A coulombian soil model applied to an offshore platform

Un modèle de sol de Coulomb appliqué aux plate-formes en mer

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SYNOPSIS: An elasto-plastic soil model based on effective stresses is developed and implemented in the finite element program: FENRIS. The paper describes the basic features of the model with its input-parameters and illustrates how it may be applied to analyze a deep skirted offshore platform founded on layered, clayey soils. In the example installation is modelled assuming undrained conditions. Installation is followed by four months of porepressure dissipation, before a quasi static wave load is applied.

1 INTRODUCTION

The work presented briefly herein is related to a research project denoted "Integrated analysis of gravity structures" run by The Foundation for Scientific and Industrial Research at the Norwegian Institute of Technology, SINTEF, and Det Norske Veritas in Oslo, Norway. The project aims at developing the non-linear, general purpose, finite element program "FENRIS" towards analyzing a gravity based offshore oil-platform, sustained to wave loading, in a manner where full integration between the structure and the supporting soil is included. The present paper addresses one out of the two effective stress based soil models implemented in the program. This soil model is called the Mobilized Friction Model (MFM) since its main controlling parameter is the degree of mobilized friction $f = \tan\rho/\tan\varphi$ as defined in Fig.1(a).

As part of the project the "FENRIS" program has been extended to include a coupled consolidation type analysis, and methods are implemented to handle a fully undrained situation simulated by zero volumetric strains. In this paper the basic principles of the soil model will first be sketched before a simplified gravity platform example is analyzed. The example includes undrained installation followed by a consolidation process before a quasi static wave load finally is applied.

A simpler version of the MFM model is previously presented at the BOSS-88 conference (Nordal and Bostrøm, 1988). The present paper gives a more complete description. However, the research project is not yet completed and some of the material presented is still under development.

2 MATHEMATICAL DESCRIPTION OF THE MODEL

The model aims at covering the following features of typical soil behaviour:

(a) The reduction in stiffness as we approach an effective stress defined failure surface.
(b) The increase in stiffness as we increase the effective mean stress
(c) The memory of a preconsolidation pressure
(d) The dilatancy or contractancy that may occur during pure shearing

To obtain these properties a two surface model is used combining a Coulomb-type criterion with an end-closing cap. The principal part of the formulation is related to the Coulomb criterion and an isotropic hardening process controlled by the degree of mobilized friction, f. A mean stress dependency is used for the elastic parameters K and G while preconsolidation and plastic volumetric strains are taken care of by a simple uncoupled cap formulation. Some more details are outlined in the following:

A Mohr Coulomb approximation is used to provide a continuous, smooth, conical yield surface:

$$F_{cone} = \sqrt{J_2} - \alpha_F(\sin\rho,\theta)\cdot(I_1 + 3a) = 0$$

$$I_1 = \sigma_1' + \sigma_2' + \sigma_3' \quad (=3\sigma_m')$$

$$J_2 = [(\sigma_1 - \sigma_3)^2 + (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2]/6$$

$$J_3 = \sigma_1'\sigma_2'\sigma_3'$$

$$\sin 3\theta = 3\sqrt{3}\, J_3/(2J_2^{3/2}) \qquad \text{Eq(1a-e)}$$

The formulation involves an extension of the Drucker-Prager Criterion as α_F is made a function of θ in addition to beeing controlled by the mobilized friction through sin ρ. The surface is sketched in Fig.1(b) and (c).

The secondary yield surface, the cap, is made up by a plane defined by:

$$F_{cap} = (I_1/3 + a) - \sigma_c' = 0 \qquad \text{(Eq.2)}$$

where the reference stress σ_c' is the isotropic preconsolidation pressure, p_c'.

In accordance with classical elasto-plastic theory the strains upon loading are assumed to be the sum of elastic and plastic contributions:

$$d\epsilon = d\epsilon^e + d\epsilon^p_{cone} + d\epsilon^p_{cap} \qquad \text{(Eq.3)}$$

The elastic strain increments, $d\epsilon^e$, are related to the stress increments, dσ, through the con-

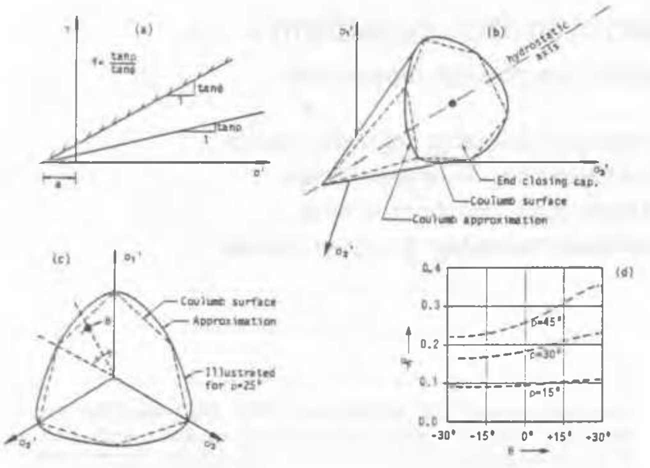

Figure 1. The yield criteria with: (a) definition of mobilized friction angle, ρ, (b) the cone and the plane cap in principal stress space, (c) the cone in the π-plane, (d) the $\alpha_F$coeff. of Eq.1a as a function of Θ and ρ.

ventional elastic constitutive matrix, $\mathbf{D}$: $d\epsilon^e = \mathbf{D}^{-1} d\sigma$. An isotropic formulation is adopted with a bulk modulus, K, and a shear modulus, G. Both K and G are made effective mean stress dependent according to:

$$K = k_0 \sigma_a \left(\frac{\sigma_c' + a}{\sigma_a}\right)^{1-n} \quad \text{(Eq.4a)}$$

$$G = g_0 \sigma_a \left(\frac{\sigma_c' + a}{\sigma_a}\right)^{1-n} \quad \text{(Eq.4b)}$$

where: σ_a = 1 atm. = 1 bar = 100 kN/m²
k_0, g_0: nondimensional moduli numbers
n = stress dependency exponent
σ_c' = mean reference stress level

By using σ_c' from the cap criterion the elastic moduli will increase for increasing reference stress (for clays: p_c') while the moduli never will be allowed to decrease again. This removes the risk of violating the thermodynamic requirements generally associated with stress dependent elastic parameters.

Two types of plastic strains are considered in the present model. The Coulomb-cone is related to plastic shear strains which grow as we approach Coulomb failure, while the cap mainly includes the plastic compressive strains observed for instance in an oedometer. The plastic deviatoric "Coulomb" strains are believed to be controlled by the degree of mobilized friction as indicated in Fig.2. The curve in Fig.2(b) is to be considered as a plastic hardening rule. For increasing sinρ the cone opens up isotropically around its hydrostatic axis, and the process is uniquely tied to the growth of plastic deviatoric "Coulomb" strains.

The "Coulomb" strains, $d\epsilon^p_{cone}$, may provide volumetric contributions through a dilatancy or contractancy mechanism, here defined as volumetric changes caused by pure shear.

A non-associated flow rule is utilized to control the dilatancy. A potential surface, Q_{cone}, having a π-plane cross section identical to the cross section of the current Coulombian yield surface is used. However, the potential surface

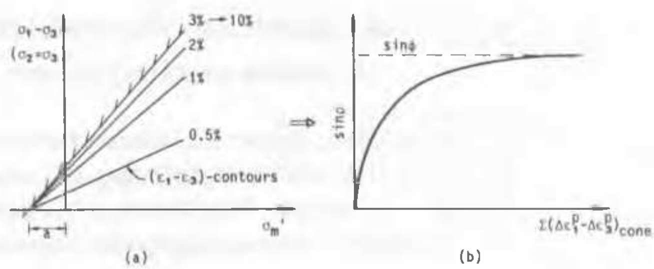

Figure 2. (a) Experimental findings indicate that the $(\epsilon_1 - \epsilon_3)$-contours are approximately straight lines. (b) Based on (a) we define a unique relationship between the mobilized friction and the plastic shear strains related to the Coulomb cone.

tilts in the direction of the hydrostatic axis (Fig.3a) according to an input controlled parameter α_Q:

$$d\epsilon^p_{cone} = d\lambda_{cone} \frac{\partial Q_{cone}}{\partial \sigma} = d\lambda_{cone}(\mathbf{n}_\pi + \alpha_Q \mathbf{n}_h)$$

$$\mathbf{n}_\pi = \frac{\partial F_{cone}}{\partial \mathbf{s}} / \left| \frac{\partial F_{cone}}{\partial \mathbf{s}} \right|$$

$$\mathbf{n}_h = (1/\sqrt{3})[1,1,1,0,0,0,] \quad \text{(Eq.5a-c)}$$

Above, $d\lambda_{cone}$ is a scalar multiplier (cfr.Eq.10) and $\mathbf{s}$ denotes the deviatoric stresses.

Since $|d\epsilon^{pd}_{cone}| = d\lambda_{cone}$ and $|d\epsilon^{pv}_{cone}| = \alpha_Q d\lambda_{cone}$ we observe that α_Q expresses the ratio between the volumetric and the deviatoric strain increments associated with the cone.

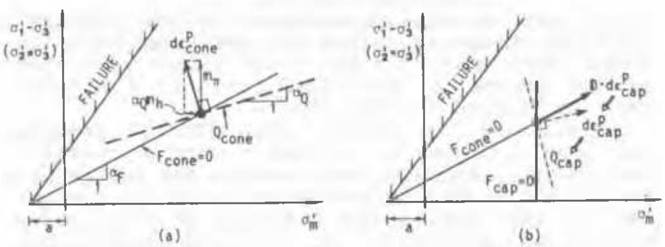

Figure 3. The plastic strain increments related to the cone (a), and the cap (b).

The α_Q parameter is a dilatancy parameter which primarely controls the direction of an effective stress path in undrained loading. The α_Q parameter is found to be dependent on the deviatoric plastic strain level expressed by $\Sigma|d\epsilon^{pd}_{cone}|$ and the effective mean stress level. Fig. 4 indicates one possible pattern for the dependency of α_Q and how it relates to the direction of an undrained effective stress path for a clay. The cap surface will not be active for this kind of loading.

The plastic strains related to the cap are also defined by a non-associated flow rule. The flow rule is determined from a requirement of keeping the cone and the cap mechanisms as uncoupled as possible. This is done according to the basic equations of the elasto-plastic theory, where:

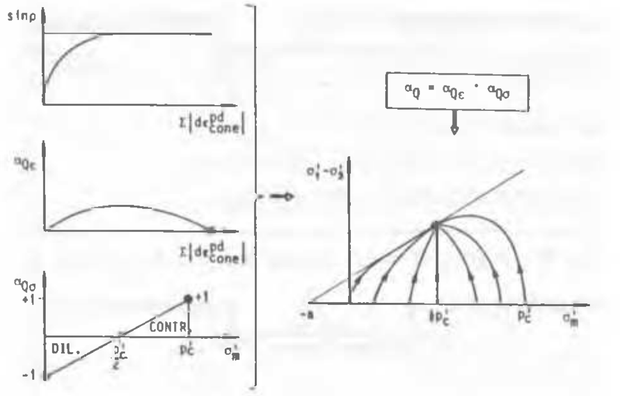

Figure 4. The dilatancy parameter α_Q controls the directions of the stress paths during undrained loading.

$$d\epsilon = d\epsilon^{e} + d\epsilon^{p}_{cone} + d\epsilon^{p}_{cap} \qquad \text{(Eq.6)}$$

$$= D^{-1} d\sigma + d\lambda_{cone}\frac{\partial Q_{cone}}{\partial \sigma} + d\lambda_{cap}\frac{\partial Q_{cap}}{\partial \sigma}$$

$$\left(\frac{\partial F_{cone}}{\partial \sigma}\right)^{T} d\sigma + \frac{\partial F_{cone}}{\partial \lambda_{cone}} d\lambda_{cone} = 0 \qquad \text{(Eq.7)}$$

$$\left(\frac{\partial F_{cap}}{\partial \sigma}\right)^{T} d\sigma + \frac{\partial F_{cap}}{\partial \lambda_{cap}} d\lambda_{cap} = 0 \qquad \text{(Eq.8)}$$

if we introduce the condition:

$$\left\{\frac{\partial F_{cone}}{\partial \sigma}\right\}^{T} D \left\{\frac{\partial Q_{cap}}{\partial \sigma}\right\} = 0 \qquad \text{(Eq.9)}$$

this leads to:

$$d\lambda_{cone} = \frac{\left\{\frac{\partial F_{cone}}{\partial \sigma}\right\}^{T} D d\epsilon}{\left\{\frac{\partial F_{cone}}{\partial \sigma}\right\}^{T} D \left\{\frac{\partial Q_{cone}}{\partial \sigma}\right\} - \frac{\partial F_{cone}}{\partial \lambda_{cone}}} \qquad \text{(Eq.10)}$$

$$d\lambda_{cap} = \frac{\left\{\frac{\partial F_{cap}}{\partial \sigma}\right\}^{T} D \left\{d\epsilon - d\lambda_{cone}\frac{\partial Q_{cone}}{\partial \sigma}\right\}}{\left\{\frac{\partial F_{cap}}{\partial \sigma}\right\}^{T} D \left\{\frac{\partial Q_{cap}}{\partial \sigma}\right\} - \frac{\partial F_{cap}}{\partial \lambda_{cap}}} \qquad \text{(Eq.11)}$$

The plastic cap-potential gradient is determined according to this requirement (Eq.9) which is illustrated in Fig. 3b.

As a result of this choise the plastic strains related to the cone-surface may be evaluated without worrying about the cap. Thereafter the plastic cap-strains may be easily evaluated once the cone-contribution is determined. Numerically this is performed in the FEM-stress routine which receives a $\Delta\epsilon$ increment and returns a corresponding $\Delta\sigma$ according to:

$$\Delta\sigma = D\Delta\epsilon - \Delta\lambda_{cone} D \frac{\partial Q_{cone}}{\partial \sigma} - \Delta\lambda_{cap} D \frac{\partial Q_{cap}}{\partial \sigma} \qquad \text{(Eq.12)}$$

3 INPUT PARAMETERS

Eqs. 13 and 14 defines the plastic stiffnesses. The flow rule may be used to relate these parameters to measured values:

$$\frac{\partial F_{cone}}{\partial \lambda_{cone}} = \frac{\partial \alpha_F}{\partial \sin\rho}\,\frac{\partial \sin\rho}{d\lambda_{cone}} \underset{\text{TRIAX SHEAR}}{=} -\frac{\sqrt{3}}{2}\,\frac{\partial \alpha_F}{\partial \sin\rho}\left(\frac{d\sin\rho}{d\epsilon^{p}_{1} - d\epsilon^{p}_{3}}\right)_{cone} \qquad \text{(Eq.13)}$$

$$\frac{\partial F_{cap}}{\partial \lambda_{cap}} = -\frac{\partial \sigma'_c}{\partial \lambda_{cap}} \underset{\text{ISOTR. COMPR.}}{=} -\sqrt{3}\left(\frac{d\sigma'_c}{d\epsilon^{pv}_{cap}}\right) \qquad \text{(Eq.14)}$$

The input parameters for the model is then the elastic parameter k_0, g_0, n and initial preconsolidation σ'_c. The plastic cap stiffness $(\partial F_{cap}/\partial\lambda_{cap})$ is defined through a curve between permanent volumetric strains and σ'_c. Finally we need the plastic cone stiffness $(\partial F_{cone}/\partial\lambda_{cone})$ defined through Eq.13 and Fig.2, together with the dilatancy parameter α_Q from Fig.4. All model parameters may be determined from conventional laboratory tests.

4 THE NUMERICAL EXAMPLE

A rigid platform including rigid soil plugs within skirt compartments down to -22 m below mudline is considered founded on a layered site with clayey soils. Due to limited capacity on our in-house computer, a rough FE-model is used as indicated in Fig.5. The soil parameters used is summarized in Figs.5 and 6. Platform installation is modelled by increasing the submerged platform weight from zero to its full value assuming undrained conditions. This leads to excess porepressures as indicated in Fig.7(a). Four months of porepressure-dissipation is thereafter modelled, using the consolidation option of FENRIS, which leads to a reduction in excess porepressures (and the degree of shear mobilization), Fig.7(b). A reasonable effective stress state is now established before the quasi static wave load causing Q_h and M (Fig.5) is applied, leading to the results of Figs.7(c) and 8(a) at design level. The wave loads are increased further towards failure and a load-deflection curve for the wave load action is depicted in Fig.9. The curve indicates that a considerable margin against turnover-failure is present.

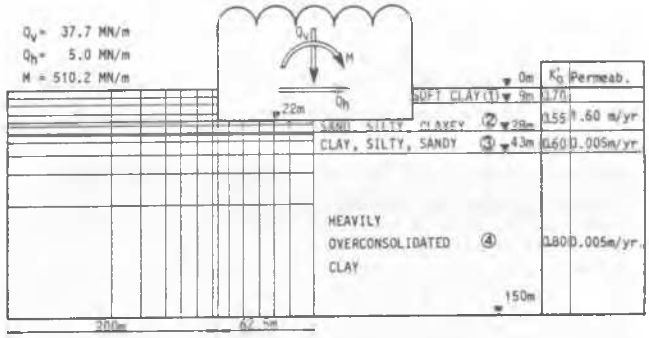

Figure 5. The finite element mesh, soil profile and loads.

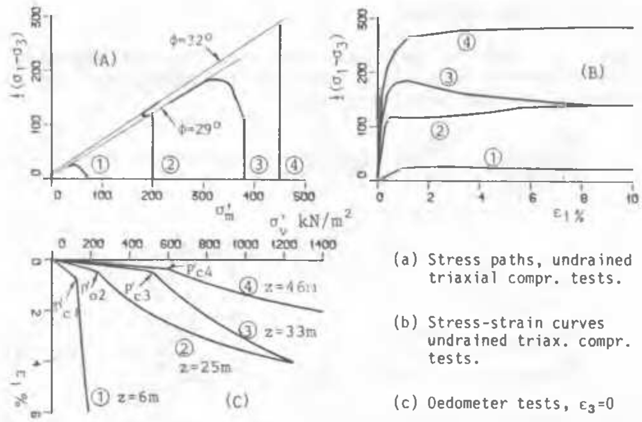

Figure 6. The soil parameters are determined from conventional triaxial and oedometer test results as reproduced above for the four layers of interest.

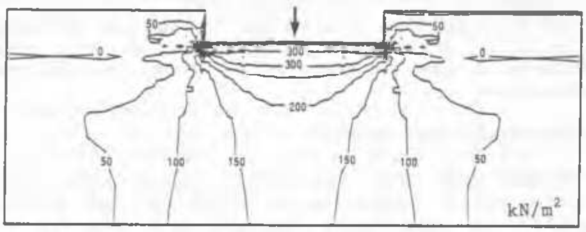

Figure 7a. Excess pore pressure from installation

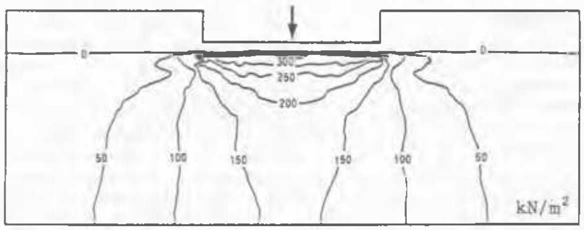

Figure 7b. Excess pore pressure after consolidation

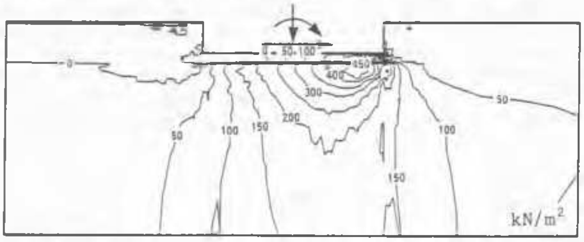

Figure 7c. Excess pore pressure for 1.3 Q_h and 1.3 M

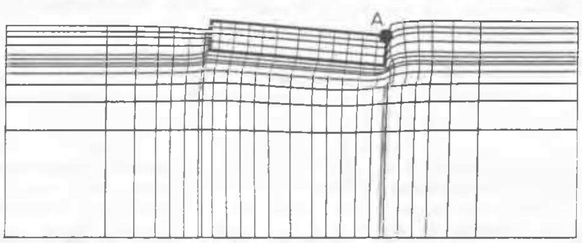

Figure 7d. Deformed mesh for 3.0 Q_h and 3.0 M

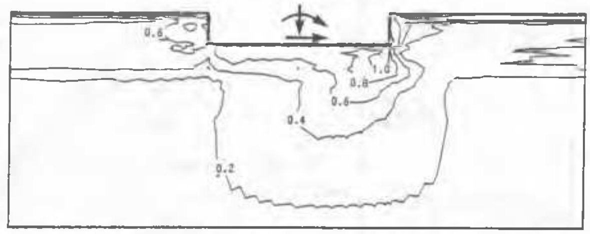

Figure 8a. Degree of mobilization for 1.3 Q_h and 1.3 M

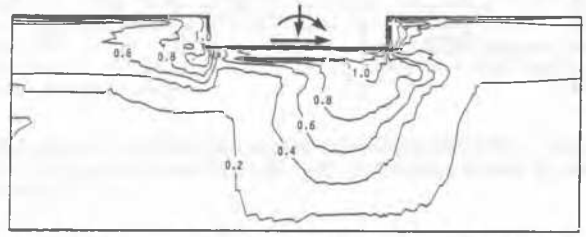

Figure 8b. Degree of mobilization for 3.0 Q_h and 3.0 M

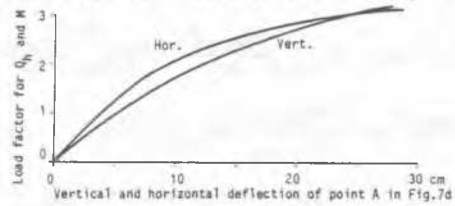

Figure 9. Load deflection curves for wave loading.

5 CONCLUSIONS

The soil model sketched couples two plastic mechanisms; one controlled by the degree of shear mobilization and the other controlled by a preconsolidation stress, σ'_c. The model is effective stress based and may be used both in undrained conditions and during consolidation by means of the computer program "FENRIS". The work is still under development but indicates that a lot of interesting aspects may be studied by means of such a tool. Comparison to classical geotechnical stability calculations are of particular interest since the degree of shear mobilization is a key parameter in both the FEM-model and the classical methods used at our University, (Janbu, 1985).

6 ACKNOWLEDGEMENT

This work is part of a research and development project paid for by the Norwegian oil companies Statoil and Hydro. Soil profile data for the numerical example is kindly provided by Statoil. The authors wish to express their gratitude to both companies.

7 REFERENCES

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