Probabilistic approach of the settlement of an arch bridge
Approche probabiliste du tassement d'un pont en arc

A.BOLLE, Lecturer, Civil Engineering Department, State University of Liège, Belgium
F.BONNECHERE, Professor, Civil Engineering Department, State University of Liège, Belgium
J.-M.CREMER, Director, Greisch Consultants, Liège, Belgium

SYNOPSIS The deformability of a rock mass with a complex geology is analyzed from pressuremeter data, taking the spatial variability into consideration. For several loading cases, the stress increments under one footing of the bridge are computed using the linear elastic FEM. The deformations of the footing are then estimated from these data through an original point estimate method, based on the information theory and using a strictly minimum number of simulations.

INTRODUCTION

Due to the geologic complexity of the slope of the valley supporting one of the inclined foundation of the arch, and considering the importance of the acting forces, an accurate estimate of the settlements, and mainly of the rotations, is difficult to obtain through a classical analysis. Particularly, usual methods are not able to take into account the scatter of the mechanical properties of the rock mass and to appreciate accurately its influence on the deformations.

According whether the estimated settlements exceed or not given limit values, special leveling devices are to be worked up at the springing course of the arch.

The available data are pressuremeter moduli measured into nine vertical and inclined borings located on the site of the projected footing. The acting forces applied on the footing by the arch bridge are also given.

STATISTICAL MODELING OF THE ROCK MASS

Basic assumptions

Considering the heterogeneity of the rock mass and the lack of a well defined geological structure, the substratum is assumed to be homogeneous on a large scale, with a possible spatial trend of the characteristics. This homogeneity on a large scale means that, on an average, the parameters of behavior remain constant (or linearly variable) throughout the rock mass, but that they exhibit random deviations around their mean value.

The pressuremeter modulus E_M and the structural coefficient α, commonly used within pressuremeter theories, describe the rheologic behavior of the rock mass. Both are considered as the sum of two components :

- a general trend, linear with x,y,z;
- a random "noise" around the general trend.

This three-dimensional "random field" is furthermore characterized by the autocorrelation of E_M and α, which is the result of the hidden internal structure of the rock mass.

Due to the lack of measured values for α, a probability density function is built on the basis of values available in the literature, assuming an autocorrelation function identical for α and E_M, considered however as uncorrelated random variables. This assumption is questionable, but without useful information on this particular point, the maximum uncertainty principle leads to such an independence.

Statistical treatment of the measured pressuremeter modulus

Experimental data, including all the values of E_M measured below the footing level, are a statistical sample of the spatial random field of E_M. The probability density function of E_M is first symmetrized through the transform

$R_M = \sqrt[4]{E_M}$, leading to a negligible skewness

$\beta_1 = 0.07$.

A linear trend with (x,y,z) is assumed for the mean value of the transformed variable, given by the expression :

$$\overline{R_M}(x,y,z) = A + B.x + C.y + D.z \qquad (1)$$

The 4 parameters A,B,C,D are obtained through a least squares procedure and a stationary random field is achieved by removing this spatial trend and retaining only random variations around the mean value. The new random variable e, with a null mean value and a standard-deviation σ_e (assumed to be constant) is estimated from the statistical sample :

$$e_i = R_{M_i} - \overline{R_M}(x_i,y_i,z_i)$$
$$= R_{M_i} - (A + B.x_i + C.y_i + D.z_i) \qquad i=1,N \qquad (2)$$

According to this procedure, numerical values (156 measurements below footing level) lead to :

$A = 6.452919 \qquad B = -9.618\ 10^{-3}$

$C = -1.488\ 10^{-3} \qquad D = -7.859\ 10^{-2}$

The factor D is prevalent, showing a rapid increase of the modulus with depth, but B and C cannot be neglected.

The statistical parameters of the reduced variable e are given by :

$e \cong 0 \qquad \sigma_e = 1.14779 \qquad \beta_1(e) = 0.070$

The input data are now the set of N values (e_i, x_i, y_i, z_i) from which the autocorrelation characteristics are to be estimated as a function $\rho(\tau)$ of the distance τ between two locations within the rock mass (Fig. 1).

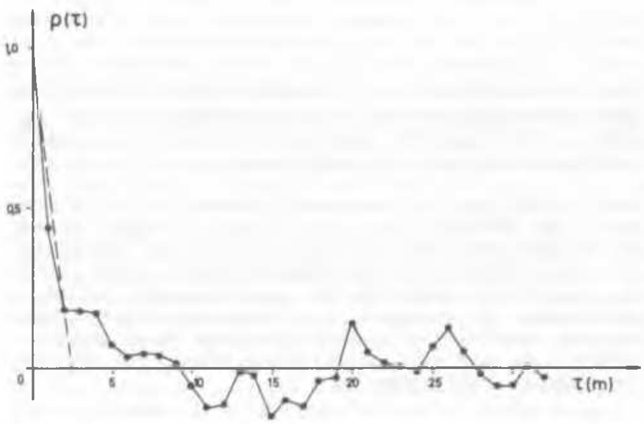

Fig. 1

Correlation function $\rho(\tau)$ of the stationary variable e versus the distance τ between any two couples of points.

A linear decreasing function with one single parameter is adopted for simplicity, with the advantage of a limited distance of influence, contrary to exponential functions extending to infinity. The observed influence distance is about 2.5 m.

Statistical modeling of the rheologic parameter α

Considering the rocky nature of the subsoil, the rheologic parameter α can vary between 1/3 and 2/3, according to the fracturation and the weathering of the rock mass.

The points $\alpha = 1/3$ and $\alpha = 2/3$ are chosen for the two lateral points of the three points estimate of a normal density function (Bolle, 1988), with a mean value at 1/2 and a variance $\sigma^2(\alpha) = 1/108$. The lower bound $\alpha = 0$ is then more than 5 standard-deviations away from the mean value and the probability for α to fall below this lower bound is negligible. Indeed, 99,9 % of the distribution are between plus and minus 3 standard-deviations, i.e. between 0.211 and 0.789 in this case.

MECHANICAL MODELING OF THE ROCK MASS BELOW THE FOOTING

Principle for computing the deformations

The deformations are estimated by computing the vertical strain ΔH of a layer with an initial thickness H loaded by horizontal and vertical stress variations $\Delta\sigma_v$ and $\Delta\sigma_h$:

$$\Delta H = \frac{H}{\alpha . E_M} \left[\Delta\sigma_v - \Delta\sigma_h \right] \qquad (3)$$

In this formula (Baguelin and al., 1978) α is the inverse of the usual structural coefficient introduced by Ménard. The formula is modified to account for the three-dimensional feature of the problem and for an inclined footing.

With a normal (to the footing) stress σ_N and two perpendicular stresses σ_{H1} et σ_{H2}, the formula becomes :

$$\Delta H = \frac{\alpha . H}{E_M} \left[\Delta\sigma_N - 0.5\ (\Delta\sigma_{H1} + \Delta\sigma_{H2}) \right] \qquad (4)$$

with α the structural coefficient according to Ménard.

Finite element analysis

Stresses throughout the rock mass below the footing level are estimated by the FEM analysis of an homogeneous elastic medium loaded by a rigid footing, for three loading cases : a normal unit load and two unit bending moments. In a first step, the stress distribution is assumed independent from the random local variations of the deformability. An arbitrary elastic modulus E is chosen, as well as the Poisson coefficient $\nu = 0.3$.

The FEM analysis was carried out by the computer code SAPLI (LMS, University of Liège), with three-dimensional 8-nodes brick elements, keeping the same discretization as for the following probabilistic method. The rock mass below footing level is limited to a volume of 80 x 60 m, 45 m thick, regularly divided into 1728 cubic blocks, 5 m aside, 216 of which are located under the footing area. Computing time was 3 to 4 hours for each loading case (VAX 750).

Compatibility of displacements

The probabilistic approach allocates a random distribution to E_M and α into each elementary block and computes, using formula (4), the deformations of the bloks. The displacements under the footing are obtained by summing these deformations on each "column" of blocks. The compatibility conditions for adjacent columns cannot be strictly obtained, but the approximation by the elastic homogeneous FEM analysis is assumed to be acceptable.

For each block immediately under the footing, and for each loading case, the settlement is thus obtained, and this for any random distribution of the mechanical characteristics throughout the rock mass. Generally, these displacements are not compatible with the "rigid footing" condition assumed initially.

That incompatibility is reduced through the hypothesis that for any "column" i of blocks under the footing, the axial displacement t_i and the mean stress σ_i applied to that column are linearly related. The displacements T_i computed with a given random distribution of the characteristics are modified in such a way that the footing moves as a rigid body, modifying the stresses of the homogeneous solution into σ_{i0} stresses, in order to agree with :

$$\sigma_i \cdot T_i = \sigma_{i0} \cdot t_i \qquad (5)$$

The movement of the footing is described by the settlement t_c at the center point and two rotations θ_x et θ_y, giving the displacement of any element as a linear function :

$$t_i = t_c + x_i \cdot \theta_x + y_i \cdot \theta_y \qquad (6)$$

The three conditions for the equilibrium are :

$$N = \iint \sigma.dx.dy = \Delta x.\Delta y.\Sigma\sigma_i \qquad (7a)$$

$$M_x = \iint x.\sigma.dx.dy = \Delta x.\Delta y.\Sigma(x_i.\sigma_i) \qquad (7b)$$

$$M_y \quad \iint y.\sigma.dx.dy = \Delta x.\Delta y.\Sigma(y_i.\sigma_i) \qquad (7c)$$

Δx and Δy being the dimensions of the elementary blocks measured in the footing plane (the cross area of the column).

Substituting the σ_i values given in (5) as a function of T_i, the relation becomes :

$$\sigma_i = \frac{\sigma_{i0}}{T_i}.(t_c + x_i.\theta_x + y_i.\theta_y) \qquad (8)$$

and the modified linear system is now :

$$N = S_{00}.t_c + S_{10}.\theta_x + S_{01}.\theta_y \qquad (9a)$$

$$M_x = S_{10}.t_c + S_{20}.\theta_x + S_{11}.\theta_y \qquad (9b)$$

$$M_y = S_{01}.t_c + S_{11}.\theta_x + S_{02}.\theta_y \qquad (9c)$$

with the short notation :

$$S_{jk} = \Delta x.\Delta y.\Sigma\left[\frac{\sigma_{i0}}{T_i}.x_i^j.y_i^k\right] \qquad (9d)$$

It is possible to include the elasticity of the bridge itself and the system remains linear and symmetrical.

The various coefficients are obtained from the results of the homogeneous elastic solution and from the settlements T_i of each "column" under the footing for any distribution of the E_M and α of the elementary blocks.

PROBABILISTIC ESTIMATION OF THE DEFORMATIONS

Point estimate method

The probabilistic method is based on a technique by "summation of independent disturbances of the moments", with a three points discretization (Bolle, 1988). This method "disturbs" the "mean" case, i.e. the case corresponding to the mean values of any random variable, by weighted point estimates of the statistical moments.

The values of the random variables to be used are two points located below and above the mean value, weighted by a coefficient. Values and coefficients are related to the statistical moments of the random variable and they are obtained through an original method based on the information theory (maximum entropy principle).

These disturbing points are taken for each independent random variable, with the other variables remaining then on the mean value.

In this particular case, with the various assumptions, the disturbing points are located at $\pm\,\sigma\,\sqrt{3}$ apart of the mean value, σ being the standard-deviation, and the weighting coefficient are equal to 1/6.

The probability density function of any function of the random variables, particularly the deformation of each block, is estimated by its first statistical moments. Contrary to the Rosenblueth point estimate method (Rosenblueth, 1975), the computed values of the function don't form a probability mass function, but the formulas are similar to the formulas derived from the Taylor series development.

The number of simulations is strictly equal to the number of elementary informations, in accordance to the information theory, resulting in a minimum computing effort.

Numerical results

The input random variables are the α_i and e_i values (to be transformed into E_M values) of each elementary block of the rock mass under the footing. The separate and cumulative effects of these two random variables are considered, and this is possible because all the variables are independent.

The results corresponding to the centered normal load (106 MN) are given as an example (Table I). Although the procedure is able to take into account the rigidity of the bridge itself, it was not considered in the actual analysis. Similar results were obtained under the extreme axial and transverse bending moments.

TABLE I

Deformations under centered loading

	Settlement t_c (m)	Rotation θ_L	Rotation θ_T
Mean value	0.0060	$1.06\ 10^{-5}$	$5.31\ 10^{-5}$
Std-dev.	0.0003	$4.46\ 10^{-5}$	$5.83\ 10^{-5}$
Skewness β_1	0.1201	0.0077	0.0279

As the computed skewness is generally very low, the probability density functions of the footing displacements are close to a normal distribution. The normal functions tables give easily the probabilities to exceed given bounds (Table II).

TABLE II

Limit deformations under permanent loads

Probability (%)	Settlement t_c (m)	Rotation θ_L	Rotation θ_T
95.0	$6.54\ 10^{-3}$	$0.84\ 10^{-4}$	$1.48\ 10^{-4}$
99.0	$6.76\ 10^{-3}$	$1.14\ 10^{-4}$	$1.88\ 10^{-4}$
99.9	$7.00\ 10^{-3}$	$1.48\ 10^{-4}$	$2.33\ 10^{-4}$

It can be observed that, following the spatial variations of E_M mean value, the footing's rotations are not null in average, even under a centered load. The estimated maximum values, as well for the rotation as for the settlement, change rather little with the chosen probability level. At 99 % level, a mean settlement of 7 mm is forecasted under permanent loads, and it doesn't exceed 8 mm under the other loading cases.

In a general manner, it appears that the results scattering obtained for the average settlement and for the rotations is mainly due to the modulus random variations, and that the effect of the rheologic coefficient α may in pratice be neglected.

CONCLUSIONS

On the footing settlement calculation

Results must be observed with a critical eye, due to the several hypotheses. Especially, a maximum uncertainty has been introduced when voluntary everything has been ignored about the geological structure. Pressuremeter moduli have been straightly considered even if they are measured in a different direction than the real loads; the rock mass anisotropy has thus been neglected and that in an unfavorable sense.

The approximations introduced about the rock mass behavior and about the calculus methods appear negligible compared to the effect of the three-dimensional spatial and random variations of the rock mass characteristics, variations which are impossible to take into account with a classical deterministic approach.

On the probabilistic approximation method

The proposed approximation technique seems to be completely satisfactory for problems with a large number of random variables, because it leads to a drastic reduction of the computing effort, compared e.g. to the Monte-Carlo simulation method. This reduction doesn't need crude approximations and doesn't require preliminary complicated analytical treatement of the studied functions.

The method allows to consider systematically the three-dimensional spatial variability of the geotechnical parameters, as it takes easily into account large numbers of random variables, even correlated or with skewed distribution functions, and behavior models which could be non-linear, or in the form of non explicit algorithms.

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