Restrained rebound of saturated clay
Gonflement restreint d'une argile saturée

A.MUSSO, Faculty of Engineering, University of Palermo, Italy

SYNOPSIS: The swell pressure growth under complete inundation of a saturated clay specimen is theoretically investigated. Different degress of restraint imposed by a stress measuring device are considered. The mathematical model, built up under usual Terzaghi's consolidation hypotheses, results in a linear integro-differential equation for pore pressures, which has been solved through a perturbation method. The solution yields pore pressure and strain isochrones, besides the swell pressure-time function, for various values of a restraint factor, which encompasses geometrical and mechanical properties of the clay specimen as well as deformability of the measuring device.

1. INTRODUCTION

The process of pore pressure equalization taking place upon inundation in clays which are restrained from rebounding, as in the case of swelling pressure measurement, still deserves considerable attention.
As far as the author knows, this specific problem has not been faced in the past, except in a paper by Baker and Kassiff (1967), who determined theoretically the function expressing the course of swelling pressure with time for a partly saturated clay.
However, those Authors did not explicitly impose the constancy of overall volume of the specimen during the process, introducing the rather arbitrary assumption that the effective stress is a constant. Under this hypothesis, pore pressure dissipation during a swelling pressure test appears identical with that occurring in usual consolidation.
The phenomenon is mainly governed by the slow rate of water absorption, which, in turn, is controlled by interparticle and intra-particle forces in the mineral constituents. The latter forces, on the other hand, are affected by several factors; among them the composition and structure of clay, the nature of pore fluid and of dissolved ions, and, in general, all those factors pertaining to physico-chemical characteristics of the sedimentary environment, are to be cited.
These complicated interrelationships somehow obscure the role of each of above mentioned components; therefore, their resultant effect is usually taken into account by introducing the so called suction in the balance between pore water pressures and stresses acting on the solid skeleton.
The restrained swelling process in a saturated clay specimen, due to dissipation of suction upon inundation is examined in the present paper.
The initial suction is induced on the clay by a state of capillary pressure, uniformly distributed on the surfaces of the specimen (Terzaghi, 1943). As a result of inundation the clay tends to expand, but its volume must be compatible with the deformation of a cell employed to record the total stress arising on account of the restrained condition.
A mathematical model, allowing the derivation of functions for both swelling pressure growth and pore pressure dissipation, is proposed. It is shown that, owing to compatibility condition imposed on the volume of specimen, the dissipation of initial suction takes place through a peculiar moving boundary process involving rebound of part of the specimen and contemporary compression of the remaining part. The location of the boundaries between these zones depends on the values of a stiffness parameter, accounting for geometric and elastic properties of the clay specimen as well as deformability of the swell pressure measuring device.
Possible implications of theoretical results on the interpretation of usual swelling tests are briefly discussed.

2. MATHEMATICAL MODEL

A saturated clay specimen, whose thickness is 2H, subjected to a capillary pressure σ_c, is placed within a consolidometer between two porous plates. The top plate is connected to a total stress cell, whose deformability, accounting for the compressibility of pipes, valves and transducer diaphragms, can be expressed in the form $\alpha = \Delta s/\Delta P$, being Δs the shortening of the measuring system under the force ΔP transmitted by the specimen.
As long as the pore water is in a state of equilibrium, the neutral

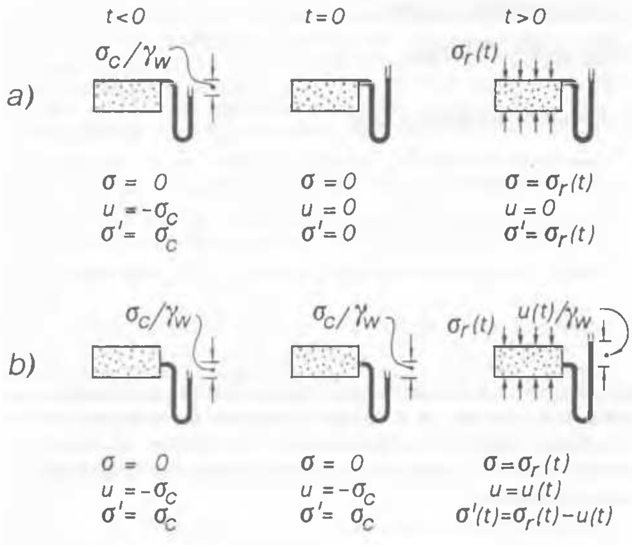

Fig. 1 - State of stress at different instants in a sample of clay initially subjected to capillary pressure and inundated at $t = 0$. a) at the boundaries; b) at midheight.

pressure is everywhere negative and equal to $-\sigma_c$. Moreover, neglecting the bulk weight of the specimen, the total stress is zero and the effective stress equals the value of capillary pressure.

As soon as the clay is flooded, the hydrostatic head at the surface suddenly increases to zero against the value $-\sigma_c/\gamma_w$ in the interior of the specimen.The corresponding hydraulic gradient would cause an infiltration of water from the ambience reservoir; as a result, rebound would tend to occur; however, the clay will actually expand just as allowed by the deformability of the measuring system.

The latter, in turn, will react on the specimen, by applying a force equal to $\Delta P = \Delta s/\alpha = A\sigma_r(t)$, where A represents the cross-sectional area of the specimen and $\sigma_r(t)$ denotes the average value of the total stress exerted on and recorded by the measuring system.

In fig. 1, the relationships among total, neutral and effective stresses on the boundary and at mid-height of the specimen, at different stages, are synthetically represented; $t = 0$ labels the instant in which the specimen is flooded. Furthermore, the stress increments at a point of the specimen can be obtained as in Table I.

Assuming the z axis upward oriented, perpendicularly to the specimen's cross section, compatibility between the deformations of the specimen and of the measuring system is satisfied by the following condition:

$$\int_0^{2H} \varepsilon_z dz = -\alpha A \sigma_r \qquad (1)$$

where ε_z, the oedometric strain, is expressed by:

	General	Associated with fig.1
$\Delta\sigma(t)$	$\sigma(t)-\sigma(0)$	$\sigma_r(t)$
$\Delta u(z,t)$	$u(z,t)-u(z,0)$	$u(z,t)+\sigma_c$
$\Delta\sigma'(z,t)$	$\sigma'(z,t)-\sigma'(z,0)$	$\sigma_r(t)-u(z,t)-\sigma_c$

Tab. I - Expressions for calculating stress increments.

$$\varepsilon_z = m_v \Delta\sigma' = m_v[\sigma_r(t) - u(z,t) - \sigma_c] \qquad (2)$$

In eq. 2, $\Delta\sigma'$ represents the increment of effective stress and m_v denotes the coefficient of volume change.

Note that m_v takes in compression a different value than in rebound; therefore, the location of the boundaries between the compressive strained zones and the expanded ones should be specified in performing the integral indicated in equation (1).

Considering, however, that the whole allowed rebound of the specimen will be small when sufficient restraint is imposed, one might reasonably assume that neither compressive nor expansive displacements will be high enough to cause appreciable variations in m_v (Ullrich, 1975).

Moreover, during rebound, effective stresses will never overcome the initial capillary stress, so that the clay specimen can be considered overconsolidated. In this situation, coincidence of unloading and reloading paths can be assumed.

The latter consideration,along with the previous one, allows to consider m_v as a constant with respect to both time and space.

This enables the integral in equation (1) to be calculated, thus eliminating the difficulties arising from a moving boundary problem.

The following expression for $\sigma_r(t)$ is easily found:

$$\sigma_r(t) = \lambda\sigma_c + \frac{\lambda}{2H}\int_0^{2H} u(z,t)\,dz \qquad (3)$$

in which the "stiffness factor" $\lambda = 2Hm_v/(\alpha A + 2Hm_v)$ is introduced. The latter depends on properties both of the specimen and of the measuring system and varies between zero ($\alpha \to \infty$) and unity ($\alpha = 0$).

The case $\alpha = 0$ depicts the situation in which no swelling is allowed; in this occurrence, $\sigma_r(t)$ attains its maximum value; on the other hand, for $\alpha \to \infty$, the measuring system imposes no restraint to the free swell of the specimen, so that $\sigma_r(t)$ is identically zero.

In order to find the unknown functions $\sigma_r(t)$, and $u(z,t)$, the continuity condition for the saturated clay must be added to equation (3):

$$\frac{\partial v_z}{\partial z} = \frac{\partial \varepsilon_z}{\partial t} \quad (4)$$

where v_z denotes the seepage velocity.
Assuming that the flow of water is exclusively due to the existence of a hydraulic gradient with a purely mechanical origin, as expressed by Darcy's law, and recalling equation (2), one obtains:

$$-k\frac{\partial^2 h}{\partial z^2} = m_v\left(\frac{d\sigma_r}{dt} - \frac{\partial u}{\partial t}\right) \quad (5)$$

in which $h = z + u/\gamma_w$ is the total head and k is the permeability coefficient.
Substituting (3) in (5) yields, finally:

$$c_v\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} - \frac{\lambda}{2H}\frac{\partial}{\partial t}\int_0^{2H} u\,dz \quad (6)$$

where $c_v = k/\gamma_w m_v$ is the consolidation coefficient.
Equation (6) is a linear integro-differential equation, which reduces to the classical Terzaghi's one when $\lambda = 0$. It must be solved according to the following initial and boundary conditions:

$$u(z, 0) = -\sigma_c \qquad 0 \leq z \leq 2H \qquad (7a)$$
$$u(0, t) = 0 \qquad t > 0 \qquad (7b)$$
$$u(2H, t) = 0 \qquad t > 0 \qquad (7c)$$

3. STRESSES AND STRAINS EVOLUTION

It is convenient to introduce the following non-dimensional parameters:

$$Z = z/H; \qquad T = c_v t/H^2; \qquad U = -u/\sigma_c \qquad (8)$$

so that dimensionless equations are rewritten in the form:

$$S = \frac{\sigma_r}{\sigma_c} = \lambda\left(1 - \int_0^1 U dZ\right) \quad (3')$$

$$\frac{\partial^2 U}{\partial Z^2} = \frac{\partial U}{\partial T} - \lambda\frac{\partial}{\partial T}\int_0^1 U dZ \quad (6')$$

$$U(Z,0) = 1 \qquad 0 \leq Z \leq 2 \qquad (7a')$$
$$U(0,T) = 0 \qquad T > 0 \qquad (7b')$$
$$U(2,T) = 0 \qquad T > 0 \qquad (7c')$$

The solution of eq.(6') under conditions (7') has been obtained by means of a pertubation method in terms of a power series of the λ factor. However, due to space limitations, all mathematical details along with a thorough numerical investigation of the influence of the stiffness parameter values on accuracy of the solution are not reported. Hereafter, some results concerning the re-distribution of stresses and strains during the dissipation of suction and the development of swell pressure with time are discussed.
Fig. 2 shows the isochrones of dimensionless pore water pressure U and strain $E = \varepsilon_z/m_v\sigma_c = [S + U - 1]$, for some values of the λ factor ranging from zero to unity.
For $\lambda = 0$, no restraint is imposed to rebound; as expected, strains are always expansive throughout the specimen; negative neutral pressures dissipate according to Terzaghi's equation of consolidation.
Pore pressure isochrones progressively deviate from those, predicted by the simple diffusion theory, and the deviation becomes more and more important, as λ increases. In the latter case, the slope of pore pressure isochrone curves does not increase monothonically approaching the drainage surfaces. On the contrary, it actually takes opposite signs along different parts of the same curve, provided the λ factor assumes sufficiently high values.
It is worth noting, in this respect, that the slope of an isochrone is not directly proportional to the hydraulic gradient, since the curves do not represent pore pressure excesses over the initial stationary values; nevertheless, the described circumstance can be taken as an indication of different directions of seepage within the specimen. As a matter of fact, strain isochrones clearly indicate the contemporary existence of expanded and compressed zones.
Moreover, it should be noticed that for $0 < \lambda < 1$, in the early phases of the process, the inner zone of the specimen compresses while the outer ones dilate; with time, expansive strains propagate from the outer to the inner zone until the whole specimen swells uniformly to the height allowed by the compliance of the measuring device.
On the other hand, for $\lambda = 1$, expansion toward the interior of the specimen progresses at a smaller rate than in former cases; in the meantime, the outer zones, which were expanding at the early phase, start compressing, in such a way as to maintain the specimen's volume unvaried throughout the process.
The influence of the stiffness factor is felt also on the growth of swell pressure (fig. 3). In fact, the higher is the value of λ, the shorter is the time factor at which the maximum swell pressure develops; furthermore,the difference between the maximum value of $S = \sigma_r/\sigma_c$ and its asympthotic value, equal to λ, increases with increasing λ. When rebound is completely prevented, the ultimate swell pressure is equal to suction, as pointed out by previous researchers (Warkentin, 1962; Baker & Kassiff, 1968).

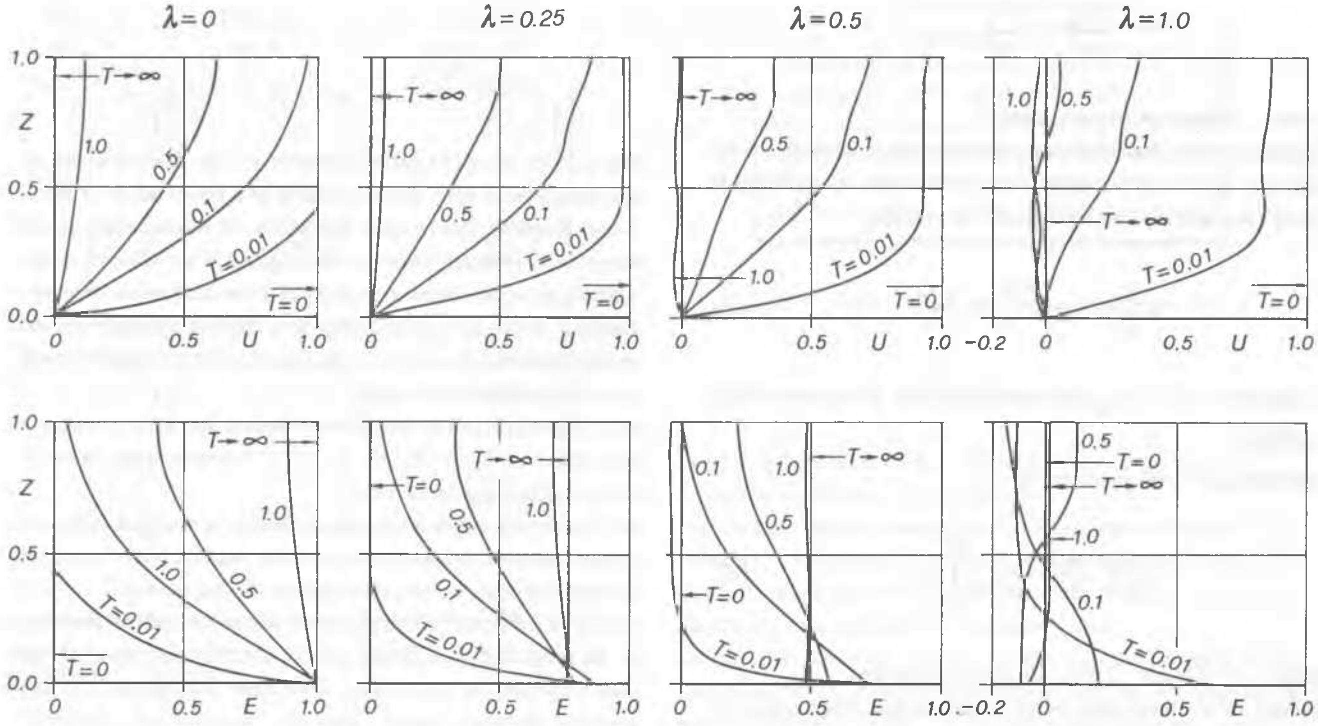

Fig. 2 - Isochrones of pore pressures ($U = -u/\sigma_c$) and strains ($E = \varepsilon_z/m_v\sigma_c$). Extensive strains are reported as positive.

As regards the influence of characteristics (m_v, H) of the clay specimen on results of a swell test, observe first that these parameters play a significant role only if α is different from zero; otherwise, λ always equals unity and the swell pressure converges to the initial suction value. In the former case, λ increases for increasing H and m_v.

Conversely, one can ask whether a high value of λ - leading to a better experimental determination of σ_r - could be obtained even if the clay material is rather stiff (m_v low) or the specimen is too thin (H small). Values of H_{min} required to ensure an admissible λ factor (λ_{adm}), for assigned values of the ratio α/m_v are reported in Tab. II.

The ratios α/m_v express the relative deformabilities of the measuring device compared to those of the clay specimen.

It is easily recognized that, assuming for α a value appropriate to a usual load ring (2.33×10^{-4} cm/Kg), very thick samples are needed if sufficiently high values of λ_{adm} are required in the measurement of the swell pressure of a stiff clay ($H_{min} = 75.6$ cm for $\lambda_{adm} \geq 0.95$; $H_{min} = 394$ cm for $\lambda_{adm} \geq 0.99$; $m_v = 0.33 \times 10^{-2}$ cm^2/Kg). On the other hand, even relatively thin samples ($H_{min} = 1.2$ cm for $\lambda_{adm} \geq 0.95$; $H_{min} = 6.5$ cm for $\lambda_{adm} \geq 0.99$) may be used, provided the clay to be tested is sufficiently soft ($m_v = 0.2$ cm^2/Kg).

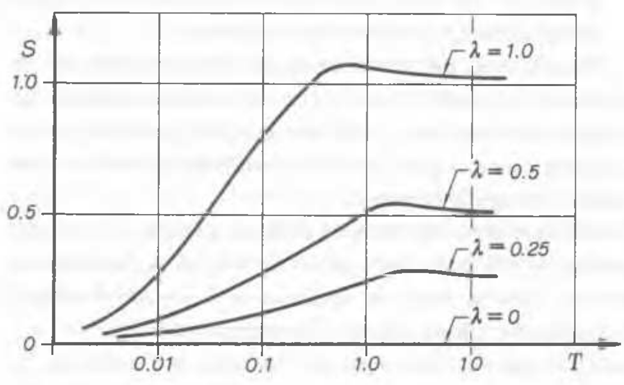

Fig. 3 - Normalized swelling pressure $S = \sigma_r/\sigma_c$ versus time factor $T = c_v t/H^2$.

Tab. II - Values of H_{min} for different values of λ_{adm} and α/m_v.

	α/m_v $\times 10^2$ (cm^{-1})	7.06	2.33	0.23	0.12
H_{min} (cm)	$\lambda_{adm} \geq 0.95$	75.6	24.9	2.5	1.2
	$\lambda_{adm} \geq 0.99$	394	130	13	6.5

4. SUMMARY AND CONCLUSIONS

The rate of dissipation of suction upon flooding in a restrained saturated clay specimen, initially subjected to capillary pressure, has been theoretically investigated.
The mathematical model describes the control of volume change, which takes place as a result of inundation, according to the compliance of a total stress measuring device.
The process of dissipation of negative pore water pressures is governed by a *linear integro-differential equation*, which has been solved by means of a perturbation method; the analytical expression governing the evolution of the swelling pressure has been obtained.
Evolution of both swelling and pore water pressures has been found to depend on a "stiffness parameter" λ, encompassing geometric and elastic properties of the clay specimen as well as deformability of the load cell employed to record the swelling pressure. The above factor may range between zero and 1.
For $\lambda = 0$, no restraint is imposed to the free swell of the clay, i. e., the swelling pressure is identically zero throughout the whole process of dissipation of the initial suction. Conversely, for $\lambda \neq 0$ the swelling pressure increases up to a maximum value; afterwards, it decreases, finally converging to the value of λ factor. Moreover, some parts of the specimen compress while remaining parts swell so as to respect the compatibility between deformations of the specimen and of the measuring system.
Corrispondently, pore pressures increase from the initial negative up to positive values; then they decrease down to zero.
As regards the role played by suction dissipation in the mechanics of restrained swelling, attention is drawn on the formal agreement between the formula for σ_r expressed by (3), after having set $\lambda = 1$, and a well known relationship proposed by Croney et al.(1958), provided that the capillary pressure represents suction measured under zero load, and that the values of pore pressures averaged over the whole height of the specimen are substituted to local values.
Finally, the analysis of the influence of the stiffness parameter discloses that for stiff clays very stiff devices are needed in order to accomplish an accurate measurement of swelling pressure without recourse to unusually thick laboratory specimens.

REFERENCES

Baker R., Kassiff G. (1968) - Mathematical analysis of swell pressure with time for partly saturated clays - Can. Geot. J., vol. V, n. 4, 217 - 224.

Croney D., Coleman J. D., Black W. P. M. (1958) - Movement and distribution of water in soil in relation to highways design and performance -Highway Research Board, Special Report 40, 226 - 252.

Terzaghi K. (1943) - Theoretical Soil Mechanics - Wiley, N.Y..

Ullrich C. R. (1975) - An experimental study of the time-rate of swelling. Ph. D. Thesis, University of Illinois, Urbana - Champaign.

Warkentin B. P. (1962) - Water retention and swelling pressure of clay soils - Cdn. J. Soil Science, vol. 42, n.1, 189.