Stability of foundations under eccentric and inclined loads

Stabilité des fondations sous charges excentrées et inclinées

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SYNOPSIS: A new analytical method for evaluating the bearing capacity of shallow strip foundations is presented. The method is a generalization of Prandtl-Novotortsev solution in the case of non-zero eccentricity. The concept of limit eccentricities is introduced and respeetive generalized formulae for the effective footing width are presented. This method gives higher safety factors in gravity platform design as compared to conventional techniques.

1. INTRODUCTION

Gravity platforms are very prospective when applied in offshore activities due to the their high ice-resistance capacity. Since such platforms experience enormous lateral and moment loads (ice, wave and seismic) the analysis of their foundations as regards the bearing capacity is of primary importance.

Currently, there are many solutions for the problem of stability of the rigid-ideally plastic foundation under the centrally loaded strip footing subject to arbitrary inclination δ of the applied load, including exact solutions for weightless soil (Prandtl, 1920; Novotortsev, 1938). While for the loads, applied with eccentricity $e \neq 0$ the what is called "effective" footing width $b' = b - 2e$ is mainly used where b is actual width. But this technique proposed by Gersevanov (1948) provides too conservative assessments of the soil bearing capacity.

Variational approach to soil slope stability analysis gives the rupture line that separates the sheared massive from the immobile one in the form of a logarithmic spiral (Kopacsy, 1961). The stress distribution along it follows Kötter equation. The solution of variational problem of strip footing soil base bearing capacity features the same properties (Fedorovsky, 1985). In this solution, however, the load eccentricity is also taken into account according to the Gersevanov's proposal, moreover, the rupture line in the form of a logarithmic spiral segment leads to overestimation of the bearing capacity.

The paper suggests a new solution that is also based on Kötter equation but the rupture line consists of three segments of straight lines and logarithmic spirals same as in Prandtl solution.

2. THE SOLUTION TECHNIQUE

The problem of the bearing capacity of a strip footing OD (plate) soil base is considered (Fig. 1). The soil base is assumed to be rigid-plastic that follows Coulomb-Mohr yield condition with parameters c and φ. Soil specific gravity is γ.

The assumed rupture line ABCD (Fig.1) consists of three segments: straight line segment AB which is the boundary of Rankine maximum stress state zone ABO, logarithmic spiral segment BC with the centre at the point O and other logarithmic spiral segment CD with downward convexity (Fig.1a) or upward convexity (Fig.1b) or another straight line segment CD (Fig.1c). Smooth connection of all the segments is ensured.

The controlling parameters of the solution are two angles β_1 and β_2, that specify the spiral CD centre position. This number corresponds to the number of loading parameters (e and δ). If $\beta_1 > \beta_2$ then the centre O is allocated above the bottom of the plate OD (Fig.1a), if $\beta_1 < \beta_2$ then it is below the plate (Fig.1b), if $\beta_1 = \beta_2 = \beta$ then logarithmic spiral degrades into straigth line whose inclination is determined by the value β (Fig.1c). The latter case corresponds to the known exact Prandtl-Novotortsev solution for the case when $\gamma = 0$ and $e = 0$.

It is assumed that upthrust is one-sided in the direction of negative semi-axis x (Fig.1). Surcharge q can be easily taken into account by introducing increased cohesion $c' = c + q\tan\varphi$ and by adding the value q to vertical component of the specific ultimate load.

The calculation of stresses along the rupture line $y = y(x)$ can be performed with the help of Kötter equation

$$\frac{d\sigma_n}{dx} - 2\tau\frac{d\theta}{dx} + \gamma\,\frac{\cos\varphi\sin(\theta-\varphi)}{\cos\theta} = 0 \qquad (1)$$

where σ_n and $\tau = \sigma_n\tan\varphi + c$ are the normal and the shear stresses along the line respectively; $\theta = \arctan(dy/dx)$ - the rupture line (slip line) inclination angle. The equation (1) is integrated along segments with the allowance of the continuity of σ_n and boundary condition at point A $\sigma_n = c\cos\varphi$.

Components of the ultimate load P and x-coordinate of its application point are obtained by means of three equations of a rupture zone ABCDO equilibrium.

The stress field in the rupture zone can be constructed by introducing the slip line grid which complies with the shape of ABCD. In the zones BOC and COD it consists of radial straight

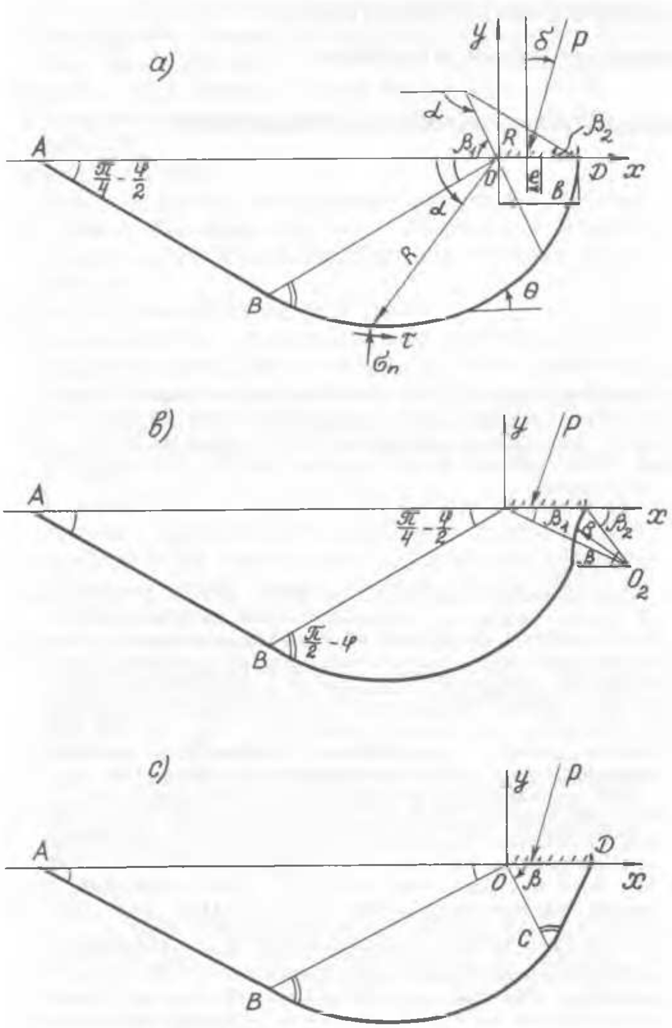

Figure 1. Failure mechanism for strip footing under eccentric and inclined load

lines and logarithmic spirals with centres at points O and $O_1\,(O_2)$, respectively. Now Kötter equation can be integrated along the segments of "active" slip lines, i.e. lines that are parallel to external boundaries of respective zones, and then σ_n and τ can be transformed into σ_x, σ_y and τ_{xy} with the help of Mohr circle. The stresses thus obtained can be represented as a sum of two components that are directly proportional to c and γ respectively. The first components everywhere comply with equilibrium conditions, while the second ones comply with them in Rankine zone ABO alone.

The displacement velocity field can be constructed together with the stress field. Velocity field in the first two variants (Fig.1a,b) is controlled by rotary movement of the plate and rigid "core" OCD around the velocity centre $O_1\,(O_2)$ and in the third variant (Fig.1c) - by translational movement of the plate and the core. In the zones BOC and AOB the velocity fields are constructed on the basis of continuity conditions in such a way that active slip line segments would move as rigid bodies (a rotation around the centre in BOC and a translational movement in AOB). The obtained velocity field is kinematically admissible from the viewpoint of the plasticity theory. The velocity discontinuity takes place along the rupture line ABCD alone. An angle of dilatation ν specified by the volume and shear strain rates ratio is equal to the angle of internal friction φ everywhere, that is equivalent to the associated rule of plastic flow.

Notably, the assumed pattern of slip lines constrains values of the controlling parameters β_1 and β_2. It is necessary that $0 \le \beta_1, \beta_2 < \varphi + \pi/2$ for the active slip lines starting from the plate bottom would end up on the free surface having passed through the three zones.

3. RESULTS OF ANALYSIS

Calculated values of vertical component V of the ultimate load P, applied to the footing, are usually represented in the form of coefficients $N_c\,(N_q)$ and N_γ of Terzaghi three-term formula while in the general case this linear formula is not correct and the author proposed additional corrective quadratic term (Fedorovsky, 1985). In the considered case it is difficult to arrange such representation since the tables of coefficients needs three inputs (φ, δ and $\bar{e} = e/b$). Therefore we shall confine ourselves by representing some small but characteristic portion of the results in a graphic form. These results correspond to one angle of internal friction ($\varphi = 30°$) and are represented in parametric form with the help of angles β_1 and β_2.

Fig. 2a shows level lines of δ and $\bar{e}$ and Fig.2b shows level lines of $\bar{V} = V/cb\,(= N_c)$ for the case of weightless soil $\gamma = 0$. In order to find $\bar{V}$ for the specified values of δ and $\bar{e}$, corresponding values of β_1 and β_2 should be determined on the graph of Fig.2a and then $\bar{V}$ could be found on the graph of Fig.2b. The symmetry with respect to diagonal $\beta_1 = \beta_2$ may be mentioned as the main feature of the solution. At this diagonal the solution corresponds to Prandtl-Novotortsev solution.

In the case of cohesionless soil $c = 0$ the picture is different (Fig.3). Here the symmetry with respect to the diagonal approximately remains for δ and $\bar{V} = 2V/\gamma b^2\,(= N_\gamma)$, while for $\bar{e}$ it completely disappears.

Very large values of V at Fig.2b and 3b for small angles β_1 and/or β_2 is the consequence of our initial assumption that the failure is one-sided (left-sided). These results correspond to negative values of δ and are valid for the case of an asymmetrical surcharge.

4. INFLUENCE OF LOAD ECCENTRICITY

The above results allow to estimate the load eccentricity influence on the bearing capacity of foundation. Usual assumption that the ultimate load onto the eccentrically loaded footing is equal to the ultimate load for centrally loaded footing with the effective width

$$b' = b - 2e \qquad (2)$$

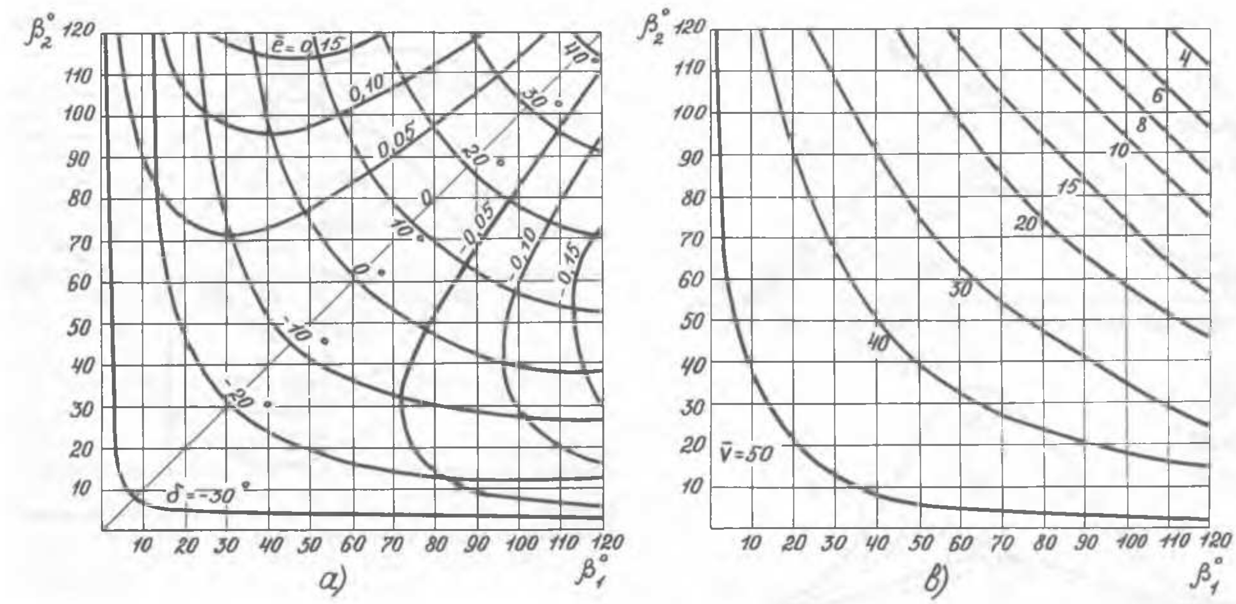

Figure 2. Level lines of (a) relative eccentricity $\bar{e}$ and inclination angle δ of ultimate load and vertical component $\bar{V}=V/cb$ of ultimate load ($\varphi=30°$, $\gamma=0$)

may be kinematically impossible and underestimates the bearing capacity. The latter has been proved by direct experiments where values of b and e were simultaneously changed in such a way that b' remained constant (Lavrov, Fedorovsky, 1988). This conclusion follows from the proposed solution, also.

Consider, for example, dependence $V(\bar{e})$ for constant b and δ (Fig.4). The graphs obtained with the help of equation (2) from the value of V for $\bar{e}=0$ are also shown for comparison. In the case of $\gamma=0$ and $\delta=0$ (Fig.4a) a divergence between both graphs increases with an absolute value of $\bar{e}$ and reaches approximately 20% at $|\bar{e}|=0.167$. For greater absolute values of the relative eccentricity $\bar{e}$ contact between footing and soil is lost at the side of the footing opposite to eccenticity. It is convenient to introduce a notion of limit eccentricities $\bar{e}_{min}$ and $\bar{e}_{max}$ such that for $\bar{e}_{min}\le\bar{e}\le\bar{e}_{max}$ there is not

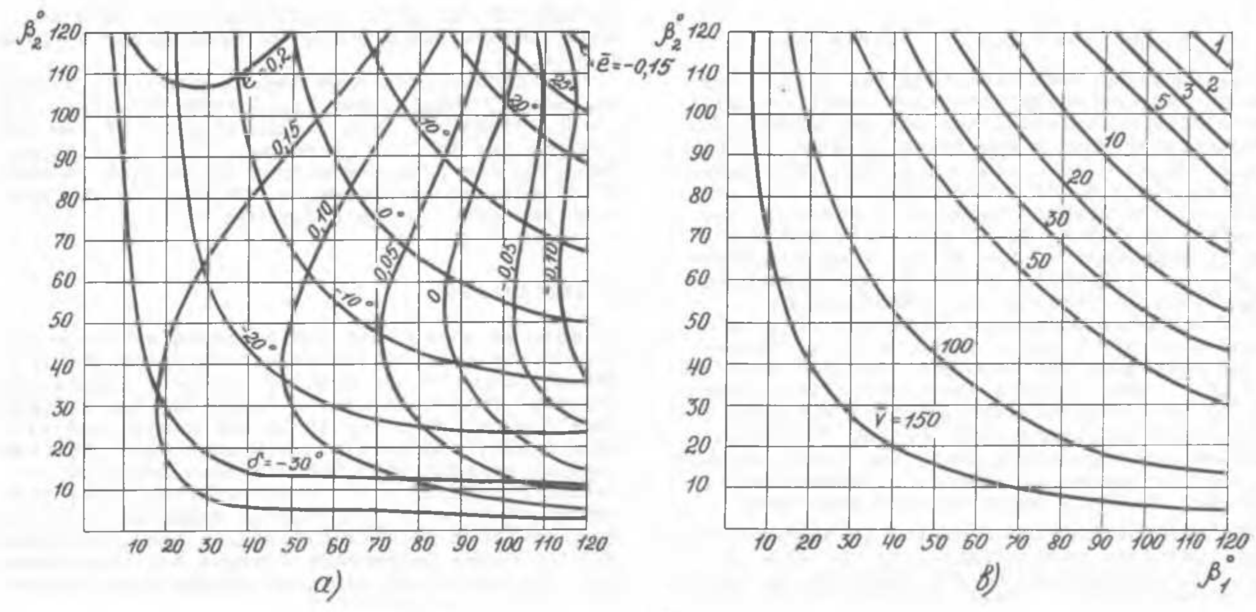

Figure 3. Level lines of (a) relative eccenticity $\bar{e}$ and inclination angle δ of ultimate load and (b) vertical component $\bar{V}=2V/\gamma b^2$ of ultimate load ($\varphi=30°$, $c=0$)

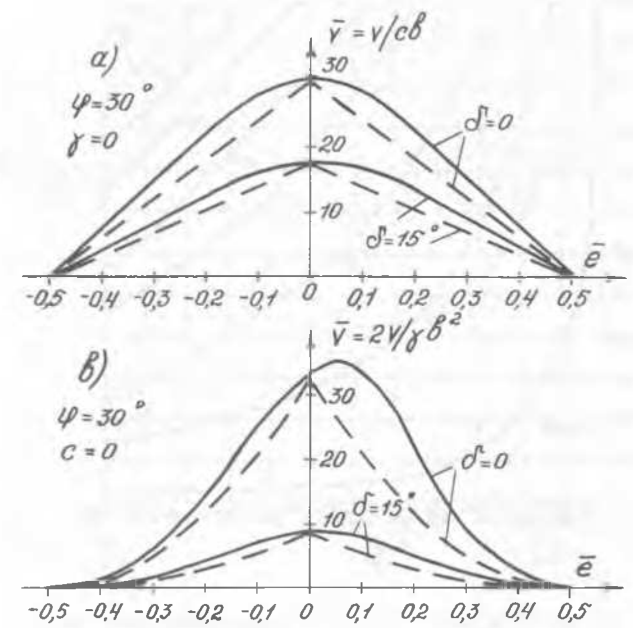

Figure 4. Ultimate vertical load versus eccentricity for different angles of load inclination. x - limit eccentricities; ———— - new method; — — — - formula (2)

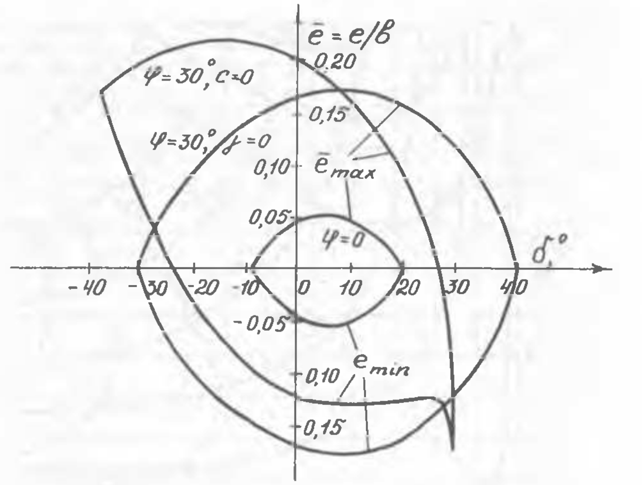

Figure 5. Maximum and minimum relative eccentricites versus load inclination angle

any contact loss while for $\bar{e} < \bar{e}_{min}$ and $\bar{e} > \bar{e}_{max}$ the loss of contact between the footing and the soil appears and the actual width of the footing reduces. It is easy to obtain the following generalization of Cersevanov formula (2)

$$b' = \begin{cases} (b+2e)/(1+2\bar{e}_{min}) & \text{if } \bar{e} < \bar{e}_{min} \\ (b-2e)/(1-2\bar{e}_{max}) & \text{if } \bar{e} > \bar{e}_{max} \end{cases} \tag{3}$$

These formulae have been obtained on the basis of an obvious assumption that when the limit eccentricity is exceeded the contact width reduces exaetly so that a new relative eccentricity would be equal to the limit one. Such calculated effective width b' is to be used in the caleulations of the bearing capacity instead of the real width b. Values of V thus obtained are smoothly dependent on the actual load eccentricity e. It is visible at Fig.4 and corresponds to experimental results and common sense.

Limit eccentricities are functions of φ and δ and also of $c/\gamma b$ - ratio (Fig.5). Therefore we must take into account the fact that the ratio of c and $\gamma b'$ would vary after limit eccentricity is exceeded. When applying Terzaghi formula to avoid iterations N_c and N_γ should be determined separately using the limit eccentricities for cases $\gamma = 0$ and $c = 0$, respectively.

Considering Fig.4 again we note that when $\gamma \neq 0$ the maximum bearing capacity is achieved for a small positive eccentricity. In the case of $\delta = 15°$ it corresponds to the experimental data (Lavrov, Fedorovsky, 1988) while for $\delta = 0$ this is explained by the assumption of one-sided upthrust.

5. APPLICATION TO GRAVITY PLATFORM DESIGN

On the basis of the above data we may recommend the following way for assessing the gravity platform foundation stability. In this case the vertical load V is practically constant and has eccentricity close to O and the horisontal load component H is applied at the permanent height h. The stability factor should be assumed to be equal to H_f/H, where H_f is the level of H that produces the failure of foundation the given V. To obtain H_f the curve of ultimate loads is plotted versus coordinates (V, H). For this purpose the set of total load inclination angles δ is taken within the range from O up to arctan $(b/2h)$. V_f and $H_f = V_f \tan\delta$ are obtained for any given δ and $e = h\tan\delta$ with account of the above considerations. Then we plot the necessary curve by joining the obtained points.

The calculations show that a gravity platform weight is usually many times less than V_f for $\delta = 0$. Therefore, the rising segment of the very segment the account of eccentricity with accordance to the proposed method gives high increase of the safety factor as compared to the calculation with the help formula (2).

6. CONCLUSIONS

It must be noted that the proposed solution is the direct generalization of Prandtl-Novotortsev solution in the case of non-zero load eccentricity. This solution is exact for the weightless foundation (i.e. it is statically and kinematically admissible simultaneously). For the case of weighty soil and non-zero angle of internal friction φ the solution gives the upper estimate of the bearing capacity. The concept of limit eccentricities has been introduced and the known Cersevanov formula for the effective footing width has been generalized respectively. The considerable increase of the design safety factor is achieved by means of the respective analysis.

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