Tidal analysis in urban areas
L'analyse des marés en sites urbains

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SYNOPSIS: In April 1987 the construction of the Willems Railway Tunnel in Rotterdam started. The tunnel is situated beneath the river and the central part of the city. On the banks the tunnel will be constructed in open pits by the cut and cover method. The performance criteria for lowering the piezometric head makes an accurate monitoring with elimination of tidal influences necessary. To predict the water pressure against the tunnel the response of the aquifer due to storm is investigated. The influence of the hydraulic resistance of the river-aquifer interface for both situations is very important. To determine the resistance of this boundary layer tidal measurements have been carried out. This paper describes the performed tidal analysis.

1. INTRODUCTION

In april 1987 the construction of the Willems Railway Tunnel in Rotterdam started. The tunnel is situated under the central part of the city and the total length is 3 km. On the banks the tunnel will be constructed in open pits by the cut and cover method. In the river the tunnel will be constructed as a submerged tunnel. This paper discusses the ground water problems during construction on the right bank. Figure 1 shows the situation.

Figure 1. Situation on the right bank.

2. GEOLOGICAL AND GEOHYDROLOGICAL CONDITIONS

The composition of the subsoil is as follows.
From the explored depth to about 30 m minus Dutch reference level (N.A.P.) loam and silty fine sand is found. On this formation a coarse pleistocene sand formation is found to a depth of N.A.P. -16 m.
The top of this pleistocene formation consists of a one meter thick stiff clay layer. On the top of the pleistocene holocene clay and peat layers are found. At some places in these soft layers fine silty sand layers

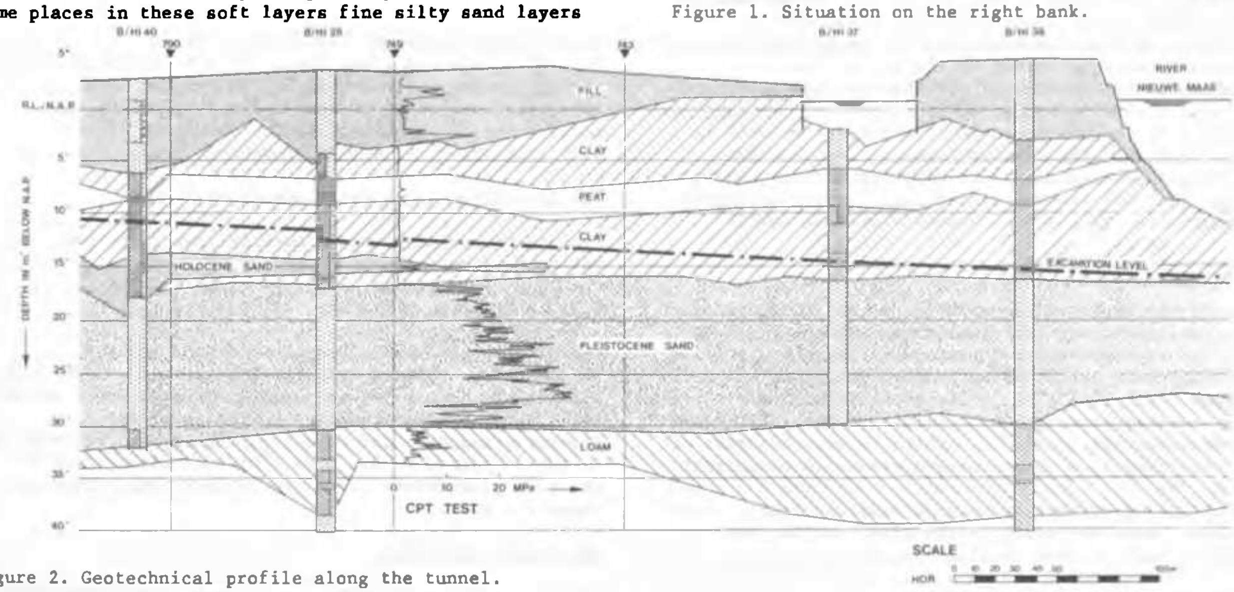

Figure 2. Geotechnical profile along the tunnel.

occur. At some locations these sand layers make contact with the pleistocene sand.
The existing ground level lays near N.A.P. Below the ground surface the formation consists of sand fill or man-made ground. The geotechnical profile and the depth of the tunnel are shown in figure 2.
The ground water conditions are as follows.
The piezometric head in the pleistocene sand varies near the river from N.A.P. -1 m to N.A.P. -2 m in the centre of the city. The phreatic level shows great variation along the tunnel: from N.A.P. -2 m to N.A.P. +2 m.
The pore pressure in the holocene clay and peat layers is generally hydrostatic and shows a sharp transition to the piezometric head in the deep sand layer.

3. GEOTECHNICAL ASPECTS AND PERFORMANCE CRITERIA

An important part of the buildings near the pit is founded on concrete piles in the deep sand layer. Generally the factor of safety is satisfactory.
Other buildings near the pit are founded on timber piles; generally also placed in the deep sand. Due to the considerable negative skin friction the factor of safety is about or just above 1.0.
A medieval church near the building pit is founded on short timber piles in the holocene layer and partly on timber piles in the deep sand layer.
It shows some damage due to unequal settlements.

The performance criteria for the ground water are as follows:
- lowering of the phreatic level is not allowed;
- lowering of the piezometric level in the deep sand layer is allowed to a level that occured during corresponding period in the past.

These criteria are based on the following considerations. Lowering of the phreatic level produces a decrease of the pore pressure in the holocene soft layers. This means settlements of streets and services, desiccation of the timber pile foundations, as well as an increase of the negative skin friction.
Lowering of the piezometric level in the deep sand layer took place in the Sixties and Seventies as a consequence of the construction of an underground railway system and industrial pumping. The piezometric head in the deep sand layer was therefore during many years some metres lower than the phreatic level. Water pressure measurements proved that, in spite of this situation, the pore pressure in the holocene layer was nearly hydrostatic. This can be explained by the presence of relatively impervious layers at the top of the pleistocene and the bottom of the holocene layers.

By adhering to the mentioned performance criteria it is achieved that the effective stresses in the zone just above the pleistocene sand will increase, but not above the level that was reached in the past (overconsolidated soil behaviour). Thus the settlements will be small.

4. OBJECTIVE OF THE STUDY

In view of the performance criteria and the presence of very vulnerable buildings in the area monitoring during pumping is very important. Therefore a reliable interpretation of the tidal influences is needed.
The normal tidal amplitude in the river is 0.80 m.
Another aspect regarding the influence of the water level in the river is the prediction of the water pressures against the tunnel in the permanent situation to prevent uplift.
So the objective of the study is to determine the influence of tide and storm in the river on the piezometric head in semi-confined aquifer.

5. HYDRAULIC CONDITIONS

The mean river level varies usually slower than the cyclic variations. Accordingly we can schematize the river level to be composed of the mean river level (stationary case), the cyclic variation and the storm surge. The last component acts independently from the two others.

$$h = h_o + A\cos(\omega t) + B\cos(\cos(\pi t_v/G)) \qquad (1)$$

where h_o is the mean river level, A is the amplitude of the astronomical tide, $\omega = 2\pi/T$ is the frequency, T is the period of the tide, B is the amplitude of the storm, t_v is an arbitrary time axis, G is the duration of the storm.

6. TIDAL PROPAGATION IN SEMI-CONFINED AQUIFER

The geohydrological schematization is shown in figure 3. A dike is present at the upper aquifer.

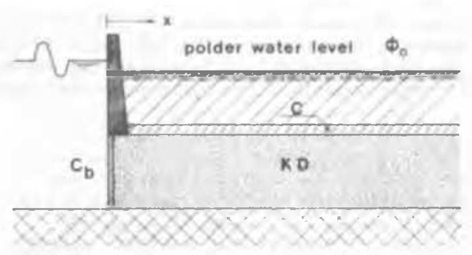

Figure 3. Semi-confined aquifer with boundary layer.

The silt layer that always occurs between the river and the aquifer is schematized by a semipervious thin layer with hydraulic resistance c_b. The piezometric head in this aquifer can be described according to the following equation:

$$kD\,\phi_{,xx} - (\phi - \phi_o)/c = S\,\phi_{,t} \qquad (2)$$

where kD is the transmissivity of the aquifer, k is the permeability of the sandlayer, ϕ_o is the polder water level, c is the hydraulic resistance of the clay layer above the aquifer, c_b is the hydraulic resistance of the semipervious thin layer, S is the elastic storativity. The boundary conditions are the continuity at the river-sandlayer interface, that is $(h - \phi)/c_b = -k\,\phi_{,x}$ and $\phi = \phi_o$ at some distance from the river. The piezometric head in the aquifer is fully determined by the river level. The response of the aquifer can be calculated by the summation of the contributions of each tidal component.

The mean river-level (stationary case)

The stationary piezometric head follows from the following equation:

$$\phi_s(x) = \phi_o + (h_o - \phi_o)\exp(-x/L)/(1 + k\,c_b/L) \qquad (3)$$

where h_o is the mean river level and is greatly influenced by the seasonal fluctuations, $L = \sqrt{(kDc)}$ is the leakage factor. It is assumed that the water polder level ϕ_o is constant and the variation of the drawdown caused by the industrial withdrawal is less than variation caused by the tide. This situation holds for the mean piezometric level over a very long period, for example a year.

The cyclic variation

Bauduin and Barends (1988) have treated the response of

a leaky aquifer for a different geohydrological schematization which takes into account the foreland and the consolidation of the clay layer. In this paper another schematization is used. In our opinion to obtain tidal responses the consolidation of the clay layer can be neglected.
This approach is less complex and the solution is easier to derive and handle. The response of the aquifer due to the cyclic variation of the river can be described as follows (the derivation of this solution is given in the appendix):

$$\phi = f\ h_o \exp(-ax) \cos[\omega t - bx - \arctan\{bkc_b/(1+akc_b)\}] \qquad (4)$$

where f, a, and b respectively reduction-, damping- and phase difference factor according to

$$f = 1/\sqrt{\{(1 + a\ k\ c_b)^2 + (b\ k\ c_b)^2\}} \qquad (5)$$

$$a = (1/L)\ \sqrt{[+\ 0.5 + 0.5\ \sqrt{\{1 + (S\omega c)^2\}}]} \qquad (6)$$

$$b = (1/L)\ \sqrt{[-\ 0.5 + 0.5\ \sqrt{\{1 + (S\omega c)^2\}}]} \qquad (7)$$

In case of $c_b = 0$ the formula (4) reduces to the tidal formula of Bosch (De Lange and Maas (1986)).

<u>The storm</u>

The response of the aquifer can be determined by assuming the storm to be a unit step function.
The solution can be derived from Carslaw and Jaeger (1959) and this gives:

$$\frac{\phi}{h_o} = +\ 0.5\ \frac{r}{r+m} \exp(-x/L)\ \mathrm{erfc}\{\frac{x}{2mL\sqrt{t}} - m\sqrt{t}\}\ +$$

$$+\ 0.5\ \frac{r}{r-m} \exp(+x/L)\ \mathrm{erfc}\{\frac{x}{2mL\sqrt{t}} + m\sqrt{t}\}\ -$$

$$-\ \frac{r^2}{r^2-m^2} \exp\{(rx/mL) + t(r^2-m^2)\}\ \mathrm{erfc}\{\frac{x}{2mL\sqrt{t}} + r\sqrt{t}\} \qquad (8)$$

where $m = 1/\sqrt{(Sc)}$, $r = L/\{k\ c_b\ \sqrt{(Sc)}\}$ and erfc(...) is the complementary error function.
Although the real curve for the storm surge (cosine increment) can be calculated by superposition, this is in the present case not necessary because of the rather stiff behaviour of the sand layer. The response seems to be immediate.
With this solution we can now predict the expected water pressure against the tunnel.

7. TIDAL MEASUREMENT AND INTERPRETATION

To monitor the progress of the drainage by the deep well system and to determine the extension of its drawdown a large number of observation wells are installed in the phreatic water and semi-confined aquifer.
The response of the leaky aquifer to the river-tide is investigated by monitoring 20 observation wells in three different directions at 40 minute intervals over a period of 14 hours. The position of the well arrays is shown in fig. 1 (section I, II, III).
The river level is also monitored at 10 minute intervals.
A Fourier analysis has been applied to the data. The Fourier series corresponding to g(t) is defined to be:

$$g(t) = 0.5\ C_o + \Sigma\ C_k \cos(\omega_k t - \theta_k) \qquad (9)$$

where 0.5 C_o is the mean value, C_k is the amplitude, $\omega_k = 2\pi k/T$ is the frequency, k = 1, 2, etc., θ_k is the phase difference, C_k and θ_k are the Fourier coefficients, T is the period of g(t). The numerical integration of the Fourier coefficients is carried out by the rectangular method. The period of the tide is 12 hours and 30 minutes. In the present case the response for three harmonic terms is determined. The results show that the amplitude and the phase difference along the sections II and II are almost the same.
The response of the tide near the tunnel can therefore be assumed to be one-dimensional. A relationship for the amplitude and the phase difference is determined.
Using this relationship the tidal parameters of the other observation wells can be derived by interpolation. The comparison between the calculated response using three harmonic terms and the observations is given in figure 4.

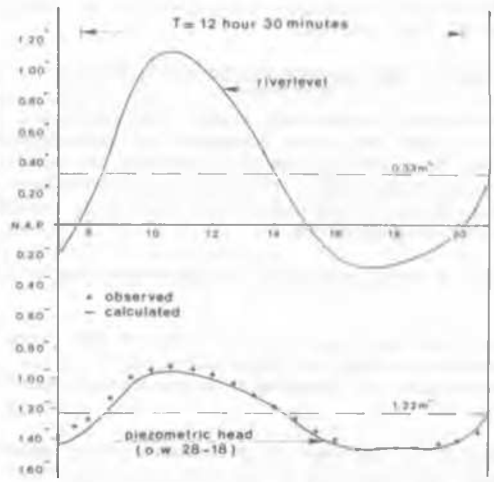

Figure 4. Response of the aquifer.

The hydraulic resistance of the boundary layer c_b can be obtained for both the stationary and cyclic part. The non-linear regression analysis is carried out to determine the unknown parameters, the polder water level and c_b (see formula (3)). The other geohydrological parameters, derived from pumping tests, are the permeability k = 50 m/day and the leakage factor L = 1,100 m. The results of this analysis are ϕ_o = N.A.P. -3,46 m and c_b = 13.9 days and summarized in table I.

Table I. Stationary piezometric head in section I.

distance from the bank (m)	measurement (m)	calculated piezo. head (m) with formula (3)
50	N.A.P. -1.22	N.A.P. -1.24
290	N.A.P. -1.63	N.A.P. -1.68
420	N.A.P. -1.85	N.A.P. -1.88
475	N.A.P. -1.96	N.A.P. -1.95
660	N.A.P. -2.33	N.A.P. -2.19
1.075	N.A.P. -2.66	N.A.P. -2.59
1.500	N.A.P. -2.75	N.A.P. -2.87

From this tidal analysis follows the dependency of the stationary piezometric head of the mean river level.
The obtained tidal parameters are reduction factor f = 0.4032, dampingfactor a = 0.0018 and phase difference factor b = 0.0003 (formula (5), (6), (7)). The boundary resistance c_b is further determined using formula (5).
The result is c_b = 16.3 days. From the observed phase difference one can also determine c_b, but the result (c_b = 5.7 days) is less accurate because of the little phase difference. The obtained c_b from both the stationary and cyclic part is of the same magnitude: 13.9 days and 16.3 days.
The graphical method to obtain the tidal parameters is by plotting the amplitude against the logarithm of the

distance and plotting the phase difference against the distance. The intersection of the first curve (amplitude diagram) to the mean river level is found to be at a distance $x_0 = -505$ m and for the second curve (phase diagram) to the x-axis at $x_0 = -188$ m. The cause of this discrepancy is already mentioned before.
The following conclusion can also be drawn from formula (4). The usual procedure to translate the origin over a distance x_0 or to substitute $(x_0 + x)$ in the tidal formula of Bosch is not justified because of the non-uniqueness of the solution.

Introducing the concept of a semipervious thin boundary layer c_b, the tidal response of a leaky aquifer for different kinds of boundary conditions can be solved using analytical methods.

8. RESPONSE OF THE AQUIFER TO THE RIVER TIDE

The relationship between the tidal river level and the piezometric head has to be expressed in tidal parameters. Using this relationship one can eliminate the tidal influence by continuously measuring the river level and once measurement at the observation well ϕ_{obs} according to the following equation:

$$\phi = \phi_{obs} - \sum_i f_i\, h_i \exp(-a_i x) \cos(\omega_i t - \theta_i - b_i x - w_i) \qquad (10)$$

In the present case digitalized values of the river level at 10 minute intervals are available.
The tide-elemination procedure can be carried out by computer.

A prediction of the expected piezometric head can also be determined for the daily situation or in case of storm using the first harmonic term according to the following equation:

$$\phi_{predict} = \phi_s(x) + f\, h_o \exp(-ax) \cos(\omega t - \theta - bx - w) \qquad (11)$$

In predicting the piezometric head some care has to be taken about the drawdown caused by some wells (industrial withdrawal). This will influence the boundary condition ϕ_s.

A prediction of the piezometric head has been carried out recently. The results are summarized in table II and the agreement is satisfactory.

Table II. Comparison observed and predicted piezometric head.

time (hour)	observed (m)	calculated (m)
9.00	N.A.P. -1.20	N.A.P. -1.04
10.00	N.A.P. -1.19	N.A.P. -1.12
11.00	N.A.P. -1.20	N.A.P. -1.13
12.00	N.A.P. -1.19	N.A.P. -1.08
13.00	N.A.P. -1.08	N.A.P. -0.97
14.00	N.A.P. -0.82	N.A.P. -0.83
15.00	N.A.P. -0.64	N.A.P. -0.71
16.00	N.A.P. -0.59	N.A.P. -0.62

9. CONCLUSION

The following conclusion can be drawn from this study:

- by using the Fourier analysis an accurate determination of the tidal parameters is possible;
- an equation for the response of a leaky aquifer due to tide has been derived using the concept of a semipervious thin layer;
- the predicted piezometric head is greatly affected by this boundary layer;
- using the obtained tidal parameters accurate monitoring and elimination of the tidal influences when interpretating pumping tests is possible;
- by determining the tidal response for the normal situation a prediction of the response due to storm can be made.

10. REFERENCES

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2. Carslaw H.S. and Jaeger J.C. (1959). Conduction of heat in solids. 2nd ed. (Oxford University Press).
3. De Lange W.J. and Maas C. (1986). On the anomalous behaviour of groundwater tides along the Hollandsche IJssel near Gouderak (in Dutch). H20 (19) nr. 2: 24-29.

11. APPENDIX: DERIVATION OF FORMULA (4)

The river level is assumed to be $h = h_o \cos(\omega t)$.
The piezometric head and the river level can be expressed by:

$$\phi = \mathrm{Re}\{\phi(x) \exp(i\omega t)\} \qquad (a.1)$$

$$h = \mathrm{Re}\{h_o \exp(i\omega t)\} \qquad (a.2)$$

The continuity condition at the river-aquifer interface ($x = 0$) demands:

$$(h - \phi)/c_b = -k\, \phi_{,x} \qquad (a.3)$$

Substitution of (a.1) in equation (1) and division by $\exp(i\omega t)$ gives:

$$\phi_{,xx} - \tau^2 \phi = 0 \text{ or } \phi = A \exp(-\tau x) \qquad (a.4)$$

where $\tau = (1/L)\sqrt{(1 + i S\omega c)} = (1/L)\sqrt{\{r \exp(i\psi)\}}$

$$\tau = (1/L)\sqrt{(r)}\,\{\cos(0.5\,\psi) + i \sin(0.5\,\psi)\}$$

$= a + i\,b$ (see formulas (6) and (7))

With (a.4) the piezometric head becomes

$$\phi = \mathrm{Re}\{A \exp(-\tau x) \exp(i\omega t)\}$$

The integration constant A follows from the continuity condition (a.3):

$$A = h_o/(1 + \tau k c_b)$$

After rearranging the piezometric head is given by:

$$\frac{\phi}{h_o} = \frac{\exp(-ax) \cos[\omega t - bx - \arctan\{b k c_b/(1 + a k c_b)\}]}{\sqrt{\{(1 + a k c_b)^2 + (b k c_b)^2\}}}$$