Instability and failure of soils with nonassociated flow

Instabilité et rupture des sols avec percolation non-associée

P.V.LADE, Professor, Department of Civil Engineering, School of Engineering and Applied Science, University of California, Los Angeles, California, USA

SYNOPSIS Materials exhibiting nonassociated flow may according to stability postulates by Drucker and by Hill become unstable when exposed to certain stress paths *inside* the failure surface. Triaxial tests designed to follow these critical stress paths have been performed to expose the type of behavior displayed by granular materials. The tests show that at low confining pressures the material dilates and exhibits stable behavior, thus violating both Drucker's and Hill's stability postulates. At high confining pressures the material compresses during shear and unstable behavior is observed *inside* the failure surface. Thus, instability is not synonymous with failure, although both may lead to catastrophic events. It is therefore important to recognize and to understand the factors that control the occurrence of unstable behavior. These are discussed in light of results of laboratory experiments.

INTRODUCTION

Two criteria for failure in soils are commonly employed in interpretation of results of triaxial tests: (1) Failure occurs when the stress difference reaches a limiting value, $(\sigma_1-\sigma_3)_{max}$, and (2) Failure occurs when the effective principal stress ratio reaches a limiting value, $(\sigma_1'/\sigma_3')_{max}$. The two conditions are reached simultaneously in drained tests. The confusion as to the definition of failure arises in interpretation of undrained tests on loose sands and sensitive clays in which the pore pressures increase monotonically during shear. For these types of soils the maximum stress difference is reached before the maximum effective stress ratio. The choice of failure criterion vary with the purpose for which the strength is to be used. Typically, $(\sigma_1-\sigma_3)_{max}$, is employed in total stress analyses, and $(\sigma_1/\sigma_3)_{max}$ is used in effective stress analyses. These issues have been discussed at length by e.g., Bjerrum and Simons (1960), Seed et al. (1960), and Whitman (1960).

It is shown in this paper that the condition of maximum stress difference does not correspond to a true failure condition, but rather to a condition of minimum stress difference at which instability may develop *inside* the true failure surface. This condition was studied in detail by Kramer and Seed (1988). Stress states can be reached above the minimum stress difference condition described by $(\sigma_1-\sigma_3)_{max}$, and instability can be induced anywhere between the $(\sigma_1-\sigma_3)_{max}$-condition and the true failure surface described by $(\sigma_1'/\sigma_3')_{max}$. Thus, instability is not synonymous with failure, although both may lead to catastrophic events.

STABILITY POSTULATES

Experimental evidence from tests on several types of soils have clearly indicated that the use of associated flow rules results in prediction of too large volumetric expansion. To characterize the volume change correctly, it is necessary to employ a nonassociated flow rule. The application of nonassociated plastic flow rules for soils have resulted in questions regarding uniqueness and stability of such materials. The stability postulates for time-independent materials due to Drucker (1951) and due to Bishop and Hill (1951) provide sufficient conditions for stability and ensures uniqueness in dynamic as well as static problems. Drucker's postulate is satisfied provided that associated plastic flow is employed in construction of constitutive models involving convex, plastic yield surfaces. Hill's stability condition is expressed in terms of total strain increments (elastic and plastic), and it extends the condition for stability a little beyond that due to Drucker. Although these conditions are sufficient to guarantee stability, theoretical considerations have suggested that they are not necessary conditions (Mroz, 1963; Mandel, 1964).

The stability postulate formulated by Drucker is suitable for solid metals which exhibit associated flow. According to this postulate, stability requires that the second increment of plastic work is positive or zero:

$$\dot{\sigma}_{ij} \cdot \dot{\varepsilon}^p_{ij} \geq 0 \qquad (1)$$

in which $\dot{\sigma}_{ij}$ = increment of stress and $\dot{\varepsilon}^p_{ij}$ = resulting increment in plastic strain. According to Hill's condition stability should be maintained as long as

$$\dot{\sigma}_{ij} \cdot \dot{\varepsilon}^t_{ij} = \dot{\sigma}_{ij} \cdot (\dot{\varepsilon}^e_{ij}+\dot{\varepsilon}^p_{ij}) = \dot{\sigma}_{ij} \cdot \dot{\varepsilon}^e_{ij} + \dot{\sigma}_{ij} \cdot \dot{\varepsilon}^p_{ij} \geq 0 \qquad (2)$$

in which $\dot{\varepsilon}^t_{ij}$ and $\dot{\varepsilon}^e_{ij}$ are the total and elastic

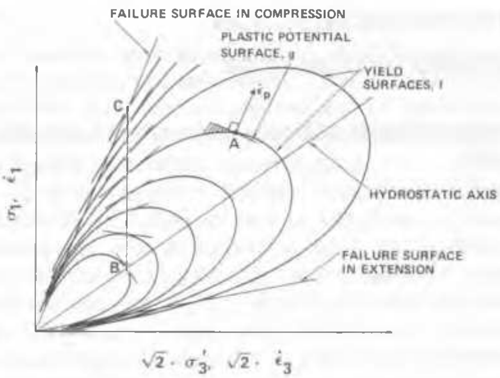

Fig. 1. Pattern of Yield Surfaces for Isotropic Soils. Stress Path for Conventional Triaxial Compression Test (BC).

strain increments, respectively. Hill's stability condition guarantees stability a little beyond the condition given by Drucker.

NONASSOCIATED FLOW AND POSSIBLE CONSEQUENCES

A typical pattern of yield surfaces for an isotropic soil is shown on the triaxial plane in Fig. 1. Plastic potential surfaces have similar shapes as yield surfaces, but for nonassociated flow the two families of surfaces cross each other. A typical plastic potential surface is shown at point A in Fig. 1.

The shaded wedge between the yield surface and the plastic potential surface defines a region in which Inequality (1) is not fulfilled for a material with nonassociated flow. Since a stress increment vector from point A located inside the wedge and the plastic strain increment vector form an obtuse angle, the scalar product of these two vectors (see Inequality (1)) is negative. According to Drucker's stability postulate the sand may exhibit unstable behavior if a stress increment lies in the shaded region. However, experiments show that the sand is perfectly stable at stress points where the normal to the yield surface points in the outward direction of the hydrostatic axis. For this condition the deviator stress can be safely increased to produce further plastic shear strains (work-hardening).

Potential instability occurs in regions where the yield surface opens up in the outward direction of the hydrostatic axis. This allows plastic strains (loading) to occur while the stresses are decreasing. Here loading occurs inside the failure surface and instability may develop in the form of inability to sustain the current deviator stresses.

Fig. 1 shows the stress path for a conventional triaxial compression test performed at constant confining pressure. As the specimen is loaded from B to C the inclination of the yield surface changes. At low deviator stresses near point B, the yield surface is inclined towards the outwards direction of the hydrostatic axis. As loading proceeds, the inclination of the yield surface changes gradually and becomes inclined towards the origin as failure is approached at point C. It is in this region of high deviator stresses where the yield surface is inclined towards the origin that instability may occur.

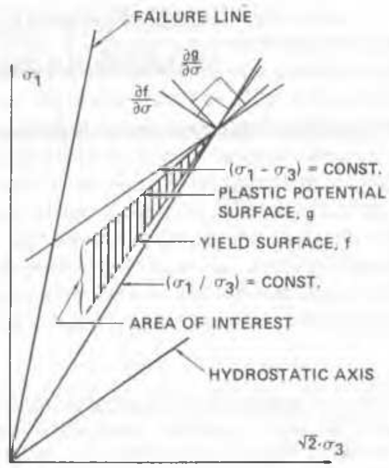

Fig. 2. Wedge-Shaped Region of Stress Paths with Decreasing Stresses in which Soils with Nonassociated Flow May Be Unstable during Hardening Inside the Failure Surface.

Fig. 2 shows a schematic illustration of the region in which Inequality (1) is not fulfilled for a material with nonassociated flow. The region is shaped as a wedge between the current yield surface f and the plastic potential surface g corresponding to the current stress point. This wedge shaped region is located within a larger region bounded by lines corresponding to (σ_1/σ_3) = const. and $(\sigma_1 - \sigma_3)$ = const. as indicated on Fig. 2. All stresses, including the stress difference $(\sigma_1 - \sigma_3)$, are decreasing within the wedge between f and g, but the stress ratio (σ_1/σ_3) is increasing in this region. By performing triaxial tests with stress paths located in this region, experimental evidence regarding the instability of materials with nonassociated flow can be obtained.

NONASSOCIATED FLOW IN UNDRAINED TESTS

A typical effective stress path observed in an undrained test performed with high confining pressure is shown in Fig. 3. Strain increments are superimposed on the stress diagram to allow analyses of strain increment vector directions and to derive the directions of the plastic potential surfaces g. These surfaces are by definition perpendicular to the plastic strain increment vector directions.

For undrained tests the total volumetric strain is zero corresponding to the total strain increment vector being perpendicular to the hydrostatic axis everywhere along the effective stress path. Fig. 3 also shows that volumetric compression is characterized by a strain increment vector pointing away from the origin, whereas volumetric dilation corresponds to a strain increment vector

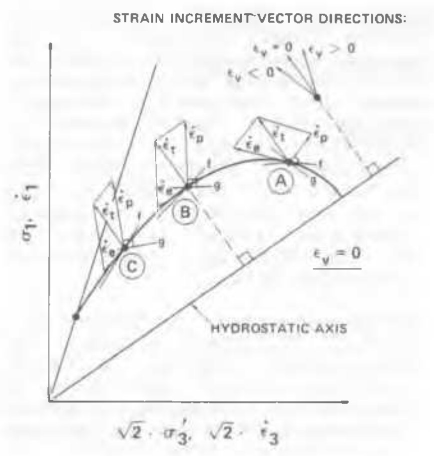

Fig. 3. Evaluation of Relative Inclinations of Yield (f) and Plastic Potential (g) Surfaces Along Undrained Effective Stress Path.

inclined towards the origin of the stress space.

The stress-strain relation corresponding to the effective stress path in Fig. 3 shows that plastic strains are produced everywhere along the undrained stress path. The yield surface must therefore be pushed out, and this requires it to be inclined relative to the effective stress path as indicated in Fig. 3.

The direction of the elastic portion of the total strain increment vector in the triaxial plane depends on the value of Poisson's ration, ν . For $\nu = 0$ the elastic strain increment vector is parallel to the stress increment vector or the stress path on the triaxial plane shown in Fig. 3. For $\nu = 0.5$ the elastic strain increment vector is perpendicular to the hydrostatic axis corresponding to no elastic volume change. For values of Poisson's ratio between zero and 0.5 the elastic strain increment vector will be between the two extreme positions. With no loss of generality, it will be assumed in the following that the elastic strain increment vector is tangential to the stress path on the triaxial plane, i.e., corresponding to $\nu = 0$.

Three points of interest are indicated along the undrained effective stress-path in Fig. 3. The length and direction of the plastic strain increment vector is obtained by vectorial subtraction of the elastic from the total strain increment vector. Only the directions of the elastic and the total strain increment vectors are known, and the derived direction of the plastic strain increment vector depends on the assumed relative magnitudes of the elastic and total strain increments.

Whether associated flow (which guarantees stability) or nonassociated flow is observed at point A, stability is obtained everywhere along the effective stress-path from the hydrostatic axis up to point B. Along this portion of the stress-path the load can be maintained constant or increased without any observable instability. Nonassociated flow is obtained at points B and C. For the limiting case where the elastic strain increments become negligible the plastic strain increment vectors become perpendicular to the hydrostatic axis. Since the yield surfaces must be inclined relative to the effective stress-paths as shown, nonassociated flow is clearly demonstrated to occur at points B and C.

INSTABILITY AND FAILURE OF SOILS

A series of stress-controlled undrained triaxial compression tests on loose Sacramento River Sand was performed to study instability in granular materials. The details of these experiments are given by Lade et al. (1988). Materials that compress or tend to compress during shear may become unstable *inside* the failure surface, and this may lead to liquefaction. The loose sand specimens were first exposed to high confining pressures at which volumetric compression was observed during shear. Fig. 4 shows the actual stress paths followed in these tests, and Fig. 5 shows the observed stress-strain, volume change, and pore pressure development in one of the tests. In each test the saturated specimen was first loaded under drained conditions to a preselected stress level S (expressed as the ratio of the current to the maximum stress difference at a given confining pressure). The drainage valve was then closed, causing the specimen to follow a stress path within the shaded wedge shown in Fig. 2. Instability developed in each specimen due to increasing pore pressures, i.e., the specimens could not sustain the applied load. The tendency for volumetric creep, however small it may be, caused the pore pressure to increase under undrained conditions, providing the small perturbation which rendered the material unstable. However, the large strains observed along the unstable stress paths could not be caused by creep or viscous flow. Thus, the undrained test was used as a tool to study stability/instability of sand that tends to compress during shear. The effective stress paths shown in Fig. 4 were within the shaded wedge in Fig. 2, the specimens exhibited nonassociated flow and plastic volumetric compression (although the total volumetric strain was zero), and instability was obtained in all cases in the hardening regime *inside* the failure surface.

Instability and failure are two different behavior aspects of soils which exhibit nonassociated flow. Although both may lead to catastrophic events, they are not synonymous. Thus, a failure surface does not go through each of the points at which instability was produced. In fact, these points are well within the failure surface as obtained from drained tests. Obviously, each of the specimens could have been loaded to higher stress differences as long as the stress paths were outside the wedge between the yield and the plastic potential surfaces. Therefore, a failure condition expressed in terms of the maximum stress differences obtained from e.g., undrained tests initiated at the hydrostatic axis (see Fig. 3) would clearly underestimate the true effective strength of the soil.

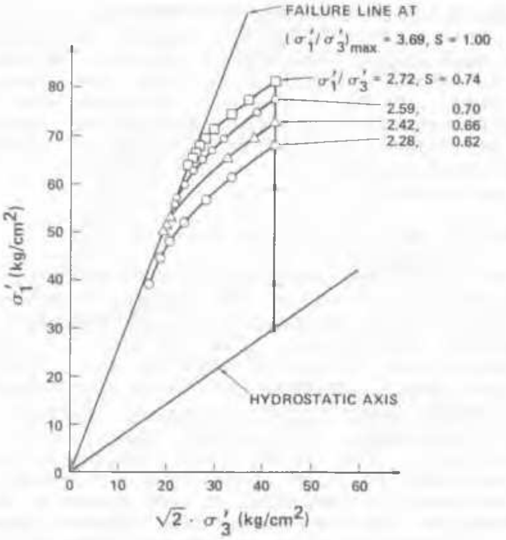

Fig. 4. Effective Stress Paths for Stress Controlled Triaxial Compression Tests on Loose Sacramento River Sand.

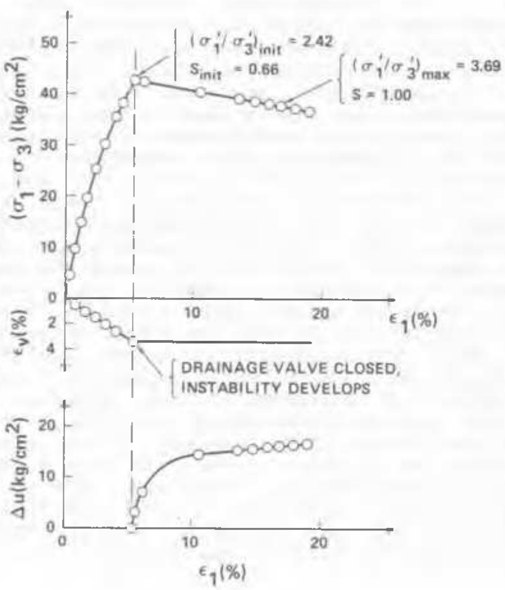

Fig. 5. Stress-Strain, Volume Change, and Pore Pressure Relations in Stress Controlled Triaxial Compression Test on Loose Sacramento River Sand

CONCLUSION

The stability conditions formulated by Drucker and by Hill provide sufficient conditions for stability and are satisfied for materials that obey an associated plastic flow rule. Granular materials and other soils exhibit nonassociated plastic flow, and the stability of such materials has often been questioned. To provide experimental evidence regarding the instability of granular materials, a series of undrained triaxial tests was performed under stress control so that any instability in the material could freely develop. In the tests reported here, specimens of loose sand were exposed to stress paths with decreasing effective stresses and increasing stress levels. The results of these tests showed that the granular material tended to compress during shear, it followed a nonassociated flow rule, and instability was obtained in all cases *inside* the failure surface. The occurrence of unstable behavior found in this study is related to the plastic, compressive volume change of the material.

It is emphasized that instability is not synonymous with failure, although both may lead to catastrophic events. A failure surface does not go through each point at which instability can be produced. Therefore, a "failure" criterion in terms of maximum stress differences obtained from e.g., undrained tests initiated at the hydrostatic axis, in fact, expresses the minimum stress differences at which instability can be produced.

REFERENCES

Bishop, J.F.W., and Hill, R., (1951). A Theory of the Plastic Distortion of a Polycrystalline Aggregate under Combined Stresses. The Philosophical Magazine, 42, 414-427.

Bjerrum, L., and Simons, N.E. (1960). Comparison of Shear Strength Characteristics of Normally Consolidated Clays, Proc. ASCE Res. Conf., Boulder, Colorado, 711-726.

Drucker, D.C. (1951). A More Fundamental Approach to Stress-Strain Relations. Proc. First U.S. Nat. Cong. Appl. Mech. 487-491.

Kramer, S.L., and Seed, H.B. (1988). Initiation of Soil Liquefaction Under Static Loading Conditions. ASCE J. Geotech. Engr., 114, 412-430.

Lade, P.V., Nelson, R.B., and Ito, Y.M. (1987). Nonassociated Flow and Stability of Granular Materials. ASCE J. Engr. Mech., 113, 1302-1318.

Lade, P.V., Nelson, R.B., and Ito, Y.M. (1988). Instability of Granular Materials with Nonassociated Flow. ASCE J. Engr. Mech., 114.

Mandel, J. (1964). Conditions de Stabilite et Postulat de Drucker. Proc. IUTAM Conf., Grenoble, 58-68.

Mroz, Z. (1963). Nonassociated Flow Laws in Plasticity. J. de Mecanique, 2, 21-42.

Seed, H.B., Mitchell, J.K., and Chan, C.K. (1960). The Strength of Compacted Cohesive Soils. Proc. ASCE Res. Conf., Boulder, Colorado, 877-964.

Whitman, R.V. (1960). Some Considerations and Data Regarding the Shear Strength of Clays. Proc. ASCE Res. Conf., Boulder, Colorado, 581-614.