to be obtained. The solution highlights how a reliable estimate of p_{lim} from SBPT can be made only provided that the expansion strains are large enough to reach the condition of critical state at the cavity wall ($\epsilon^{cv} \geq 15\%$).

Simplified procedures, previously reported in the literature, can, in this context, lead to a significant overestimate of p'_{lim}.

2 PROPOSED METHOD

Referring to a cylindrical co-ordinate system, the basic stress and strain definitions are:

σ'_r, σ'_θ : respectively radial and hoop principal effective stresses;
p' : radial effective stress at the cavity wall;
$\epsilon_r = -d\xi/dr$: radial principal strain;
$\epsilon_\theta = -\xi/r$: hoop principal strain;
ξ : radial displacement;
$r = r_o+\xi$: current radial distance;
r_o : radial distance before expansion;
$\epsilon = -\xi_R/R$: hoop principal strain at cavity wall;
$R = R_o+\xi_R$: current radius of expanding cavity;
R_o : cavity radius before expansion;
$\epsilon_v = \epsilon_r+\epsilon_\theta$: volumetric strain in plane strain conditions;
$\gamma = \epsilon_r-\epsilon_\theta$: shear strain.

The general equations of equilibrium and strain compatibility around the cylindrical cavity are:

$$\frac{d\sigma'_r}{dr} = \frac{\sigma'_\theta - \sigma'_r}{r} \quad \ldots (1)$$

$$\frac{d\epsilon_\theta}{dr} = \frac{\epsilon_r - \epsilon_\theta}{r} \quad \ldots (2)$$

The same equations (1) and (2) in terms of r/dr become:

$$\frac{\sigma'_\theta - \sigma'_r}{d\sigma'_r} = \frac{\epsilon_r - \epsilon_\theta}{d\epsilon_\theta} \quad \ldots (3)$$

When the critical state condition ($d\epsilon_v=0$) is reached:

$$d\epsilon_r = -d\epsilon_\theta \quad \ldots (4)$$

and upon integrating:

$$\epsilon_r = \epsilon_\theta + C_1 \quad \ldots (5)$$

and moreover:

$$\frac{\sigma'_\theta}{\sigma'_r} = K_a^{cv} = \frac{1 - \sin\phi_{cv}}{1 - \sin\phi_{cv}} \quad \ldots (6)$$

where:
ϕ_{cv} = constant volume friction angle.

Combining eqs.(4), (5) and (6) with eq.(3) and rearranging it follows:

$$\frac{d\sigma'_r}{\sigma'_r} = \left[\frac{K_a^{cv} - 1}{2}\right]\left[\frac{d\epsilon_\theta}{\frac{C_1}{2} - \epsilon_\theta}\right] \quad \ldots (7)$$

Solving eq.(7) one can write:

$$\left[\frac{2}{1 - K_a^{cv}}\right] \ln \sigma'_r = \ln\left[\frac{C_1}{2} - \epsilon_\theta\right] + C_2 \quad \ldots (8)$$

where from eq. (5):

$$C_1 = \epsilon_r + \epsilon_\theta = \epsilon_v^{cv} = \text{constant}$$

Table 1. Result of proposed method from SBPT's in calibration chamber.

Sand: Ticino Sand; ϕ'_{cv} = 34°

Test No.	D_R %	OCR –	σ'_{vo} kPa	σ'_{ho} kPa	K_o –	q_c kPa	ϵ^{cv} %	ϵ_v^{cv} %	γ^{cv} %	p'_{cv} kPa	p'_{lim} kPa
208	43.2	1.00	112.8	45.13	0.400	5491	−8.93*	−1.92*	15.93*	312*	770
209	49.2	1.00	116.7	51.99	0.441	6840	−8.93	−1.88	15.97	409	1008
210	53.3	1.00	511.1	244.27	0.479	17582	−7.93	−1.65*	14.22*	1403.1*	3680
211	67.4	1.00	512.1	242.31	0.473	25385	−8.93*	−3.65*	14.20*	1516.9*	3891
212	64.6	2.86	110.9	82.40	0.747	12262	−8.85	−2.01	15.69	797.5	1980
213	47.5	2.78	112.8	83.39	0.740	7883	−8.93*	−0.50*	17.41*	655.5*	1573
214	42.4	1.00	113.8	53.96	0.476	5788	−8.93*	−2.54*	15.31*	455.6*	1140
215	92.3	1.00	514.6	225.63	0.439	47457	−8.93*	−2.78*	15.07*	2096.4*	5272
216	46.3	7.57	60.8	56.90	0.927	5593	−8.93*	1.17	19.02	472.1	1100
218	65.4	7.66	59.8	59.84	0.980	10077	−9.92	0.64	20.47	640.5	1452
219	65.9	5.46	112.9	101.04	0.902	13815	−8.48	−0.92	16.03	958.0	2365
220	47.2	1.00	313.3	150.09	0.481	11472	−8.93*	−0.95*	16.90*	709.6*	1719
221	44.6	2.88	108.9	81.42	0.751	7199	−8.93*	−1.65*	16.20	604.6*	1485
222	46.2	5.50	111.8	95.16	0.850	8027	−9.92*	−0.73	19.10	757.2	1756
224	74.6	5.38	113.8	93.20	0.816	16822	−8.93*	2.17*	15.68*	1014.0*	2517
225	74.6	5.46	111.8	87.31	0.775	16342	−9.92*	−2.50*	17.33*	967.9	2316
228	77.0	1.00	518.0	215.82	0.417	31217	−9.92*	−2.81*	17.02*	1710.3*	4117
233	79.6	1.00	512.1	224.65	0.439	33915	−7.93*	−2.63*	13.24*	1606.8*	4234
234	76.1	5.34	115.8	103.90	0.904	18337	−7.93	−1.83	14.03	995.7	2573
235	48.5	1.00	516.0	239.36	0.465	15393	−4.96*	−0.64*	9.27*	982.8*	2954
236	75.2	2.72	114.8	78.48	0.686	15959	−6.94*	−2.13*	11.76*	748.4*	2060
237	74.6	2.90	96.1	81.42	0.850	15548	−6.68	−1.20	12.16	820.1	2234
238	74.8	2.83	101.0	83.39	0.828	15894	−6.97	0.96	14.89	761.6	1937
239	74.8	2.84	101.0	86.33	0.856	16118	−5.44	−0.24	10.64	752.0	2153
241	91.8	2.76	104.0	86.33	0.829	25293	−5.27	−0.63	9.90	964.8	2832
242	40.1	1.00	103.0	49.05	0.475	5168	−8.96*	−1.91*	16.00	240.8*	594
243	42.7	3.10	95.2	74.56	0.785	6484	−8.96	−0.38	17.53	214.6	513
244	42.8	6.12	97.1	94.18	0.970	7166	−8.96	−1.57	16.34	382.1	936
245	40.0	1.00	102.0	54.94	0.539	5390	−6.75	2.75	16.25	286.0	707
246	43.0	1.00	102.0	52.97	0.523	5746	−8.96*	−2.68*	15.23*	363.1*	910
247	43.0	4.19	190.3	147.15	0.776	9496	−8.96*	−4.14*	13.77*	799.2*	2071
250	43.0	1.00	480.7	219.74	0.457	12741	−8.96*	−2.16*	15.75*	1379.0*	3418
251	41.0	1.00	100.1	51.01	0.508	5355	−8.96	−1.10	16.81	189.1	459
252	75.0	1.00	101.0	52.97	0.518	13295	−7.34	−0.77	13.90	579.0	1505
253	71.0	1.00	103.0	52.97	0.517	12003	−3.98*	−1.14*	6.82*	488.5*	1637
254	71.0	6.16	97.1	88.29	0.912	14639	−2.99*	−1.24*	4.73*	503.1*	2077
255	65.0	1.00	108.9	55.92	0.514	10564	−7.96*	−3.01*	12.91*	643.0*	1709
257	87.0	1.00	130.5	77.50	0.597	22044	5.64	0.01	11.29	588.3	1649
258	86.0	1.00	495.4	226.61	0.458	40074	−8.73	−2.52	14.94	2099.3	5299
259	92.0	4.63	138.3	139.30	1.008	32138	−6.03	−0.99	11.07	1106.1	3118
260	89.0	1.00	131.5	78.48	0.595	23376	−8.96*	−4.70*	13.21*	956.8*	2514
261	91.5	3.99	199.1	157.94	0.797	35146	−8.96	−2.24	15.67	1881.3	4670
262	88.7	1.00	113.8	45.10	0.398	18155	−6.60	−2.00	11.20	605.3	1696
263	89.1	1.00	112.8	103.00	0.913	25602	−8.06	−2.15	13.96	1179.0	3051

D_R : relative density just before expansion;
OCR : overconsolidation ratio
σ'_{vo}, σ'_{ho}: vertical and horizontal effective stress at midheight of the specimen just before expansion;
K_o : coefficient of earth pressure at rest;
q_c : cone resistance from cone penetration tests;
(*) : values at the end of expansion phase for SBPT's where constant volume strain has not been reached

being:

ϵ_v^{cv} : attained total volumetric strain when constant volume behaviour of soil around the expanded cavity is reached.

Rewriting eq.(8) at the cavity wall in terms of measured parameters p′ and ϵ one gets:

$$\left[\frac{2}{1 - K_a^{cv}}\right] \ell n\ p' = \ell n \left[\frac{\epsilon_v^{cv}}{2} - \epsilon\right] + C_2 \quad \text{... (9)}$$

To find the value of the constant C_2 one can refer to the method of interpretation of SBPT's in sands proposed by Manassero (1988). This method incorporates the non linear nature of the stress–strain behaviour of sand, and assumes that Rowe's stress–dilatancy concept applies. Therefore:

$$\frac{\sigma'_r}{\sigma_\theta} = -\frac{1}{K_a^{cv}} \frac{d\epsilon_\theta}{d\epsilon_r} \quad \text{...(10)}$$

Combining eq.(10) with eq.(3) the following final solution is obtained (Manassero, 1988):

$$\frac{d\sigma'_r}{d\epsilon_\theta} = -\frac{\sigma'_r\,[1 - K_a^{cv}(d\epsilon_r/d\epsilon_\theta)]}{\epsilon_r - \epsilon_\theta} \qquad ...(11)$$

The above equation has been integrated using the finite differences technique under the assumption that:

- At the start of a SBPT $\epsilon_r=\epsilon_\theta=0$ and $\sigma_r=\sigma_\theta=\sigma_{ho}$ (σ_{ho}=initial in situ horizontal stress).
- The relation between σ'_r and ϵ_θ can be derived directly from the SBPT expansion curve as function of measured p' and ϵ (Manassero, 1988).

The above mentioned procedure allows the complete stress-strain and effective stress path of a sand to be computed from the results of a drained SBP expansion test knowing ϕ'_{cv}.

Figure 1 shows an example of the stress-strain, volumetric vs. shear strain relationships, and effective stress paths computed on the basis of a SBPT performed in CC in dense Ticino sand [Bellotti et al. (1988)].

Returning to the assessment of c_2, by substituting $\epsilon=\epsilon^{cv}$ and $p'=p_{cv}$ into eq.(9) one can find:

$$c_2 = \ell n \left\{ 2 \, {\frac{p'_{cv}}{\gamma^{cv}}}^{[2/(1-K_a^{cv})]} \right\} \qquad ...(12)$$

being:

$\gamma^{cv} = \epsilon_r^{cv} - \epsilon^{cv}$: cavity shear strain at the point where the sand reach the critical state ($\Delta\epsilon_v$=0) named constant volume point in Figure 1.

where:

ϵ^{cv}= hoop cavity strain	at point where constant volume behaviour is reached, see Figure 2
ϵ_r^{cv}= radial cavity strain	
p'_{cv}= cavity effective stress	

Now considering that when $\xi_R = \infty$:

$$\epsilon = -\frac{\xi_R}{R_o + \xi_R} = -1 \quad \text{and} \quad p' = p'_{lim},$$

one obtains the equation for the effective limit cavity stress:

$$p'_{lim} = p'_{cv} \left[\frac{\epsilon_v^{cv} + 2}{\gamma^{cv}}\right]^{(1-K_a^{cv})/2} \qquad ...(13)$$

An illustration of the proposed procedure to assess p'_{lim} from SBPT is shown in Figure 2.

Table 1 gives the results obtained after applying the above outlined procedure for the evaluation of p'_{lim} to the results of 45 SBPT's performed in the CC in Ticino sand (Bellotti et al., 1988). With the values of ϵ_v^{cv} reported in the Table 1 one can infer that the use of eq.(13) under the assumption that ϵ_v^{cv}=0 leads to an underestimate of p'_{lim} which does not exceed 8%.

Therefore, according to eq.(9) p'_{lim} can be evaluated following a simplified procedure consisting of plotting the experimental p' vs. ϵ data on a double logarithmic scale and then constructing a line with a slope of 0.5 $(1-K_a^{cv})$ from the last point until the intersection with the point corresponding to $-\epsilon$=100%. An example of application of this simplified procedure to an expansion curve obtained from CC tests in medium dense Ticino sand is shown in Figure 3. It is important to stress that this simplified approach assumes implicitly that the pressuremeter membrane has been expanded enough to reach the critical state condition at the cavity wall.

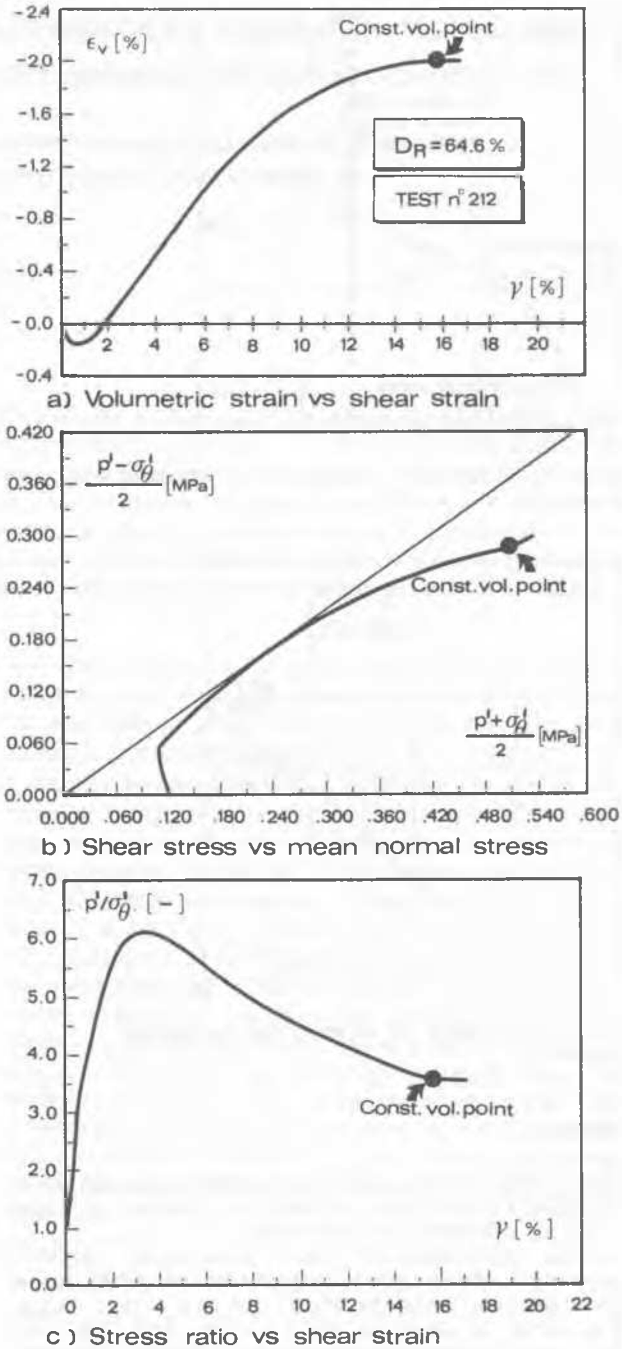

Figure 1. Strain and stress paths from SBPT (Manassero, 1988).

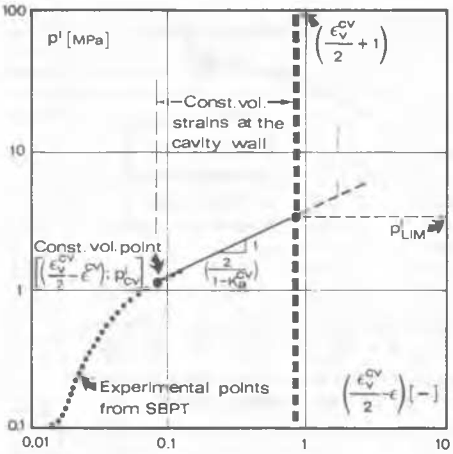

Figure 2. Graphical explanation of proposed method.

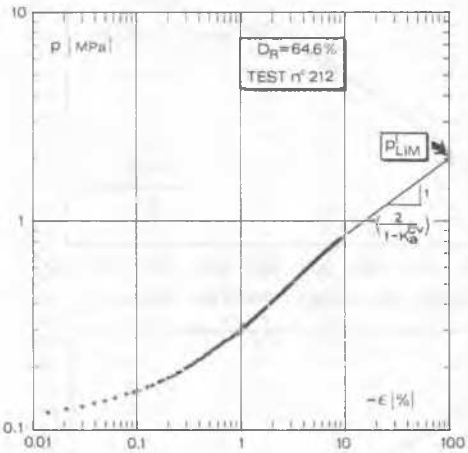

Figure 3. Example of simplified proposed procedure.

3 REMARKS

On the basis of the previous statements and with reference to the data given in Table 1, the following comments can be made:

a. The analysis of the available SBPT's following the complete procedure outlined above have indicated that in about 50% of the tests the maximum attained cavity strain (10%) was not sufficient to reach the critical state in the sand at the cavity wall. However, the ϵ_v observed at the end of these tests was already very small. This highlights the importance of using pressuremeter devices which allow higher expansion strains than that of the Camkometer probe used in this research if one wants to evaluate p'_{lim} in a reliable manner.

b. In the tests where the critical state condition at the cavity wall has not been reached, the assessment of p'_{lim} from the slope of the apparently straight line in the log p' vs. log ϵ plot corresponding to terminal part of expansion tests lead to an overestimate of the ultimate cavity stress.

c. The fact that during an expansion test the maximum ϵ measured was not sufficient to induce critical state conditions at the cavity wall can be perceived from the fact that the slope of the terminal part of log p' vs. log ϵ plot is greater than 0.5 $(1-K_a^{cv})$.

Similar solutions have been obtained in the past by Ladanyi (1963) and Hughes et al. (1977). Ladanyi (1963) assumed the relationship between volumetric and shear strain idealized in Figure 4. Failure was assumed to occur at point B and the volumetric strain v_f is obtained by trial and error, until a straight line is found when plotting log p' vs log $(\Delta V/V + v_f)$. Hughes et al. (1977) assumed the stress-dilatancy theory is valid and the failure occurs under constant ratio of principal stresses. Their eq.(18) is almost identical to the eq.(9) proposed in this note.

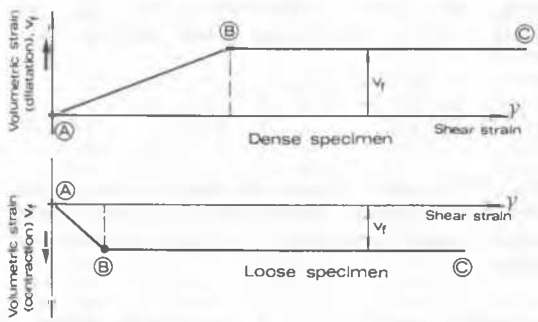

Figure 4. Idealization of shear behaviour of granular materials as proposed by Ladanyi (1963).

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