A study of sand migration in gravel

Une étude de la migration des sables dans les graviers

D.K.ATMATZIDIS, Professor of Civil Engineering, University of Patras, Greece

SYNOPSIS: Improvements and modifications were introduced to a method which is based on probability theory and is used to describe the process of sand migration in gravel. The validity and limitations of the theory were investigated experimentally. In general, good agreement was obtained between predicted and observed migration patterns. An explanation is offered for the detrimental effects of sand migration into openwork gravel in the alluvial foundation soils of a large dam and it is shown that these effects can be predicted and anticipated using the available theory.

1 INTRODUCTION

The nature and function of a dam inevitably associate it with alluvial soils, which sometimes contain layers or lenses of openwork gravel. High hydraulic gradients at the upstream margins of these layers may cause migration of large amounts of the surrounding soil into the gravel, formation of cavities in the foundation, and significant settlement of the ground surface. A theoretical approach was developed and used to analyse the process of sand migration in a gravel deposit. A size distribution of the voids in the gravel is obtained and is compared to the grain size distribution of the surrounding soil to yield the probability of a grain encountering a void of a particular size; finally, the probable migration distances of invading grains are determined. Theoretical predictions are checked against results of laboratory experiments. The case of a large earth and rock fill dam which rests on alluvial soils containing openwork gravel, sand, and sand chocked cobble gravel is investigated. Migration distances are computed theoretically and are compared with the results of exploratory grouting with grout containing alluvial sand. The predicted sand migration process offers an explanation for the development of a large number of sinkholes in the upstream impermeable blanket of the dam which appeared after filling of the reservoir.

2 THEORETICAL CONSIDERATIONS

The effectiveness of a granular material as a filter depends on numerous factors, most importantly on the geometric characteristics, including particle sizes and shapes of the filter and base materials, the velocity and direction of seepage, and the time dependency of the filter clogging mechanism. Void sizes have been studied extensively using idealized models of uniform spheres, and uniform and graded aggregates. For a material with a given grain size distribution, different void sizes and void size distributions can occur with different states of packing. Increasing uniformity of grain size results in an increase of the void sizes and of the uniformity of the void size distribution. Angularity and sphericity of the grains also affect the void sizes. The ability of base material grains to migrate into the matrix of a filter depends on their size, shape, and uniformity. However, limited information is available concerning the geometric relationships between migrating base material grains and filter voids and the effect of such relationships on the process of migration. Accordingly, an attempt is made herein to develop a method for the quantification of this relationship based on probability theory.

To study the process of migration of base material grains into a filter, the geometry of the filter matrix should be quantified by obtaining a void size distribution curve; this requires estimates to be made of all the possible void diameters, $\overline{d}$, and their respective probabilities of occurrence, $\overline{p}$. As proposed by Silveira (1965), the grain size distribution of the filter is transformed into a discontinuous curve by dividing it into m parts, each one represented by its average diameter, d_i. By assuming that the probability of occurrence p_i, can be directly related to the percentage of occurrence for a representative sample, a probability of occurrence, p_i, is obtained for each representative diameter, d_i, and the gradation curve can be represented by a series of m diameters, d_i, with their respective probabilities of occurrence, p_i, as shown in Figure 1.

It is then assumed that the filter material has spherical grains and is at its maximum density, and that relative positions occupied by the filter grains are random. Accordingly, at any point in the interior of the filter there must be a corresponding group formed by three tangent spheres, represented in a plane by three tangent circles with diameters d_i, d_j and d_k and respective probabilities of occurrence p_i, p_j and p_k as shown in Figure 1. For each of these groups, an internal void results that can be represented by the internal tangent circle with diameter, $\overline{d}$, which is easily computed (Soddy, 1936; Atmatzidis, 1973). The probability of occurrence, $\overline{p}$, of a void with diameter $\overline{d}$ is equal to the probability of occurrence of a combination of three specific grain sizes d_i, d_j, and d_k, and is computed by adopting a theorem from the "independent trial process" (Kemeny et al., 1959; Silveira, 1965; Atmatzidis, 1973) as:

$$\overline{p} = \frac{3!}{r_i!\ r_j!\ r_k!}\ p_i^{r_i}\ p_j^{r_j}\ p_k^{r_k} \qquad (1)$$

where r_i, r_j, and r_k are the number of times that d_i, d_j, and d_k, respectively, occur in a group and are non-negative integers such that their sum is equal to three. After the various void diameters, $\overline{d}$, and their corresponding probabilities of occurrence, $\overline{p}$, have been computed, the void size distribution curve can be plotted as shown in Figure 1.

According to the void size distribution obtained, it is assumed that the probability of encountering a void of a given diameter is equal to its volume probability. However, it appears to be more realistic to assume that the encounter probability is related to the cross-sectional areas rather than the volume of voids. Therefore, the continuous curve of the void size distribution by volume is

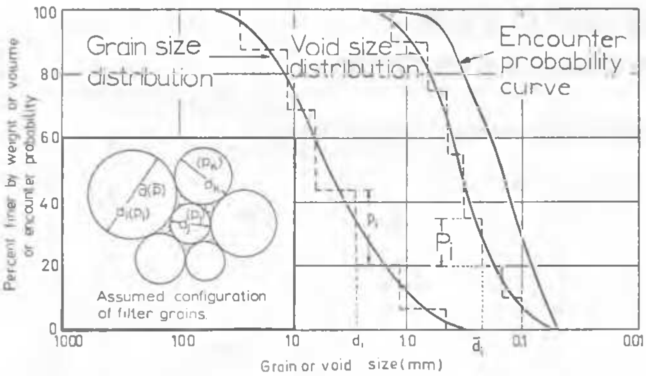

Figure 1. Void Size Curves of Filter Material

transformed to a discontinuous curve by dividing into m parts, each one represented by its average diameter, $\bar{d}_i$, and having a percentage of occurrence, P_i (Figure 1). In a single trial, the probability that a base material grain will encounter a void with diameter $\bar{d}_i$ is:

$$P_i^m = \frac{n_i\bar{d}_i^2}{\sum_{j=1}^{m} n_j\bar{d}_j^2} = \frac{\text{cross sectional area of voids } \bar{d}_i}{\text{cross sectional area of all voids}} \qquad (2)$$

where n_i is the number of voids of diameter $\bar{d}_i$. P_i can be expressed as the ratio of the volume of voids $\bar{d}_i$ to the volume of all voids

$$P_i = \frac{n_i\bar{d}_i^3}{\sum_{j=1}^{m} n_j\bar{d}_j^3} = \frac{\frac{\pi}{6} n_i\bar{d}_i^3}{V_{voids}} = \frac{\frac{\pi}{6} n_i d_i^3}{\frac{\pi}{4} K \sum_{j=1}^{m} n_j\bar{d}_j^2} \qquad (3)$$

if it is considered that the total volume of voids is equal to the cross sectional area of the voids multiplied by a geometric factor K, having units of length. Division of Equation (2) by Equation (3) yields:

$$\frac{P_i^m}{P_i} = \frac{3}{2}\frac{K}{\bar{d}_i} \text{ or } P_i^m = c\frac{P_i}{\bar{d}_i}, \text{ where } c \times \sum_{j=1}^{m} \frac{P_i}{\bar{d}_i} = 1 \qquad (4)$$

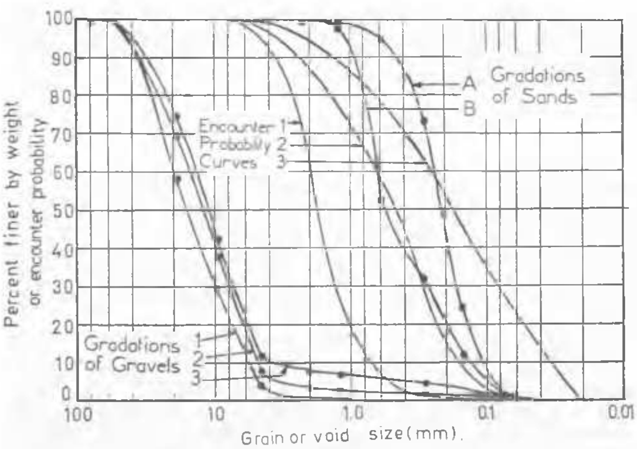

Figure 2. Grain and Void Size Distribution of Materials Used for Theoretical Predictions and Laboratory Testing

A comparison of the encounter probability curve and the grain size distribution curve of the base material indicates that any base material grain with diameter between the maximum and minimum estimated filter void size will be captured in the filter after it traverses a certain distance of migration, S. The number of voids, n, through which a base material grain moves before it encounters a void of size smaller than its diameter is (Silveira, 1965) $n = \log(1-\bar{P}/100)/\log P$, where $\bar{P}$ is the confidence level ($\bar{P}$ per cent of confidence that a base material grain of a given diameter will be stopped within n trials or $\bar{P}$ per cent of the grains of a given diameter will be stopped within n trials) and P is the probability that a base material grain encounters a filter void larger than its diameter and is equal to the percentage of voids larger than the particular base grain. The actual migration distance, S, can be computed if the average migration distance, s, per trial is quantified. If the average diameter of filter grains, $d_{50,f}$, is assumed to represent s, the total migration distance can be estimated as $S = n \cdot d_{50,f}$.

3 APPLICATION

The procedure described in the foregoing paragraphs was employed in order to analyze the migration process of sand grains invading gravel layers having the gradations shown in Figure 2. The computed void size-encounter probability curves shown in Figure 2 indicate that void sizes decrease significantly with increasing sand content of the gravel. For example, in gravels having residual sand contents, 0%, 4%, and 8%, the encounter probability for a void with diameter equal to 0.5 mm is 3%, 45%, and 80%, respectively. This implies that the finer fraction of the gravel gradation controls the void sizes in the gravel. This observation is in good agreement with observations made on protective filters. Kenney et al. (1985) conducted analytical and experimental studies and concluded that the fine range of filter particles (expressed by a representative diameter D_5 or D_{15}) controlled the constriction sizes in the network of voids in the filters; similar findings have been reported by Sherard et al. (1984).

Estimated migration distances of sand grains with diameters ranging from 0.1 mm to 1.0 mm are shown in Figure 3. Migration distances range within very wide limits and are significantly affected by the void size distribution of the gravel which, in turn, is controlled by the residual sand content. For example, the average migration distance of a sand grain with diameter 0.5 mm is 200 cm, 20 cm, and 5 cm when it enters a gravel with 0%, 4%, and 8% residual sand, respectively. It is further indicated that the migration process can result in significant segregation of the mi-

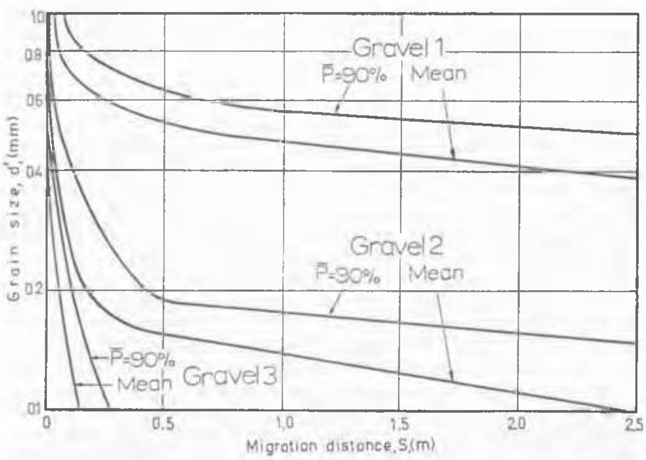

Figure 3. Predicted Migration Distance of Sand Grains

grating sand grains. However, it appears that the degree of segregation depends on the composition of the gravel (residual sand content) as well as on the composition of the migrating sand. Since the migration process is a time-dependent phenomenon, initially invading sand grains that are captured in the gravel alter the composition of the gravel and reduce the available void sizes; this results, in shorter migration distances and lower degree of segregation for subsequently invading sand grains.

4 EXPERIMENTAL OBSERVATIONS AND COMPARISON WITH THEORY

A series of laboratory experiments was performed in order to evaluate the predictive capabilities of the theoretical model. Water was allowed to flow horizontally, under a constant gradient of 0.25, through various combinations of sand and gravel having the gradations shown in Figure 2. The equipment used, the operating procedures for a typical test, and the results obtained have been presented and discussed in detail elsewhere (Atmatzidis, 1973 and 1987), but information on four of the tests conducted, which are most pertinent to this presentation, is reported next. Table 1 gives a summary of test conditions and results. Figure 4 shows the sand content, by weight, of the gravel at the end of each test, plotted against distance from the sand-gravel interface. For tests 5 and 6, the amount of sand reported has been adjusted to reflect only the amount of sand actually entering the gravel and not the residual sand in the gravel before the start of the migration process.

It can be observed that the predicted migration distances of most sand grains invading a "clean" gravel (0% residual sand content) are much larger than the experimentally observed distances in tests 7 (125 cm) and 8(215 cm). Good agreement between predicted and observed migration distances was obtained for the coarser sand grains. For example, a sand grain with a diameter of 0.5 mm has a predicted mean migration distance of about 200 cm which is approximately equal to the observed migration distance of most sand grains in test 8. This grain size corresponds to the D_{90} and the D_{45} of the sands used in tests 8 and 7, respectively, and represents only a fraction of the invading sand grains. It should be noted that, although the sands used in tests 7 (sand B) and 8(sand A) had the same range of grain sizes, a larger percentage of coarser sand grains resulted in a drastic decrease of the total observed migration distances.

Observed and predicted migration distances are in good agreement for practically the whole range of sand sizes used when the gravel had a 4% residual sand content. Sand grains with diameter equal to or larger than 0.15 mm have a predicted average migration distance of about 110 cm which is practically equal to the observed migration distance in test 6. These grains constitute about 87% by weight of the sand and, therefore, may well dominate the migration process. For the case of sand invading gravel with 8% residual sand content (test 5) the predicted migration distances were smaller than those observed. This may be attributed to the fact that the packing of the gravel did not correspond to the densest possible packing, as assumed in the theory, and resulted in larger voids than those predicted. Nevertheless, both predictions and observations yield the same pattern for the effect of the mechanical composition of the gravel: increasing residual sand content from 0% to 8% resulted in decreasing migration distances of the invading sand grains.

In addition to shortening of the migration distances, an increase in the relative amount of coarse grains in the invading sand shortened significantly the time required for reaching a stable condition.

Finally, it can be observed that at the end of the migration process, in all tests conducted, the sand content in the first 5 to 7 cm of the gravel adjacent to the sand-gravel interface was extremely high (on the order of 20%, by weight). This observation indicates that a "filter" formed at the sand-gravel interface which prevented further piping of sand into the gravel and caused the stabilization of the migration process. This experimental observation is in good agreement with theoretical predictions of maximum migration distances of 2 cm to 3 cm when the gravel has a residual sand content of 20%.

Table 1. Summary of Test Conditions and Results

	Test Number			
	5	6	7	8
Gradation of Sand and Gravel (Fig.2)	A,3	A,2	B,1	A,1
Sand Content of Gravel (% by weight)	8%	4%	0%	0%
Maximum Sand Migration Distance (cm)	75	110	125	215
Amount of Sand Entering Gravel (kg)	3.17	7.88	9.25	14.5
Negligible Migration after (hrs)	3.25	3.50	3.00	6.50

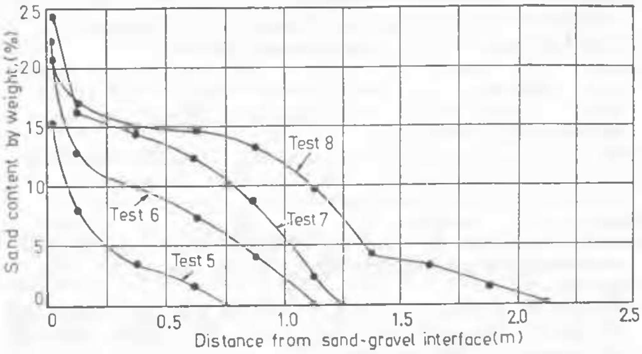

Figure 4. Sand Content of Gravel at End of Tests

5 CASE STUDY

The Tarbela Dam Project is a major irrigation storage and hydroelectric project on the Indus River in northern Pakistan. The main embankment is an earth and rockfill dam with a maximum height of 145 m and a volume of 116 million cubic meters and is founded on alluvium of variable thickness with a maximum depth of 230 m. An upstream impervious blanket with a length of about 1700 m is tied into the main dam inclined core to minimize under-seepage losses and foundation gradients. Following collapse of one of the two power tunnels and failure of its central intake gate, the reservoir was emptied and more than 600 sink holes or craters were discovered in the upstream impermeable blanket (Apleton, 1975). An explanation for the formation of these sink holes is offered herein in terms of sand migration processes in the foundation alluvium under the blanket and the dam.

A variety of subsurface exploration methods were employed to investigate the alluvial material filling the valley at the dam site (Lowe and Sandford, 1982). It was concluded that the alluvium is composed of essentially two components: fine sand and cobble gravel. In places the cobble gravel is choked with sand but there is evidence of extensive deposits of openwork gravel in contact with fine sand layers. Typical gradations of valley alluvium are shown in Figure 5 (after Lowe and Sandford, 1982). The grain size distributions of the gravels shown in Figure 5 were used to obtain void size-encounter probability curves and these curves were compared with the grain size distribution of the sand to yield the predictions of migration distances shown in Figure 6. It can be observed that (a) sand can very easily enter into an openwork gravel layer with low residual sand content and (b) quantities of residual sand

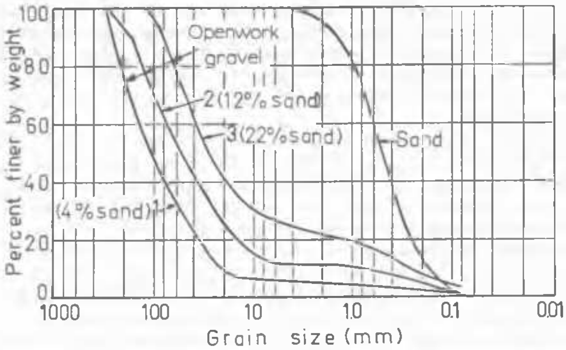

Figure 5. Typical Gradation of Valley Alluvium

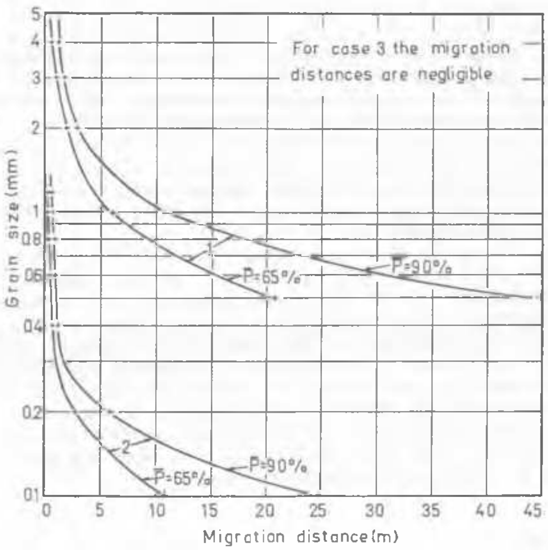

Figure 6. Sand Migration Distance in Openwork Gravel

are able to move from place to place within this layer. Internal migration, according to the latter observation, can result in a reduction of the sand content of a choked gravel which would then be capable of accepting sand from surrounding sand layers thus further increasing the damaging effects of sand migration into openwork gravel layers

An extensive exploratory grouting program using sanded grout (1.0 cement, 0.2 bentonite, 5.0 sand, and 11.0 water) indicated that wide zones of openwork gravel with continuity in the upstream-downstream direction existed at the site (Lowe and Sandford, 1982). Grout takes in the openwork varied from 75 to 230 m^3/m of solids. Using an average migration distance of 20 m (according to results shown in Figure 6) it can easily be computed that the amount of solids (mostly sand) pumped into the openwork layers during grouting resulted in an increase of the gravel's sand content by about 6% to 18% by weight. Since the residual sand content of the openwork ranged from about 4% to about 12%, the final sand content could easily have reached values of 18% to 20% or greater. The migration study indicates that for such amount of residual sand the invading sand grains have negligible migration distances, and this prediction is in very good agreement with the field observation of grout takes equal or less than 14 l/min (Lowe and Sandford, 1982) when the openwork had been choked with sand.

The substantiation of the theoretical predictions by the field observations and measurements permits the following picture of the development of the migration process to be postulated. Under the high hydraulic gradients caused by filling of the reservoir, large volumes of sand migrate into adjacent openwork gravel layers. Due to the large dimensions of the voids in the openwork, even the largest sand grains can move through most of them. The totally migrating sand leaves a space which is filled in by the surrounding sand. The net reduction in the volume of the foundation soils is reflected by settlement of the ground surface which can cause direct formation of a sinkhole or crater or may result in cracking of the impervious blanket which is then eroded by seeping water and forms a crater. The rate of migration is reduced with time as the coarser sand grains get caught in the voids of the openwork; void sizes and sand grain sizes that can be caught in the voids decrease continuously until a "filter" is formed near the sand-openwork interface which prevents further sand movement and causes the migration process to terminate. This, however, is no consolation since severe damage has already been sustained.

6 CONCLUSIONS

Based on the information and discussion presented herein, and within the limitation of the theory developed and experimental observations made, the following conclusions can be advanced:

(1) A method based on probability theory has been developed in order to describe the process of sand migration in gravel. In general, predictions of the migration distances of invading sand grains were in good agreement with laboratory and field observations. Best predictions were obtained for common and openwork gravel with modest sand content (4% and 8%, respectively) while agreement between prediction and observation was not as good for "clean" gravel with high sand content and this may be attributed to the limitations of the theory.

(2) The phenomenon of sand migration in gravel is a time-dependent process which terminates when a thin layer of gravel with high sand content (on the order of 20%) is formed next to the sand-gravel interface.

(3) Migration distances, as well as duration of the migration process, are controlled by the finer fraction of the gravel gradation (amount of residual sand) and by the coarser fraction of the invading sand grains.

REFERENCES

Apleton, B. (1975). Tarbela Dam. Insurers Reject Largest-ever Civils Claims, New Civil Engineer, October, pp. 16-18.
Atmatzidis, D.K. (1973). A Study of Sand Migration in Gravel, M.S. Thesis, Northwestern University, Evanston, Illinois, U.S.A., 72 p.
Atmatzidis, D.K. (1987). An Experimental Study of Sand Migration in Gravel, Proc. 9th ECSMFE, Dublin, Ireland, pp. 887-890.
Kemeny, J.G., Mirkil, H., Shell, J.L., and Thompson, G.L. (1959). Finite Mathematical Structures, Prentice-Hall Inc. Englewood Cliffs, New Jersey, U.S.A.
Kenney, T.C., Chahal, R., Chiu, E., Ofoegbu, G.I., Omange, G.N., and Ume, C.A. (1985). Controlling Constriction Sizes of Granular Filters, Canadian Geotechnical Journal, 22, pp. 32-43.
Lowe, J.III, and Sandford, T.C. (1982). Special Grouting at Tarbela Dam Project, Proc. Grouting in Geotechnical Engineering, ASCE, New York, U.S.A., pp. 152-171.
Sherard, J.L., Dunnigan, L.P., and Talbot, J.R. (1984). Basic Properties of Sand and Gravel Filters, Journal of the Geotechnical Engineering Division, ASCE, 110, 6, pp. 684-700.
Silveira, A. (1965). An Analysis of the Problem of Washing through in Protective Filters, Proc. 6th ICSMFE, Montreal, Canada, 2, pp. 551-555.
Sody, F. (1936). The Kiss Precise, Nature, 137, p. 1021.