Limit analysis for cracked and layered soils

L'analyse limite des sols fissurés et stratifiés

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SYNOPSIS: The method of limit analysis is applied to a problem of a rigid foundation on cracked and layered soil. The kinematically admissible failure mechanism is constructed and upper bound to limit load is specified. The predicted values of limit load agree fairly well with available experimental data.

1 INTRODUCTION

The problem of evaluating limit load for a foundation resting on cracked or layered soil is of foundamental importance for engineering design. However, despite early solutions of this problem for a homogeneous soil (Prandt 1920; Hill 1950), the effective and occurate methods of solution for cracked and layered soils are still not available. Some attempts to use static (lower-bound) approach to this problem were presented by Rychlewski (1966), Stroganow (1971) and Giroud (1971). In view of difficulties associated with generating analytical solutions, some empirical formulas are used for the assessment of limit load.

In this paper, the kinematic approach will be presented and the proper failure mechanisms will be constructed. The associated values of the limit load will be obtained by equating the internal rate of dissipation to the rate of external work.

2 KINEMATIC APROACH TO LIMIT ANALYSIS

The theorems of limit analysis are based on the assumption of a rigid-plastic material model and the associated flow rule. The upper bound theorem based on a kinematically admissible failure mechanism is then valid (see Chen, 1975; Izbicki and Mróz, 1976) and will be applied in this paper.
Within plastic domain, the yield condition

$$f(\sigma_{ij})=0 \qquad (1)$$

and the associated flow rule

$$\dot{\varepsilon}_{ij}=\dot{\lambda}\frac{\partial f}{\partial \sigma_{ij}}, \qquad \dot{\lambda}>0 \qquad (2)$$

occur at each material point undergoing plastic flow. Here σ_{ij} and $\dot{\varepsilon}_{ij}$ are stress and strain rate tensors, $\dot{\lambda}>0$ is a plastic multiplier specified in terms of the strain rate. In some cases, the failure mechanism is constructed of rigid domains sliding over each other along the velocity discontinuity surfaces. The velocity discontinuities along these surfaces in normal and tangential directions are denoted by $[V_n]$ and $[V_t]$. Assuming the contact limit condition along the discontinuity surface in a form

$$f(\sigma_n,\tau_n)=|\tau_n|-\sigma_n \mathrm{tg}\varphi+c \qquad (3)$$

Where σ_n and τ_n are the normal and tangential contact stress on the discontinuity surface, whereas c and φ denote cohesion and the friction angle. The associated flow rule (2) then provides

$$[V_n]=-\dot{\lambda}\frac{\partial f}{\partial \sigma_n}=\dot{\lambda}\mathrm{tg}\varphi, \quad [V_t]=\dot{\lambda}\frac{\partial f}{\partial \tau_n}=\dot{\lambda} \qquad (4)$$

and the velocity discontinuity vector $[V]$ is now inclined at the angle φ to the discontinuity line. The kinematically admissible failure mechanism composed of rigid blocks should satisfy the condition of positive internal dissipation rate and the external work rate. The specific dissipation rate along the discontinuity line per unit length is provided by the formula

$$D^s=c\,[V]\cos\varphi \qquad (5)$$

and the total dissipation rate should be integrated along all discontinuity lines. The limit load multiplier λ_K is obtained from the equality of external work and internal dissipation rates

$$\lambda_K\int_s P_i V_{P_i}\,ds+\int_v G_i V_{G_i}\,dv=D^t=\sum_i D_i^s l_i \qquad (6)$$

where P_i are external loads specified to within a multiplier λ_K, G_i are the body forces, V_{P_i}, V_{G_i} are the velocity components along P_i and G_i, and D_i^s, l_i are the specific dissipation rates and lengths of particular discontinuity lines.

3 LIMIT ANALYSIS OF A CRACKED SOIL

It is quite common to encounter a cracked soil or rock for which a parallel set of densely distributed cracks exist. This set imposes a anisotropic structure on soil with specific weak orientation along which the preferral slip may occur. Let us concentrate on a rigid footing problem and present a proper modification of classical solution.

3.1 Model of a cracked soil

The soil is defined by its specifie weight γ, cohesion c and angle of internal friction φ. Assume that the set of parallel craks is inclined at the angle $\varkappa$ to the horizontal axis. Fig. 1a, and the material parameters along this direction are c^* and φ^*, where $c^* < c$ and $\varphi^* < \varphi$.

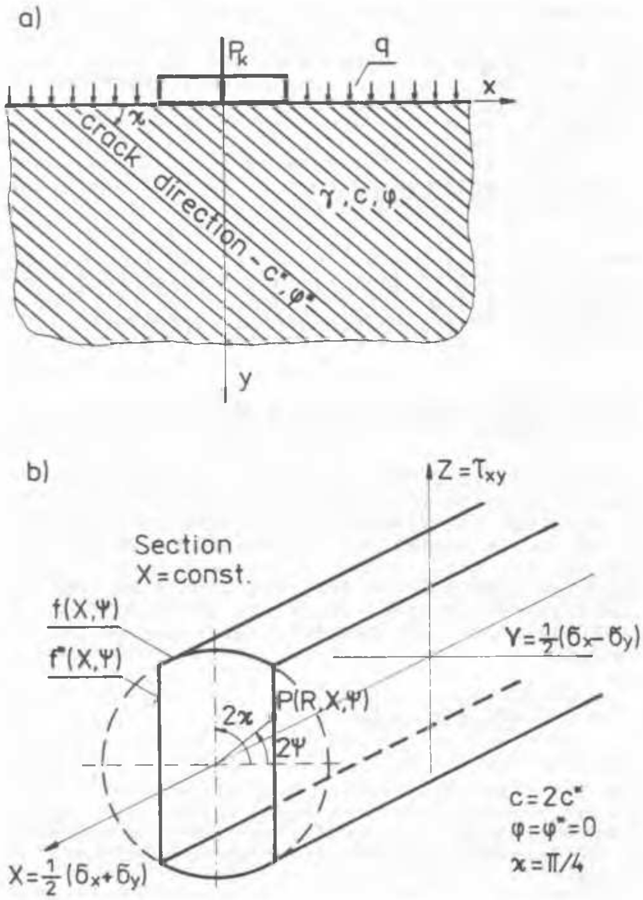

Figure 1. Strip footing on cracked subsoil - (a). Yield criterion in X,Y,Z - space for cracked material of Tresca type - (b).

This type of material can be described by the limit condition which in the stress space

$$X=\frac{1}{2}(\sigma_x+\sigma_y),\quad Y=\frac{1}{2}(\sigma_x-\sigma_y),\quad Z=\tau_{xy} \qquad (7)$$

is represented by a surface each point of which belongs to a set

$$A=\{R,X,\psi : R_{min}[f(X,\psi),\ f^*(X,\psi)]\} \qquad (8)$$

where

$$R=\frac{1}{2}[(\sigma_x-\sigma_y)^2+4\tau_{xy}]^{\frac{1}{2}},\qquad tg2\psi=\frac{2\tau_{xy}}{\sigma_x-\sigma_y}$$
$$f=X\sin\varphi+c\cos\varphi,\qquad f^*=\frac{\mp X\sin\varphi^*\mp c^*\cos\varphi^*}{\sin(2\varkappa+\varphi^*-2\psi)} \qquad (9)$$

The condition (8) is on intersection of two contact limit conditions specified by $f^*=0$ and $f=0$, where

$$f^*=|\tau_n|-\sigma_n tg\varphi^*-c^*=0,\qquad f=|\tau_n|-\sigma_n tg\varphi-c=0 \qquad (10)$$

and condition $f^*=0$ is satisfied along the crack direction, wheras $f=0$ is satisfied along the Coulomb slip directions. Such conditions are discussed in detail by Florkiewicz (1986). In Fig. 1b, this condition is pressented in the space X,Y,Z for a cracked material of Tresca type.

3.2 Rigid strip on a cracked soil

Let us now consider the kinematic approach to the limit analysis of a rigid, rough footing of width B on a cracked soil. The footing is loaded vertically by the vertical force P_k, and the distributed loading q acts on soil on both sides of the footing. In view of anisotropic propierties of soil, the classical type of Prandtl mechanism was assumed - Fig. 2.

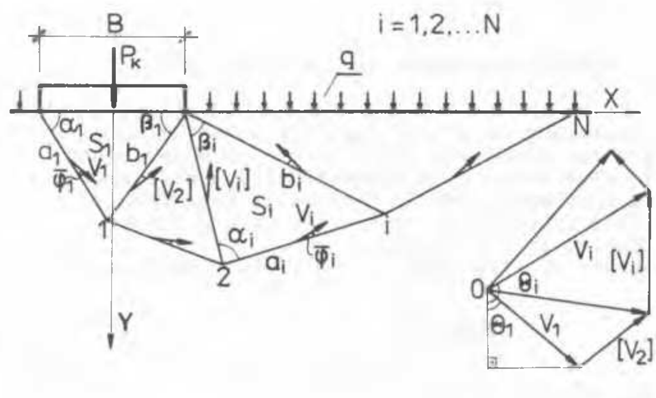

Figure 2. Failure mechanism of soil below footing and hodograph.

The value of P_k is obtained from the expression following from the equality of internal dissipation and work rates:

$$P_k = \frac{1}{V_1 \cos\theta_1} [\sum_1^N \bar{c} a_i V_i \cos\bar{\varphi}_i + \sum_1^{N-1} \bar{c}_i b_i [V] \cos\bar{\varphi}_i + \quad (11)$$
$$- q b_N V_N \cos\theta_N - \gamma \sum_1^N S_i V_i \cos\theta_i]$$

Where S_i denotes the areas of N rigid triangular blocks limited by the sides a_i and b_i and moving with the velocities V_i. Here $[V_i^i]$ are the velocity discontinuities between the rigid blocks. The vectors V_i and $[V_i]$ are inclined at anglee $\bar{\varphi}_i$ to the discontinuity lines. The material parameters are

$$\bar{c}_i, \bar{\varphi}_i \begin{cases} c^*, \varphi^* - \text{if the discontinuity lines coincide with the crack direction,} \\ c, \varphi \;\; - \text{if the discontinuity lines are oriented differently from crack direction.} \end{cases}$$

The expression for P_k is now a function of a set of parameters a_i, b_i, so that $\alpha_i = \alpha_i(a_i, b_i)$ $\beta_i = \beta_i(a_i, b_i)$. Numerical solution was obtained assuming $\beta_i = \pi/N$ and determining the final values of α_i through the optimalization procedure. Applying the gradient method, the values of P_k were minimized for all values of α_i and $\alpha_i^* = \varkappa - \pi + \Sigma\alpha_{i-1}$ specyfying the stripes of rigid blocks of Fig. 3.

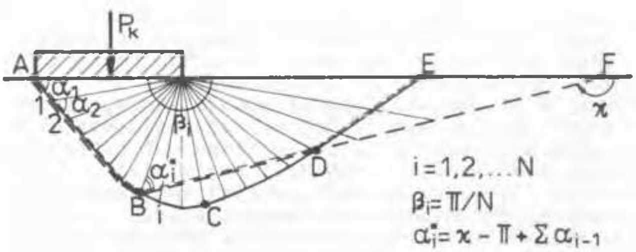

Figure 3. Explanation of a optimalization procedure used for limit load computations.

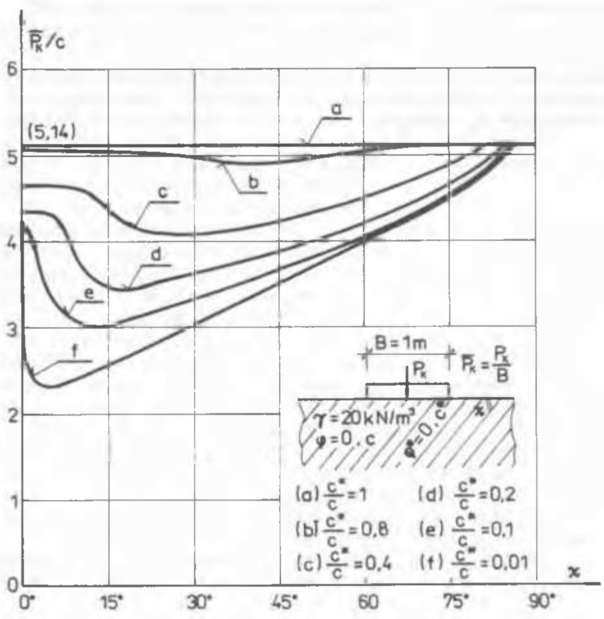

Figure 4. Relation between a limit load and crack orientation $\varkappa$.

The failure mechanism so specified is shown in Fig. 3 where the bounding discontinuity line ABDE coincides partly with the crack orientation. For other crack orientations the external discontinuity line is ABDE.

The limit load variation with the angle $\varkappa$ - crack orientation - is shown in Fig. 4. It was assumed that $B = 1{,}0$ m, $c^*/c = 0$, $\varphi = \varphi = 0$. Other examples are discussed by Florkiewicz (1988).

4 LIMIT ANALYSIS FOR A LAYERED SOIL

Now we consider the case when the soil parameters may change discontinuously in consecutive layers, whose orientation may be arbitrary, Fig. 5.

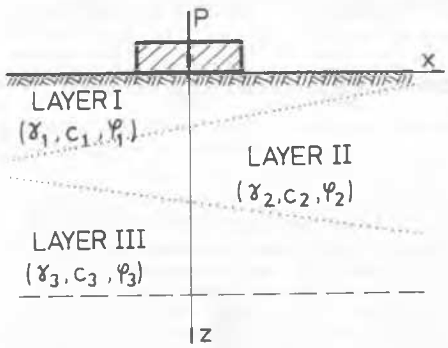

Figure 5. Strip footing on layered subsoil.

For the case of a two-layer continuum the assumed failure mechanism is that shown in Fig. 6.

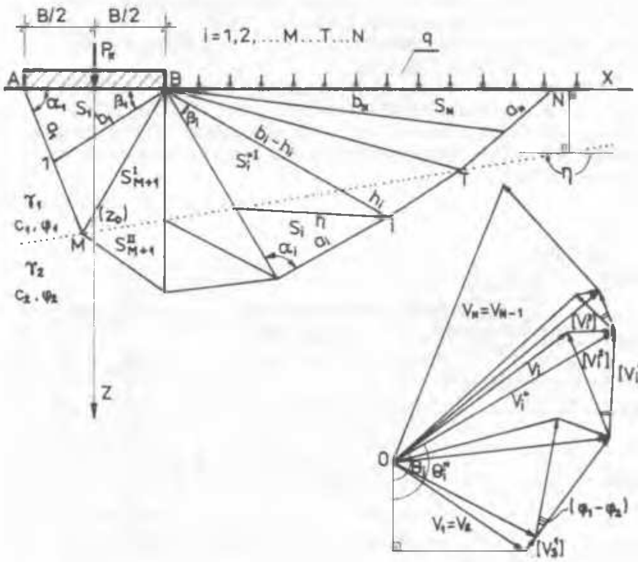

Figure 6. General failure mechanism for layered subsoil and hodograph.

The parameters z_0 and η specify the position of the straight interface line between the two layers.

The upper bound on the limit load is specified from equation

$$P_k = \frac{1}{V_i \cos\theta_i} \{c_1 \cos\varphi_i [\sum_1^M a_i V_i + \sum_{T+1}^N a_i V_i + \sum_1^M b_i [V_{i+1}] + \sum_{M+1}^{T-1} (b_i - h_i)[V_{i+1}^1]] + c_2 \cos\varphi_2 (\sum_{M+1}^{I} a_i V_i + \sum_{M+1}^{T-1} h_i [V_{i+2}^2] + \sum_{M+2}^{I} r_i [V_i^3]) - q b_N V_N \cos\theta_N - \gamma_1 (\sum_1^M S_i V_i \cos\theta_I + S_{M+1}^I V_{M+1} \cos\theta_{M+1} + \sum_{M+2}^{I} V_i^* S_i^{*I} \cos\theta^* + \sum_{T+1}^{N} S_i V_i \cos\theta_i) - \gamma_2 (S_{M+1}^{II} V_{M+1} \cos\theta_{M+1} + \sum_{M+2}^{I} S_i V_i \cos\theta_i + \sum_{M+2}^{I} S_i^{*II} V_i^* \cos\theta_i^*)\} \quad (12)$$

Where a_i, b_i, $h_i(a_i, b_i, z_0, \eta)$, $r_i(a_i, b_i, z_0, \eta)$ are the geometric parameters specifying the partition of soil onto rigid domains moving with the velocities V_i and V_i^* with respect to fixed soil mass lying outside of the slip line AMTN. The slip vectors V_i and V_i^* are inclined at the angles θ_i and θ_i^* to the - axis. The velocity discontinuities of V_i and V_i^* between the blocks S_i and S_i^* are denoted by $[V_i^1]$ along the segments b_i or $b_i - h_i$, by $[V_i^2]$ along the segments and by $[V_i^3]$ along the segments r_i, of Fig. 6. Let us note that because of discontinuity of along the interface line, secondary slip segments r_i occur, whose directions may coincide with the interface line in some domains , of Fig. 5.
The velocity hodograph in Fig. 6 corresponds to the case when $\varphi_1 > \varphi_2$.

The present analysis was applied in particular to a cohesionless soil ($c_1 = c_2 = 0$) with horizontal interface line ($\eta = 0$), and the results compared with the experimental data repored by Hanna (1981).

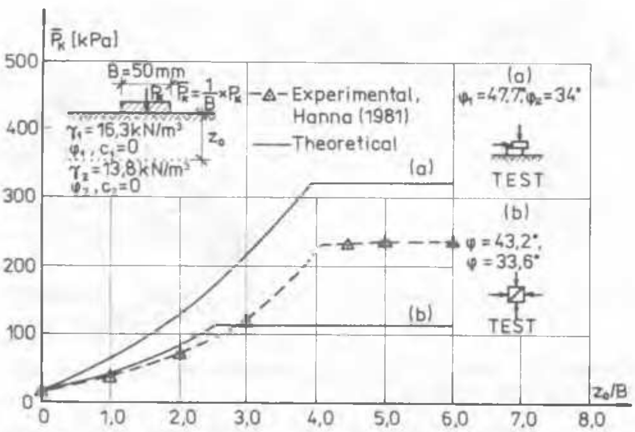

Figure 7. Comparision of theoretical results with test data.

It should be noted that the material parameters specified in that work were tests and were different for particular tests. In Fig. 7 the theoretical predictions were presented for two values of friction angles in both layers and compared with the experimental results.

5 CONCLUDING REMARKS

The presented kinematic approach, complemented with optimization techniques, provides an effective tool in handling limited analysis problems for both cracked and layered soils. The method can be applied to a wide variety of geometrical configurations and loading conditions.

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